RAYLEIGH-TAYLOR INSTABILITIES IN INERTIAL-CONFINEMENT FUSION TARGETS

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ABSTRACT

This paper reports calculations of the growth of Rayleigh-Taylor instabilities (1) in the ablator-pusher region such as may be caused by irregularities in an electron beam, and (2) in the pusher fuel interface, a problem common to all inertial confinement fusion targets. For the first case, it is found that shallow density gradients and scattering of the beam by the target, both stabilize the shorter wavelength instabilities, which would otherwise grow the most rapidly. In the second case, it is found that moderately short wavelength instabilities may not degrade the target performance as much as has previously been supposed.
I. Introduction

Inertial confinement fusion targets are susceptible to hydrodynamic instability during implosion. Figure 1 shows a radius-time plot for various regions of a typical spherical target and identifies three topics of concern. The longest wavelength disturbances (low $\ell$ number, where $\ell$ is the index of the spherical harmonic) belong in the domain of target symmetry and have been described elsewhere.\textsuperscript{1,2} Perturbations near the ablator-pusher interface, region 2, are unstable during the period of acceleration of the target. These instabilities, generally assumed to be initiated by surface perturbations, have received considerable study.\textsuperscript{1-7}

We report here work done to study the stability of the ablator-pusher region to perturbations in the electron beam energy deposition due, for instance, to beam filamentation.\textsuperscript{9,10} These instabilities are of a somewhat different character from those due to surface perturbations because of the stabilizing effects of beam scattering.

As the compressed fuel decelerates the pusher, the interface between the fuel and the pusher, shown as region 3 in Fig. 1, becomes unstable to perturbations. These instabilities can arise even in targets which are stable in the ablator-pusher region, such as thin, uniformly-heated "exploding-pusher" targets. They are also a feature of most targets driven with ion or laser beams. We report initial results obtained from numerical computations of these instabilities. These results provide insight into the nature of the non-linear development of the perturbations, and estimates of the time at which instabilities might degrade the implosion.
II. Method of Computation

The growth of the instabilities has been computed by direct simulation using the two-dimensional hydrodynamics code CSQ.\textsuperscript{11} A similar approach was used by Lindl and Mead\textsuperscript{5} and, more recently, by Taggart,\textsuperscript{12} et al. CSQ has a number of features which make it attractive for these problems. First, it is an Eulerian code which can compute the non-linear spike development without the mesh tangling difficulties of Lagrangian methods. The code uses Lagrangian difference methods to advance the variables in time, rezoning to the Eulerian mesh at each time step. The Lagrangian fluid positions in each mesh are, however, stored at the time of rezoning to retain sub-grid resolution of interfaces. This information is also used in the code to avoid any numerical material mixing which plagues many two-material Eulerian treatements. CSQ includes energy flow with both thermal conduction and radiation diffusion, a complete materials physics package, and an electron beam deposition package.

The use of hydrocode simulation to study Rayleigh-Taylor problems was first described by Harlow and Welch\textsuperscript{13} and further developed by Daly.\textsuperscript{14} The ability of CSQ to compute the Rayleigh-Taylor instability was established by computing a test case close to the classical incompressible fluid problem. Figure 2 shows the configuration studied. A dense fluid (dotted area) is initially in equilibrium in a gravitational field. A high sound speed is used to approximate an incompressible fluid and a depth $d \gg \lambda$ is chosen to reduce finite boundary effects. The instability is initiated by a small cosinusoidal velocity perturbation. The linear growth of the spike is given by\textsuperscript{15}
where $\eta$ is the spike displacement from the equilibrium position, $\gamma$ is the growth rate, $\lambda$ is the wavelength, and $g$ is the gravitational acceleration.

Figure 3 shows the results of the calculations using a finite difference mesh with fifty zones per wavelength. Non-linear effects cause the spike to begin to narrow by $t = 3 \mu \text{sec}$. At $t = 6 \mu \text{sec}$, the tip of the spike is free-falling; the computed tip acceleration is within 1% of the correct value. Figure 4 plots the normalized spike displacement vs. time. The linear analytic result, Eq. (1) is shown as a solid line. The computed results exhibit a growth rate early in time within 2% of the linear growth rate, Eq. (2). It is seen that non-linear reduction of the growth occurs at a time somewhat after $k\eta = 1$, but well before $\eta = \lambda$. The ability to compute well past this time is one of the advantages of direct simulation over strictly linear methods. Additional test cases were run in which the wavelength and the number of meshes employed per wavelength were varied. Cases using four zones per wavelength gave slightly reduced growth rates and poorer spike resolution, although the increased computational speed provided was useful for parameter variations.
III. Ablator-Pusher Instability

