Prediction of Microstructural Evolution during Grain Growth of a Pure Aluminum by means of Monte Carlo Simulation *

Masakazu Kobayashi¹, Yoshimasa Takayama² and Hajime Kato²

¹Satellite Venture Business Laboratory, Utsunomiya University, Utsunomiya 321-8585, Japan

²Department of Mechanical Systems Engineering, Utsunomiya University, Utsunomiya 321-8585, Japan

The modified Monte Carlo simulation of grain growth has been performed in the model applying actual orientation data by SEM/EBSP analysis in a pure aluminum to an initial microstructure. Changes in microstructural features and textures have been compared between experimental and simulated results. The SEM/EBSP analysis reveals that the pure aluminum has a residual rolling texture after annealing. The texture becomes sharper as the annealing time increases. The simulations are carried out under various conditions of grain-boundary energy and mobility. Microstructural and textural developments in the simulation taking the dependence of grain-boundary energy on misorientation into consideration were in good agreement with those in the experiment. The present model is expected to make it possible to predict the microstructural evolutions precisely.

(Received May 23, 2001; Accepted July 10, 2001)

Keywords: aluminum, grain growth, grain-boundary energy, grain-boundary mobility, Monte Carlo simulation, scanning electron microscopy/electron backscatter diffraction pattern analysis

1. Introduction

The scanning electron microscopy/electron backscatter diffraction pattern (SEM/EBSP) method is an effective technique of gathering information on crystallographic orientation.¹⁾ Not only quantitative but also the geographical distribution of orientation can be obtained by SEM/EBSP analysis. On the basis of an array of local crystallographic orientation data, the SEM/EBSP system also enables us to investigate grain boundary features, grain boundary character distribution, grain size distribution and phase distribution. It is noted that the data format of the SEM/EBSP system is similar to that of the Potts model in the Monte Carlo (MC) simulation.²⁾ The MC simulation has been applied successfully to reproduce grain growth,²⁻⁷⁾ static⁸⁾ or dynamic recrystallization⁹⁾ and so on.

Grain growth is one of the important phenomena in microstructural evolution. The driving force for the grain growth is a reduction in the energy of grain boundaries. However, the kinetics of the process in polycrystalline material has not been understood completely because the migration of grain boundaries depends on several factors influencing each other, and is accompanied with interference in connecting other boundaries.

In order to reproduce and predict grain growth precisely by using the simulation, we should take account of the several factors governing grain growth such as temperature, grain-boundary energy and mobility. We have already developed the simulations of grain growth having taken into consideration the misorientation dependence of grain-boundary energy,¹⁰⁾ the influence of temperature,¹¹⁾ and variations in mobility¹²⁾ using novel algorithms of the MC method. Next, a consideration of the influence of an initial microstructure is required for more precise reproduction and prediction. The

preferred orientation in the initial microstructure should be connected with the final microstructure and texture.

Recently, the simulation of abnormal grain growth based on the actual orientation data in Fe-3%Si was reported by Baudin *et al.*^{13,14)} They performed the simulation by applying EBSP orientation data to the initial microstructure by classifying the data into several orientation classes. In the present study, in order to predict microstructural evolution, the modified MC simulation of grain growth is performed using the Potts model. In the modified simulation, reorientation accompanied with activated state is employed to account for temperature dependence. Misorientation dependence of grain-boundary energy is taken into consideration while including coincidence site lattice (CSL) relations. Mobility described in relation to the effect of impurities on grain boundary is also considered in the simulation. It should be noted that the EBSP orientation data represented as Euler angles is directly applicable to an initial microstructure of the simulation. We compare changes in microstructural features and texture between experimental and simulated results. Moreover, how each factor plays an important role during grain growth is discussed based on the local balance of grain boundary energy.

To be able to predict microstructural evolution the development of a simulation model is early needed. If the prediction of microstructural evolution is performed by the simulation with the actual orientation data, the similarity of both data formats makes it possible to develop a microstructural image data base for material design, which include grain orientation, microstructural features and distribution and, moreover, temporal evolution.

2. Experimental Procedure and Simulation Method

2.1 Experimental procedure

The experimental sample used in this study was an annealed sheet of a 99.99% pure aluminum (A1N99) after rolling. The chemical compositions of the aluminum are listed in Table 1. The samples were prepared for

*A part of this paper will be presented at ReX & GG the First Joint International Conference on Recrystallization and Grain Growth, held in Aachen, Germany on August 27-31, 2001.

