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ORIGINAL PAPERS

**Mattauch-Herzog Type Mass Spectrograph with
Two-Stage Electrostatic Field**

($\alpha\beta$ -Focusing for all Masses)

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An earlier work on a Mattauch-Herzog type mass spectrograph with two-stage electrostatic field is extended to investigating the conditions of simultaneous elimination of $\alpha\beta$ aberration for all masses. The design parameters are numerically calculated. The other coefficients of the second-order aberrations are examined and the total image defects are calculated for typical examples. An instrument according to the present study has sufficiently small and approximately constant image defects and has a high potential for use in the chemical analysis of solid.

1. Introduction

A Mattauch-Herzog type mass spectrograph is unique in that it is double-focusing for all masses along a straight line. Because of this remarkable character this type of instrument equipped with a spark ion source is being widely applied to the chemical analysis of solid material. One disadvantage, however, is such that there exist no available slit for controlling the velocity spread (β) independently of beam divergence (α). In earlier papers¹⁾²⁾, the author showed that this disadvantage was removed by substituting two-stage electrostatic field for the usual one without failing the first-order double-focusing condition for all masses, and that by a suitable choice of the deflecting fields the second-order angular aberration could be completely eliminated simultaneously for all masses.

This paper describes the conditions for simultaneous correction of $\alpha\beta$ aberration on the first-order double-focusing plane. The numerical calculation has been made by the MADIC-II A in the author's laboratory.

**2. Second-Order Aberrations and Conditions of Eliminating
 $\alpha\beta$ Aberration for all Masses**

An earlier paper reported in detail the first-order focusing conditions of a Mattauch-Herzog type mass spectrograph which consists of a two-stage electrostatic field followed by a magnetic field.

The present study is restricted to the two-stage electrostatic field consisting of a pair of cylindrical electrostatic fields in an equal radius (r_c) of center beam which deflects in the same direction. Since

ions of different masses are required to suffer the same deflection in a magnetic field, the magnetic field is necessary to be homogeneous and to satisfy the following geometrical relation:

$$\theta'' = \frac{\varphi_m}{2} - \frac{\pi}{2}, \quad (1)$$

where θ'' is the exit angle and φ_m is the deflection angle.

The schematic diagram of this optical system and the meaning of the symbols are shown in Fig. 1.

Fig. 1. Geometry and notations of Mattauch-Herzog type mass spectrograph with two-stage electrostatic field. S_1 : Source slit, S_2 : Energy slit, E, F: Electrostatic field, H: Magnetic field, F.P.: Focal plane.

For tracing the trajectory of ion beam, the matrix representation³⁾⁴⁾⁵⁾⁶⁾ is most suitable. The over-all transfer matrix $[T]$ of this system is expressed as the product of each matrix of component fields:

$$[T] = [l_4] [B] [d] [A] [l_1], \quad (2)$$

where $[A]$ and $[B]$ are the transfer matrices of the two-stage electrostatic field and the magnetic field, respectively, and $[l_1]$, $[d]$ and $[l_4]$ are the transfer matrices of field-free regions in a length of l_1 , d , and l_4 , respectively. The transfer matrix is expressed by nine-dimensional one in the second-order approximation.

An ion beam starting with α and β from a point source passes through a point on the plane vertical to the optical axis at the first-order double-focusing point. The point is defined by a lateral displacement (x) from optical axis as follows:

$$x = T_{16}\alpha^2 + T_{18}\alpha\beta + T_{19}\beta^2, \quad (3)$$

where T_{ij} 's are the matrix elements of the transfer matrix $[T]$.

These terms are called the second-order aberrations. As shown in the earlier paper³⁾, these coefficients are given by the following equations;

$$T_{16} = -\frac{r_e^2 r_m}{a_{21}^2 b_{21}} \left\{ \frac{D_4}{r_m^2} + (a_{24}a_{22}^2 - a_{25}a_{21}a_{22} + a_{26}a_{21}^2) \frac{1}{r_e^2} \right\}, \quad (4)$$

$$T_{18} = \frac{r_e r_m}{a_{21} b_{21}} \left\{ -\left| \frac{a_{21}a_{27}}{a_{22}a_{28}} \right| \frac{1}{r_e} + 2D_4 \left(a_{13} - \frac{d}{r_e} D_3 \right) \frac{r_e}{r_m^2} + (D_7 - D_3 D_5) \frac{1}{r_m} \right\}, \quad (5)$$

$$T_{19} = -\frac{r_m}{b_{21}} \left\{ a_{29} + D_4(a_{13} - \frac{d}{r_e} D_3)^2 \frac{r_e^2}{r_m^2} + (D_7 - D_3 D_5)(a_{13} - \frac{d}{r_e} D_3) \frac{r_e}{r_m} + D_3^2 D_6 - D_3 D_8 + D_9 \right\}. \quad (6)$$