Studies were made of the stability of the ablator-pusher region during target acceleration. In contrast to previous work which has concentrated on instabilities arising from surface perturbations, the present work examines the instabilities arising from perturbations of the beam current density. Such perturbations might be caused by beam filamentation.\textsuperscript{9,10}

There are a number of significant differences between instabilities initiated by surface perturbations and power perturbations. One of the most important differences arises from electron scattering in the target. Scattering has the effect of reducing the perturbation in the deposited power, leading to significantly reduced growth for wavelengths which are shorter than an electron scattering length. This effect has been included in these calculations by first performing a series of Monte Carlo electron transport deposition computations using the CYLTRAN\textsuperscript{16} code to relate the energy deposition perturbation amplitude to the beam current density perturbations. These computed amplitudes were then used in a parameter variation to study the growth of the various wavelengths.

A second difference between surface and power perturbations results from the fact that the source perturbation can remain applied for significant intervals of time. If the perturbation is applied for a significant fraction of the beam pulse, target deformations will, of course, grow even for perfectly stable cases. Although the non-linear phase of the beam filamentation instability may consist of wandering and coalescing filaments, we have elected to study the worst case in which a spatially stationary current density perturbation is applied for the duration of the beam pulse.

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A third feature of power perturbations arises from the beam deposition. The beam deposition provides a means for supersonically coupling stable and unstable regions of the target. Lateral motion of target material in the unstable region affects the propagation of the beam into stable regions deeper in the target. As a result, perturbations can grow in stable regions of the target due to perturbed beam transmission through unstable regions. However, this effect diminishes in roughly a scattering mean free path of the beam electrons.

Actual electron beam fusion targets are usually spherical high-Z shells filled with gaseous fuel. For reasons of economy we have studied a planar slab high-Z target with the same thickness as the spherical shell. This limits us to wavelengths and target translation short compared to the radius of the actual spherical target. This is not a serious limitation as the instability growth rates are greatest early in time before the target inner surface travels more than one initial target thickness. Figure 5 shows results for the growth of distortion at the slab surface opposite to the beam, corresponding to the inner surface of the sphere. These calculations were done for a 0.02 cm thick gold slab driven by \(4.4 \times 10^{15}\) W/cm\(^2\) of 1 MeV electrons, which is similar to previously computed\(^{17}\) breakeven-level, ablatively-driven, spherical targets. The beam source perturbation was considered to consist of cosinusoidal harmonics with amplitudes of 10% of the zero-order current density, and the numerical resolution was four zones per wavelength. These perturbations were multiplied by an appropriate reduction factor for each wavelength, determined by separate Monte Carlo calculations, to account for beam scattering effects. The amplitudes and the wave numbers \(k = d/\lambda\), plotted in Fig. 5 are normalized to the initial slab thickness \(d\).

The results show that the maximum growth occurs for the longest
wavelengths which can be studied with the slab approximation. Both scattering and finite density gradients tend to stabilize the shorter wavelengths. The inner surface of the slab had moved a distance equal to its initial thickness at about 3 ns. Perturbation amplitudes of the surface had reached about 10% of the initial slab thickness for the $\bar{k} = 0.625$ case by that time. Growth of short wavelengths to saturation, seen in computations for laser driven targets, was not seen for any of the cases studied.

The quantitative effects of the beam scattering can be seen in Figure 6. This graph plots the reduction factor $f$ vs. $\bar{k}$, the normalized wave number. Equal amplitude beam current density perturbations were assumed for each harmonic in the series of Monte Carlo calculations. The reduction factor is defined to be the ratio of the energy deposition perturbation amplitude to the beam perturbation amplitude. Even the longest wavelengths studied showed some reduction, while the amplitude of the perturbation for a wavelength of one-tenth the initial slab thickness was reduced to only a few percent of the beam perturbation.

IV. Pusher-Fuel Interface Instability

The simplest calculations of thermonuclear output from inertial confinement fusion targets neglect instability effects. More conservative calculations make use of the concept of a free-fall time. This idea is based on the assumption that the shortest wavelengths of the pusher-fuel interface instability grow up to non-linear saturation very rapidly and free-fall at the maximum pusher velocity. Thermonuclear output in the
calculations is assumed to cease when the free-falling material reaches the center of the sphere.

We have investigated the concept of the free-fall time and the non-linear behavior of the unstable pusher-fuel interface using the CSQ code. Once again, a planar slab model was used to simplify the computations. Such a model, consisting of two slabs compressing gaseous fuel, omits convergence effects, but enhances the separation between the free-fall and implosion times, thereby providing a test problem in which the inner surface instability problems are most severe.