SEM/EBSP analysis as shown Fig. 1. Small specimens of $15 \times 10 \times 1.57 \text{ mm}^3$ were cut from this sheet, and were then isothermally annealed in a salt bath at 673 K for 60 s $\sim$ 57.6 ks. After annealing, the specimens were polished both mechanically and electrochemically in the RD-TD section. Orientation mapping data of the specimens were investigated using an SEM/EBSP analysis system, OIMTM (TexSEM Laboratories). The specimens were $2 \times 2 \text{ mm}^2$ in area and had a step size of $10 \mu\text{m}$. The number of measuring points totaled about 46000.

2.2 Simulation method

An MC simulation using the Potts model has already been performed in relation to reorientation accompanied with an activated state for grain boundary migration.¹¹⁾ The model in this study is composed of 2-dimensional triangular discrete lattices, which is the same arrangement of orientation mapping data obtained by OIMTM. A unit of discrete lattices is called a site, and orientation data is stored for each site. The orientation of sites is changed as follows.

A re-orientation trial of a site is carried out in two steps (Step 1 and Step 2). A site is selected at random, and then the first step (Step 1) is carried out. It is the activation step of the site that determines whether site is activated depending on the activation probability. The probability is represented with the following equation.

$$W_a \propto \exp(-E_a/\alpha^3 k_B T), \quad (1)$$

Table 1 Chemical compositions of the sample aluminum.

Material	Compositions, ppm									
A1N99	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Al (mass %)	
	33	28	17	2	Tr.	1	6	0.5	99.99

Fig. 1 Experimental procedure.

where W_a is the activation probability in the selected site, E_a is the activation energy of the site, α is a constant representing the ratio of site to atom, k_B is Boltzman's constant, and T is absolute temperature. Furthermore, the activation energy, E_a , is given as the difference between the potential energy of the activated state of the site, E_{act} , and the potential energy of the site at present, U . We regard the potential energy of a site as the energy of a state stabilized by segregation of impurities at the grain boundary.¹²⁾ The activation energy when affected by impurities is larger than that without impurities because the potential energy, U' , decreases as a result of the interaction energy, E_m , between the impurities and the grain boundary. Therefore, the activation energy E_a of the site is given by

$$E_a = E_{act} - (U - E_m) = 18J - \sum_{\langle xy \rangle} (j_{s_x s_y} - \mu_{s_x s_y}) / k_s, \quad (2)$$

where E_{act} is the potential energy of the activated state of the site $18J$, which is the maximum value calculated as grain boundary energy for the neighboring shells; $k_s \leq 3$ Model B¹⁰⁾ in this study. J is a unit of grain-boundary energy and k_s is the number of the neighboring shells. U is the sum of grain-boundary energy in each site $j_{s_x s_y}$ for the neighboring shells, and E_m is defined as sum of interaction factor, $\mu_{s_x s_y}$ in each site. The factor $\mu_{s_x s_y}$ is the magnitude of the interaction at a grain boundary between sites s_x and s_y , and represents mobility of grain boundary migration.¹²⁾ When eq. (2) is inserted into eq. (1), we obtain the following equation.

$$W_a \propto \exp \left[- \frac{18J - \sum_{\langle xy \rangle} (j_{s_x s_y} - \mu_{s_x s_y}) / k_s}{\alpha^3 k_B T} \right]. \quad (3)$$

The right side of the eq. (3) is arranged as follows:

$$W_a \propto \exp \left(- \frac{\sum_{\langle xy \rangle} \mu_{s_x s_y} / k_s}{\alpha^3 k_B T} \right) \exp \left(- \frac{18J - \sum_{\langle xy \rangle} j_{s_x s_y} / k_s}{\alpha^3 k_B T} \right), \quad (4)$$

where the term of temperature dependence of $\mu_{s_x s_y}$ is regarded as mobility M with an Arrhenius relation as follows,

$$M \propto \exp \left(- \frac{\sum_{\langle xy \rangle} \mu_{s_x s_y} / k_s}{\alpha^3 k_B T} \right) = \exp \left(- \frac{E_m}{\alpha^3 k_B T} \right). \quad (5)$$

The interaction factor $\mu_{s_x s_y}$ is given for $0 \leq \mu_{s_x s_y} \leq j_{s_x s_y}$ so as not to exceed grain-boundary energy. The mobility of grain boundary migration is reduced with increasing of the interaction factor $\mu_{s_x s_y}$ that is the decrement of energy arising from the presence of impurities. If the selected site is activated according to the probability (1), then it goes on to the next site. In practice, the equation (1) is normalized by the probability at melting point T_m as follows:

$$W_a = \frac{\exp(-E_a/\alpha^3 k_B T)}{\exp(-E_a/\alpha^3 k_B T_m)}. \quad (6)$$

Fig. 2 (a) Illustration of nearest neighboring sites: 1-6 and objective site: 7. (b) Schematic view of energies on orientation number in reorientation process of Step 2.