The D_j is designated as follows:

$$D_j = \left| \frac{b_{11}b_{1j}}{b_{21}b_{2j}} \right|, \quad (7)$$

where a_{ij} and b_{ij} are the dimensionless factors of the matrix elements A_{ij} and B_{ij} , respectively.

In order to eliminate $\alpha\beta$ aberration for all masses, the coefficient of the second term of Eq. (3) is required to be zero identically:

$$T_{18} = 0. \quad (8)$$

This condition is fulfilled when the following three relations hold simultaneously:

$$C_1 \equiv D_7 - D_3 D_5 = 0, \quad (9)$$

$$\begin{vmatrix} a_{21}a_{27} \\ a_{22}a_{28} \end{vmatrix} = 0, \quad (10)$$

$$D_4(a_{13} - \frac{d}{r_e} D_3) = 0. \quad (11)$$

An ion trajectory in a homogeneous magnetic field with straight boundaries are determined by three parameters: Incident angle θ' , exit angle θ'' and deflection angle φ_m . In Mattauch configuration, the exit angle is determined by Eq.(1) as a function of φ_m . In the present case Eq.(9) is another requirement for these parameters. Then only one of these can be chosen arbitrarily. One may choose φ_m or θ' as an independent variable. D_3 , however is a function of these parameters, and is more suitable for mathematical analysis as already shown in the earlier paper²⁾.

Substituting for θ'' from Eq. (1), D_3 is expressed as follows:

$$D_3 = \frac{2 \sin \varphi_m / 2}{\cos \theta'} \cos (\varphi_m / 2 - \theta'). \quad (12)$$

The elimination of θ' from Eq. (9) by means of Eq. (12) gives

$$\frac{D_3}{4 \sin^4 \varphi_m / 2} \{ 4D_3 \sin \varphi_m - (3D_3^2 - 1) \cos \varphi_m - 2D_3^2 - 1 \} = 0. \quad (13)$$

To solve φ_m as a function of D_3 , the following expression is useful:

$$\cos \varphi_m = \frac{-(6D_3^4 + D_3^2 - 1) \pm 4D_3^2 \sqrt{5D_3^2 + 6}}{9D_3^4 + 10D_3^2 + 1}. \quad (14)$$

This solution, however, does not always satisfy Eq. (13). It is necessary to examine whether the solutions of Eq. (14) satisfy Eq. (13) or not. Fig. 2 shows the relations between φ_m and D_3 which satisfy Eq. (13). The two branches correspond to the double sign in Eq (13), respectively. The shaded area shows the region for negative l_4 . Therefore, the branches in the unshaded area are only available. The earlier study shows that D_3 should be positive provided the deflection angles φ_{e1} , φ_{e2} are limited within $[0, \pi/\sqrt{2}]$. When the direction of deflection in the magnetic field is inverted, that is, deflections in the magnetic field and the first electrostatic field are in the opposite directions, the sign of D_3 is changed. Then the branch below abscissa in Fig. 2 is also available. Fig. 3a and 3b show plots of θ' , θ'' , D_3 , D_4 , l_4/r_m in connection with φ_m ; the former is for deflections in the same direction and the latter for deflections in the opposite direction.

Fig. 2. Plots of D_3 vs. φ_m satisfying Eq. (9). The shaded area represents the region where $l_4 < 0$.

Fig. 3. Calculated curves for various constants of magnetic field satisfying $C_1 = 0$ as a function of φ_m : (a) The deflections in the magnetic field and the two-stage electrostatic field are in the same directions; (b) the deflections in the opposite directions.