In order to compute the effect of short wavelength disturbances on a spherically converging target, we would need to use a conical segment of a sphere a few wavelengths wide in order to do the calculation in a reasonable amount of time. This, however, requires a spherical coordinate grid, which is not presently available in CSQ. The problem of the pusher-fuel interface instability can be subdivided into two questions: (1) How fast do the instabilities grow—is the free fall time appropriate? (2) How do the instabilities affect the thermonuclear yield of the target. It would appear that the first question can be adequately investigated in planar geometry, while the second question requires spherical geometry to obtain the appropriate compression and temperature.

Recent 1-D calculations of Clauser have indicated that about $10^5$ neutrons could be obtained from a spherical iron target 1.5 mm in radius and 70 μm thick, assuming 0.2 TW of 750 keV electrons from the Hydra accelerator. We study here the analogous planar case of 70 μm iron slabs compressing deuterium fuel using $3.4 \times 10^{12}$ W/gm power deposition. Figure 7 shows the radius versus time for the pusher-fuel interface and
the average fuel temperature from a 1-D planar calculation. The time of
deceleration $t_d$ is at 60 ns, the free-fall time $t_f$ is about 64 ns, and the
implosion time $t_i$ is about 85 ns. One observes that the peak temperature
is achieved after the free-fall time, indicating that little thermonuclear
output would be expected by the free-fall time.

Another computational problem relates to the various time scales in the
problem. For the planar example the perturbations do not begin to grow
until about 60 ns when the slabs begin decelerating, at which time they
begin to grow on a subnanosecond time scale. It is thus both inefficient
and unnecessary to carry out full 2-D calculations until after $t_d$, the
time at which deceleration of the slabs begins. We have thus used the
1-D Lagrangian code CHARTD,\textsuperscript{19} with identical physics to CSQ, to compute the
behavior until deceleration time. At that time the CHARTD solution is
interpolated onto the CSQ mesh to begin the 2-D instability calculations.
The instabilities are initiated by using a pusher velocity perturbation,
typically about 0.05% of the zero order velocity.

Figures 8a and 8b show a sequence of frames from the 2-D CSQ calculations which
were initiated at the deceleration time. A 10 $\mu$m wavelength cosinusoidal
velocity perturbation was applied. The plane of symmetry between the two
slabs is at the bottom of each frame and the right and left boundaries are
reflecting. The frames show only the region of interest. Additional zones
are present above the upper boundary, but are not plotted. Sixteen radial
zones per wavelength and an axial zoning of $k_\alpha z \approx 0.5$ were used to resolve
the spikes. The higher density iron slab is observed to enter from the
top, compressing the lower density $D_2$ fuel. The calculations are
cylindrically symmetric about a vertical axis in the center of the frame.

The first frame is at 4 ns after \( t_d \), the deceleration time. At this time, the perturbation amplitude is about 0.1 \( \mu \text{m} \). By 6 ns after \( t_d \), \( k \eta \approx 1 \), where \( \eta \) is the surface displacement, so that non-linear effects are becoming important. As the perturbation grows, the spikes begin to sharpen and neck off behind the tip. Splashing of the iron at the symmetry plane is seen in the final frame. The problem was repeated using an initial surface perturbation of 0.1 \( \mu \text{m} \) and no velocity perturbation; very similar growth was observed.

Figure 9 again shows the pusher-fuel interface and fuel temperature versus time after the deceleration time \( t_d \). The dashed line is the free-fall line, and the triangles indicate the extent of the 10 \( \mu \text{m} \) wavelength spikes. One observes that the spikes take a finite time to grow and reach a free-fall condition at about 66 ns, or 6 ns past \( t_d \). This results in the spikes reaching the symmetry plane well past free-fall time.

Figure 10 plots the thermonuclear output versus time from the original 1-D spherically symmetric calculations of Clauser. It is seen that if thermonuclear output continues a bit past free-fall time, then most of the output should occur. Comparisons of the planar and spherical cases indicate that the deceleration and density gradients are similar (despite the much different convergence ratios) so that similar instability growth rates are expected. One thus concludes that instabilities on the order of 10 \( \mu \text{m} \) or above might not be a problem at least for the initial perturbation amplitude studied.
Preliminary studies at 1 μm wavelengths indicate a similar qualitative behavior. The spikes grow somewhat faster and reach the symmetry plane sooner, but still not as fast as the simple free-fall time argument. As one goes to shorter wavelengths, this trend should continue, although stabilization should occur at some sufficiently short wavelength. Viscous stabilization has been described by Chandrasekhar. If surface tension is absent, and the density ratio of the pusher to the fuel is large ($\rho_p/\rho_f \gtrsim 10$), the mode of maximum growth is given by

$$k = 0.49 \left( \frac{\epsilon}{\nu} \right)^{1/3},$$

where the kinematic viscosity $\nu = \mu/\rho$ is assumed equal for both fluids. For classical viscosity and the conditions of the target under investigation, Eq. (4) indicates maximum growth at wavelengths of $\lambda \approx 0.1$ μm.