The second step (Step 2) is a selection of next orientation corresponding to the grain-boundary energy on the site. The orientation is changed with the following probability,

$$W_i = f_i / \sum_q f_q, \quad (7)$$

where q is a number of orientations and

$$f_i = \frac{\exp(-U_i/\alpha^3 k_B T)}{\exp(-U_i/\alpha^3 k_B T_m)}. \quad (8)$$

Here, there is a possibility that the orientation of the selected site can be changed into any orientation. However, in order to reduce calculating time, the next orientation is chosen only among the 7 orientations of the nearest neighboring 6 sites and the selected site itself, as shown schematically in Fig. 2. This is supported by the fact that these orientations occupy almost all the probabilities distribution of reorientation in the Step 2. If the energy of the site after changing orientation is equal to that of the activated state, the probability of changing into such orientation is zero. The simulation proceeds by repeating the above trial of Step 1 and Step 2. The unit of time is defined as 1 Monte Carlo step (MCS) per site, which corresponds to trials of the number of sites in the Potts model. The flow chart of simulating grain growth is shown in Fig. 3. In this study, the governing parameter of the simulation, $J/\alpha^3 k_B T_m$ was given as an appropriate value of 0.8 for a pure Al,⁽¹¹⁾ and the temperature of the simulations, $T = 0.72 T_m$ was selected to be consistent with the experimental one.

2.3 Simulation models

The simulations based on orientation mapping data have been performed to compare changes in microstructure feature and texture between actual and simulated results. The orientation mapping data was obtained in the pure aluminum sample annealed for 60 s. The following four types of simulations were carried out:

- 1) Basic model, which has a constant energy for all of the grain boundaries (Fig. 4(a)). This setting was often seen in early studies²⁾ on the simulation of grain growth.
- 2) LAGB/HAGB model, in which dependence of grain boundary energies on crystallographic misorientation is taken into account. The low and high angle grain boundaries (LAGB and HAGB) are defined by the misorientation setting a transition angle of 15° in this study. The energy of LAGB

Fig. 3 Flow chart for the simulation of grain growth.

Fig. 4 Illustration of the dependence of grain boundary energies on misorientation in each model simulated in this study: (a) basic model, (b) LAGB/HAGB model, (c) LAGB/HAGB & CSL model and (d) GBE + Mobility model.

is assumed to be proportional to the misorientation angle. On the other hand, the energy of HAGB is given as a constant value 1.0 J. (Fig. 4(b)). The misorientation refers to the minimum rotation angle around one crystallographic axis in common between two orientations of the grains.

- 3) LAGB/HAGB & CSL model, in which coincidence site lattice (CSL) boundaries are considered to extend the

Fig. 5 Inverse pole figure maps in the normal direction (IPF Map [ND]) in the specimen annealed at 673 K for 60 s and 57.6 ks.

Fig. 6 Pole figures in the specimen annealed at 673 K for 60 s and 57.6 ks.

Fig. 9 IPF map [ND] and pole figures simulated by the basic model at 1000MCS.

Fig. 11 IPF Map [ND] and pole figures simulated by the LAGB/HAGB model at 5000MCS.

Fig. 13 Comparison of grain boundaries simulated at 5000MCS by the model with and without consideration of CSL boundaries.

Fig. 15 IPF map [ND] and pole figures simulated at 5000MCS by the GBE + Mobility model, in which the interaction factor $\mu_{HAGB} = 0.5$ is set up for HAGBs with low mobility.

LAGB/HAGB model. In this model, we assume the V-shape energy cusps in misorientations corresponding to the CSL boundaries as shown in Fig. 4(c). The energies are supposed to increase gradually with an increase in the deviation angle from the exact CSL relation having the same gradient as the dependence in the LAGB. The CSL boundaries are generally defined by the Brandon criteria,¹⁵⁾ $\theta_{\min} \leq \pi/12 \times (\Sigma)^{-1/2}$, where Σ is the Σ -value of the coincidence relation. Consequently, the cusp of small Σ is deeper than that of large Σ owing to a large tolerance angle. The 67 coincidence relations from Σ 3 to Σ 21 about ten types of axes of $\langle 100 \rangle$, $\langle 110 \rangle$, $\langle 111 \rangle$, $\langle 210 \rangle$, $\langle 211 \rangle$, $\langle 221 \rangle$, $\langle 310 \rangle$, $\langle 311 \rangle$, $\langle 320 \rangle$ and $\langle 321 \rangle$ are used in the model.