Eq.(10) is the condition which determines the configuration of the two-stage electrostatic field. This equation contains D_3 as a parameter, since the electrostatic field is bound by the Mattauch-Herzog condition⁷⁾, i.e.

$$a_{23} + D_3 = 0. \quad (15)$$

Using the following abbreviations,

$$\begin{aligned} \sqrt{2} \varphi_{e1} &= \varphi_1, \\ \sqrt{2} \varphi_{e2} &= \varphi_2. \end{aligned} \quad (16)$$

Eq. (10) is expressed as follows:

$$\begin{aligned} & \frac{\sqrt{2}}{3} [3 \sin \varphi_2 \cos \varphi_2 - \sin \varphi_1 (8 + \cos^2 \varphi_2) \\ & + \frac{\sin^2 \varphi_2}{\sin \varphi_1} (8 + 3 \cos \varphi_1 + \cos^2 \varphi_2) \\ & - \frac{D_3}{\sqrt{2}} \{5 \cos \varphi_2 + 4 - \frac{\sin \varphi_2}{\sin \varphi_1} (11 + 13 \cos \varphi_1)\} \\ & + \frac{2D_3^2}{\sin \varphi_1} (2 \cos \varphi_1 + 1) = 0. \end{aligned} \quad (17)$$

Since this equation is very complex and difficult to be solved analytically, the values of φ_1 and φ_2 satisfying Eq. (17) are numerically calculated for various D_3 values and plotted in Fig. 4. All these

Fig. 4. Relation between φ_1 and φ_2 satisfying Eq. (10) with D_3 as the parameter.

combinations of φ_1 and φ_2 give the two-stage electrostatic fields which satisfy Eq. (10). Practical significance, however, requires positive l_1 , l_2 and l_3 , because the ion source and the intermediate image are not allowed to be imaginary. These are given as follows:

$$\frac{l_1}{r_e} = \frac{\frac{1}{\sqrt{2}} D_3 \cos \varphi_1 + \sin \varphi_2}{D_3 \sin \varphi_1}, \quad (18)$$

$$\frac{l_2}{r_e} = \frac{\frac{D_3}{2} + \frac{1}{\sqrt{2}} \cos \varphi_1 \sin \varphi_2}{\sin \varphi_1 \sin \varphi_2}, \quad (19)$$

$$\frac{l_3}{r_e} = \frac{1}{\sqrt{2}} \cot \varphi_2. \quad (20)$$

Fig. 5. Position of source with respect to the first electrostatic field as a function of φ_1 for various D_3 values.

Fig. 6. Position of intermediate image with respect to the first electrostatic field as a function of φ_1 for various D_3 values.

Fig. 5 shows the plots of l_1/r_e and Fig. 6 shows that l_2 becomes negative with increasing φ_1 and decreasing D_3 . The condition required for positive l_3 is easily proved to be $0 \leq \varphi_2 \leq \frac{\pi}{2}$.

The last condition (11) is fulfilled by choosing d as follows:

$$\frac{d}{r_e} = \frac{a_{13}}{D_3}, \quad (21)$$

where a_{13} is expressed by the following equation:

$$a_{13} = \frac{\sin \varphi_1}{\sin \varphi_2} + \frac{D_3 \cos \varphi_2}{\sqrt{2} \sin \varphi_2} + 1 \quad (22)$$

To be physically significant, d should be also positive. The result of numerical calculation are given in Fig. 7.

Fig. 7. Distance between the second electrostatic field and the magnetic field as a function of φ_1 for various D_3 values. The shaded area shows the region where $l_2 < 0$.

It is interesting to note that for a given D_3 , the effect of the magnet on the configuration of the two-stage electrostatic field is equal and independent of whether the deflections in the magnetic field and the two-stage electrostatic field are in the same direction or opposite.

3. Coefficients of α^2 aberration and β^2 aberration

The preceding description in the present paper deals with the conditions of eliminating $\alpha\beta$ aberration for all masses and with the configurations which satisfy these conditions. In order to find the most favorable configuration, it is necessary to evaluate the other coefficients of the second-order aberrations.

Putting Eq. (18) in Eq. (5),

$$T_{19} = -\frac{r_m}{b_{21}}(a_{29} + D_3^2 D_6 - D_3 D_8 + D_9). \quad (23)$$

The last three terms are expressed as follows:

$$D_3^2 D_6 - D_3 D_8 + D_9 = -D_3 \left\{ \frac{1}{2} + \left(\frac{D_3}{2 \sin \frac{\varphi_m}{2}} \right)^2 \right\}, \quad (24)$$

and a_{29} can be written in the form

$$\begin{aligned} a_{29} = \sqrt{2} \left[\sin(\varphi_1 + \varphi_2) + \frac{D_3}{\sqrt{2}} \left(\frac{7}{6} + \frac{4}{3} \cos \varphi_1 \right) \right. \\ \left. + \frac{1}{\sin \varphi_2} \left\{ \frac{3}{2} (\sin^2 \varphi_2 - \sin^2 \varphi_1) - \frac{D_3}{\sqrt{2}} \left(\frac{4}{3} + \frac{5}{3} \cos \varphi_2 \right) \sin \varphi_1 \right. \right. \\ \left. \left. - D_3^2 \left(\frac{1}{12} + \frac{2}{3} \cos \varphi_2 \right) \right\} \right]. \end{aligned} \quad (25)$$