Thermal conduction also appears not to be an important stabilizing mechanism except at comparably small wavelengths. The basic reason for this is that the pusher-fuel interface is unstable while the pusher inner surface is still relatively cold ($T \sim 100$ eV). Although 1-D calculations indicate pusher temperatures at implosion high enough to improve the thermal and viscous stabilization, this occurs too late in time to be important.

Another possible stabilization mechanism is Kelvin-Helmholtz growth at the surface of the spikes as they stream through the lower density fuel. This effect was reported by Daly. The growth rate is given by

$$\gamma = k \Delta \nu \sqrt{\frac{\sigma_p \sigma_f}{\rho_p \rho_f}},$$

where $\sigma_p = \rho_p/(\rho_p + \rho_f)$, $\sigma_f = \rho_f/(\rho_p + \rho_f)$, $\Delta \nu$ is the difference between the pusher spike and fuel velocities (typically about equal to the spike
velocity, \( v_p \), and \( k \) is the mode number, which is generally larger than the Rayleigh-Taylor mode number. For the case under study, \( \alpha_p \approx .9 \), \( \alpha_f \approx .1 \), and \( v_p \approx 4 \times 10^6 \) cm/sec. For \( \lambda = 2\pi/k = 1 \) \( \mu m \), the growth rate is \( \gamma \approx 7.5 \times 10^{10} \) sec\(^{-1} \). The 1-D spherical calculation indicates that at the deceleration time \( t_p \), the radius of the pusher-fuel interface is about \( 5 \times 10^{-2} \) cm, so that the spikes take at least 12 ns to reach the center. This is clearly enough time to expect significant Kelvin-Helmholtz growth, which could lead to mixing, density gradient formation, and slowing of the spikes.

V. Summary

The hydrodynamic stability of electron beam fusion targets during two phases of their implosion has been studied. We find, in agreement with previous work,\(^1\)\(^2\) that instabilities in the ablator-pusher region during acceleration of the target grow slowly. A 10% beam current density perturbation results in about a 10% perturbation in the shell thickness of a breakeven-level spherical gold shell target irradiated with 1 MeV electrons. Scattering of electrons in the target tends to wash out beam power perturbations.

Two-dimensional hydrodynamic calculations were presented for the pusher-fuel interface instability which results as the dense pusher begins to be decelerated by the compressed lower density fuel. The ability
to compute the growth of the spikes well past the non-linear limit was established. The results indicate that moderately short wavelengths, about one-tenth of the radius of the final fuel volume of spherical targets probably cause little problem. The time required for their growth results in free-streaming spikes which arrive at the origin late enough to permit significant thermonuclear output from the design.

The growth of shorter wavelengths remains a cause for concern. Neither viscosity nor thermal conduction, if assumed classical, appear to have any significant stabilizing effect except for very short wavelengths, \( \lambda \approx 0.1 \) μm. Kelvin-Helmholtz growth at the surface of the free-streaming spikes remains a possibility for mixing and stabilization.

The results for the two types of instability were calculated independently. One might expect that perturbations which grow in the ablator-pusher region during target acceleration would be further amplified during fuel compression. However, this particular coupling may not prove important, since the dominant wavelengths in the ablator-pusher region, for beam power perturbations, are much longer than the dominant wavelengths in the pusher-fuel interface instability.

Acknowledgements

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References


LIST OF FIGURES

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1. Irradiation Symmetry (low $\lambda$)

2. Ablator - Pusher R - T Instability (moderate $\lambda$)

3. Pusher - Fuel R - T Instability (high $\lambda$)

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**FIGURE 1**

**FIGURE 2**
FIGURE 3
FIGURE 4

- LINEAR ANALYTIC RESULT
- COMPUTED RESULTS

\[ e^{\gamma t} \]

\[ \tilde{\eta} = \tilde{\lambda} \]

\[ \gamma = 0.94 \]

\[ \gamma = 0.98 \]

\[ \tilde{t} = \gamma t \]
FIGURE 9

FIGURE 10

NEUTRONS