4) GBE + Mobility model, in which grain-boundary mobility is considered, in addition to the LAGB/HAGB & CSL model. It is well known that the grain boundary migration rate strongly depends on impurities in the grain boundary.^{16,17)} It is also expected to decrease the grain-boundary energy on account of segregation of impurities at the boundaries. The mobility of grain boundary migration can be given as any value in the model on the basis of experimental results. Here, we give low mobility to the HAGBs because it is generally understood that impurities affect migration of HAGBs rather than LAGBs.¹⁸⁾ The interaction factor for HAGBs μ_{HAGB} is set up as 0.2 or 0.5 in contrast with $\mu_{\text{LAGB}} = 0$ for LAGBs. Further, the setting of grain boundary energies related to Step 2 is the same as in the 3) LAGB/HAGB & CSL model. The depth of energy cusps in CSL is unaffected by the mobility (Fig. 4(d)).

Because of applying the actual orientation data to the initial microstructure in this simulation, a special procedure is needed for calculation of the interaction energy. We use only the measured orientation data in the case of the border sites of the microstructure ignoring a lack of neighboring shells in order to calculate the interaction energy.

3. Results and Discussion

3.1 Experimental results

Isothermal grain growth in pure aluminum was investigated by the SEM/EBSP method. An initial mean grain size before annealing was 79.9 μm in the equivalent area diameter. Figure 5 shows inverse pole figure maps in the normal direction (IPF Map [ND]) in the specimens annealed at 673 K for 60 s and 57.6 ks, where fine lines show LAGBs with misorientation of 5–15°, bold lines HAGBs above 15° and red lines CSL boundaries of Σ 3– Σ 21. Grains grow with the increase in annealing time to reach a grain size of 175.4 μm for 57.6 ks as shown in Fig. 5(b). Figure 6 displays pole figures in the same microstructures as shown in Fig. 5. It is found that the aluminum sample has a residual rolling texture after annealing or recrystallization. It was reported that the recrystallization texture of rolled sheets is analogous to the rolling texture.^{19,20)} The texture becomes sharper than it was before annealing was carried out. Both of the grain area distributions exhibit near lognormal form during grain growth (Fig. 7). Normal grain growth is confirmed by the scaling rule²¹⁾ observed in grain area distributions during annealing. Further, distributions of grain misorientation angles between adjacent grains after excluding noise from the EBSP image data are shown for the specimens in Fig. 8. The figure in-

Fig. 7 Grain area distributions in the specimen annealed at 673 K for 60 s and 57.6 ks.

Fig. 8 Distributions of grain misorientation in the specimen annealed at 673 K for 60 s and 57.6 ks.

icates that the distribution of grain misorientation has two peaks in the ranges of 5–10° and 50–60°, and moreover, remain unchanged during grain growth.

3.2 Simulation results

The first simulation is carried out by a basic model, which has often been employed in early MC simulation studies. Figure 9 shows (a) IPF Map [ND] and (b) Pole Figures at 1000 MCS simulated using the basic model. The grain boundaries are indicated in the same way as they are in Fig. 5; fine, bold and red lines are LAGBs, HAGBs and CSL boundaries, respectively. Here, we should keep it in mind that these boundaries are indistinguishable from one another in the model. The evolved grains are equiaxed and have the same size as shown in Fig. 9(a). Long slender grains are observed near on the border as the boundaries, which are normal to the border are stable in terms of boundary energy. Every grain is painted with a single color or orientation in the figure. It was also found that the grain area distribution was lognormal and narrow in the evolved microstructure. As seen in Fig. 9(b), the texture develops at 1000MCS with formation of peaks of the different orientations from the recrystallization texture. Such randomization of texture is associated with a theoretical distribution of the grain misorientation with a single peak at about 50°, which has already been acquired by the calculation for random texture.²²⁾ The grain misorientation distribution becomes close to the theoretical one for the random texture as the microstructure develops (Fig. 10). It is, therefore, obvious that the texture is randomized during grain growth in this model.