Fig. 8. Plots of the relative aberration coefficient T_{19}/r_m vs. φ_1 with D_3 as the parameter: (a) The deflections in the magnetic field and the two-stage electrostatic field are in the same directions; (b) the deflections in the opposite directions. The shaded area shows the region where $l_2 < 0$.

In the present case the relative aberration coefficient T_{19}/r_m is independent of r_m and is plotted in Fig. 8a and 8b for deflections in the same and opposite directions, respectively.

The coefficient of a^2 aberration is written as follows:

$$\frac{T_{16}}{r_m} = \frac{1}{a_{21}^2 b_{21}} \left\{ \frac{D_4}{\tau^2} + (a_{24} a_{22}^2 - a_{25} a_{21} a_{22} + a_{26} a_{21}^2) \right\}, \quad (26)$$

where $\tau = r_m/r_e$. This equation converges for $\tau = \infty$. The limiting value $\bar{T}_{16}$ is given by the following equation

$$\frac{\bar{T}_{16}}{r_m} = \frac{a_{24}a_{22}^2 - a_{25}a_{21}a_{22} + a_{26}a_{21}^2}{a_{21}^2 b_{21}} \quad (27)$$

Fig. 9. Limiting values of the relative aberration coefficient T_{16}/r_m plotted as a function of φ_1 for various D_3 values: (a) The deflections in the magnetic field and the two-stage electrostatic field are in the same direction; (b) the deflections in the opposite directions. The shaded area shows the region where $l_2 < 0$.

and plotted in Fig. 9a and 9b. When D_4 and $a_{24}a_{22}^2 - a_{25}a_{21}a_{22} + a_{26}a_{21}^2$ have different signs, Eq. (26) may possibly be zero for particular r_m . This condition is

$$\frac{r_{m0}}{r_e} = \sqrt{\frac{-D_4}{a_{24}a_{22}^2 - a_{25}a_{21}a_{22} + a_{26}a_{21}^2}} \quad (28)$$

where r_{m0} denotes the particular r_m which satisfies this condition.

The real solutions are derived only in the case where the deflections in the magnetic field and the two-stage electrostatic field are in the opposite direction. The results are shown in Fig. 10. The deflection in the same direction makes the solution imaginary. However, if it is possible to use the magnets with curved boundary at entrance, Eq. (28) is written as follows:

$$\frac{r_{m0}}{r_e} = \sqrt{\frac{-D_4 - \frac{1}{2k \cos^3 \theta'}}{a_{24}a_{22}^2 - a_{25}a_{21}a_{22} + a_{26}a_{21}^2}} \quad (29)$$

where $k = R/r_{m0}$ and R is the radius of curvature of the field boundary at entrance. R is taken as

Fig. 10. Ion path radii in the magnetic field which correspond to the elimination of α^2 aberration at one mass as a function of φ_1 for various D_3 values.

positive for convex boundary and as negative for concave. Therefore, for a given r_{m0} one can find the appropriate curvature of field boundary which satisfy Eq. (29) without changing the field configuration, since the other D_j 's are not modified by using the magnet with curved boundary.

4. Discussion

The present paper have been carried out for clarifying the conditions for correction of $\alpha\beta$ aberration at all masses. A numerous variety of mass spectrograph satisfying these conditions are possible. These mass spectrographs are divided into two groups: The one is the case where the deflections in the magnetic field and the two-stage electrostatic field are in the same directions and the other is the case for deflections in opposite directions.

In the former case the area of the magnetic field is relatively small. The most interesting case is the vertical entry which is achieved by taking the deflection angle $64^\circ 14'$. The exit angle, however, is always large. In the latter case the deflection angle of the magnetic field becomes greater than 180° , and then the ion beams cross together at a point in the magnetic field. This fact is likely to result to the space-charge defocusing. The advantage of this type is that both the entrance angle and exit angle can be small. In this case, α^2 aberration at one mass can be corrected without using the magnet with curved boundary.

As for the β^2 aberration, T_{19} , however, is smaller generally in the former case. It should be

noted that the relative aberration coefficient T_{19}/r_m is constant. If the constant is zero, the correction of β^2 aberration for all masses is achieved. Unfortunately it is impossible to find such configuration under the practical condition. Fig. 8a shows that it decrease with a decrease in D_3 , and be small for $D_3=0.7$. The smaller aberration is expected with the smaller D_3 . However, the magnet with extremely small D_3 has not the practical geometry.

Since the total ion path is preferable to be as short as possible, the length of l_1 , l_2 , l_3 and d are required to be examined. They are functions of φ_1 which decrease with increase in φ_1 in general and are dependent on D_3 . Though l_2 become shorter with a decrease in D_3 , l_1 and d increase dominantly. Accordingly, the total ion path becomes longer as D_3 becomes smaller.

Typical design parameters are numerically computed and tabulated in Table 1 for several favour-

Table 1. Examples of Mass Spectrographs with corrected $\alpha\beta$ image defect for all masses.

No.	D_3	φ_m	θ'	θ''	φ_{e1}	φ_{e2}	l_1/r_e
1	1.5	$85^\circ 13'$	$28^\circ 46'$	$-47^\circ 23'$	$91^\circ 55'$	$62^\circ 27'$	0.277
2	0.9	$64^\circ 14'$	$-3'$	$-57^\circ 53'$	$91^\circ 55'$	$49^\circ 14'$	0.766
3	0.7	$53^\circ 44'$	$-14^\circ 35'$	$-63^\circ 8'$	$84^\circ 51'$	$47^\circ 8'$	1.106
4	1.2	$187^\circ 1'$	$-28^\circ 25'$	$3^\circ 30'$	$99^\circ 0'$	$59^\circ 34'$	0.447
5	1.0	$195^\circ 36'$	$-20^\circ 25'$	$7^\circ 48'$	$91^\circ 55'$	$50^\circ 58'$	0.649
No.	l_2/r_e	l_3/r_e	l_4/r_m	r_{m0}/r_e	d/r_e	T_{16}	T_{19}
1	0.386	0.021	0.384		1.198	0.939	-2.472
2	0.033	0.262	0.273		2.281	0.692	-1.524
3	0.032	0.305	0.223		3.081	0.662	-1.137
4	0.095	0.071	1.538	0.288	1.443	1.945	-3.464
5	0.093	0.229	1.695	0.314	2.034	2.388	-4.283

Fig. 11. Second-order aberration coefficients as a function of $\tau \left(= \frac{r_m}{r_e} \right)$ for the mass spectrograph No. 2 in Table 1.

— Takeshita No. 2 in Table 1

..... Mattauch ($d=r_e$)

able examples. In Fig. 11 the change in aberration coefficients along with the focal planes is illustrated as a function of τ for the mass spectrograph No. 2 in Table 1. The change in these coefficients for the usual Mattauch-Herzog type mass spectrograph are also shown for comparison. Typical configurations are shown in Fig. 12. The total image defects due to the second-order aberration are dif-

Fig. 12. Typical configurations of mass spectrographs with corrected $\alpha\beta$ image defects for all masses. S_1 : Source slit, S_2 : Energy slit, F.P.: Focal plane.

ferences between the maximum and minimum values of x given by Eq. (3) for any values of α and β of the collected beam. This type of instrument exhibit a minimum of the relative image width when $\alpha/\beta = \sqrt{T_{19}/T_{16}}$, under the constant luminosity. The minimum value is $2\sqrt{T_{16}T_{19}}$. In this case the image defects are symmetrical with respect to the central beam ($\alpha=0$, $\beta=0$). In Fig. 13 the total image defects divided by r_m are illustrated in connection with the typical examples in Table 1, and with a usual Mattauch-Herzog type mass spectrograph for comparison. This figure shows that the image defects of the present systems are considerably small and approximately constant for a wide mass range. Such image defects are preferable for use in the instrument for chemical analysis of solid.

Fig. 13. Calculated total image defects as a function of $\tau(\alpha_s=\beta_s=1)$.

- (a) Takeshita No. 2 in Table 1
- (b) Takeshita No. 4 in Table 1
- (c) Mattauch $d=d'=0$
- (d) Mattauch $d=d'=r_e$

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