Actual grain-boundary energy depends on crystallographic misorientation.²³⁾ Figure 11 shows the microstructure evolved

Fig. 10 Distributions of grain misorientation in the basic model.

Fig. 12 Distributions of grain misorientation in the LAGB/HAGB model.

in the LAGB/HAGB model. Highly curved and broken-off boundaries are observed in the microstructure as shown in Fig. 11(a). Some grains have several regions painted with separate similar colors or orientations. These grains are regarded as consisting of subgrains. The simulated microstructure is analogous to the experimental one. The grain area distributions indicated a relatively wide range of grain sizes, as was observed in the experiment. It is also found that the texture is sharpened and this corresponds to the experiment as shown in Fig. 11(b). It was evident that the sharp texture is formed owing to the dependence of grain boundary energies on misorientation. The distributions of grain misorientation possess two peaks in the low angle range from 5 to 10° and in the high angle range from 50 to 60° as shown in Fig. 12. They are confirmed to be fairly similar to the experimental results (Fig. 8). There is high frequency in the range from 5 to 10° compared with the experimental results. The increase in the frequency at low misorientation angles is owing to the stable low energies of the low angle boundaries.

Next, we carried out the simulation in the LAGB/HAGB & CSL model. The CSL boundaries are well known to have special properties caused by their good coherency. It is thought that the boundary energy-misorientation relation has energy cusps corresponding to some CSL boundaries. Therefore, it is logical to postulate that grain growth is influenced by the CSL boundaries with the lower energy. Figure 13 is a comparison between microstructures with and without the CSL boundary effect. A slight difference is observed between (a) LAGB/HAGB & CSL and (b) LAGB/HAGB model. The CSL boundaries in the former look to be relatively longer than those in the latter because of the stability of the CSL boundaries, which have low energies.¹²⁾ The frequency of CSL boundaries in the microstructures has been investigated more quantitatively in the experimental and simulated results as displayed in Fig. 14. In the experimental results of (a) and (a'), length fractions of the $\Sigma 3$ boundary and the other CSL boundaries are about 5% and 1%, respectively. The experimental frequency is almost unchanged with an increase in the annealing time. As concerns the simulation results, the frequency of the CSL boundaries of (b) and (b'), which take the CSL boundary effect into account, is larger than that of (c) and (c') without the CSL. The frequencies in the LAGB/HAGB & CSL model are in relatively good agreement with the experimental result. The CSL boundaries in the simulations decrease with increasing time. Consequently, it is confirmed that the fraction of CSL boundaries improved by

Fig. 14 Frequency of CSL boundaries: (a) and (a') are experimental results, (b) and (b') are simulation results with consideration of CSL boundaries, (c) and (c') are simulation results without consideration of CSL boundaries.

the LAGB/HAGB & CSL model is almost equivalent to that of the experiment.

In the GBE + Mobility model, slow microstructural evolution is observed owing to low mobility set for HAGB. IPF map [ND] and pole figures simulated by the GBE + Mobility model, in which the interaction factors $\mu_{\text{HAGB}} = 0.5$ for HAGBs as low mobility and $\mu_{\text{LAGB}} = 0$ for LAGBs are set up, are shown in Fig. 15. The simulation tendency in the microstructural evolution can be seen in Fig. 15(a) with the LAGB/HAGB model or the LAGB/HAGB & CSL model although the growth rate is slower. The simulated mi-

Table 2 Grain growth exponents, n in the experiment and the simulations.

Experimental at 673 K	0.119
Basic model (a constant GB energy)	0.327
LAGB/HAGB model	0.320
LAGB/HAGB & CSL model	0.362
GBE + Mobility with $\mu_{\text{HAGB}} = 0.2$ model	0.287
GBE + Mobility with $\mu_{\text{HAGB}} = 0.5$ model	0.128

Fig. 16 Correspondence of the simulation time to the real time based on mean grain size.

crostructure is also analogous to the experimental one owing to the finding of highly curved and broken-off boundaries, subgrain structures and so on. Moreover, the textural evolution is confirmed in Fig. 15(b) as well as in Fig. 11(b). There are some differences in microstructural details among the LAGB/HAGB, the LAGB/HAGB & CSL and the GBE + Mobility models, except for the textural evolution.

Grain growth exponents, n in the experiment and the simulations performed in the present study are listed in Table 2. The experimental exponent of 0.119 is comparatively small because of already large grain size before annealing and its correspondence to the latter term of grain growth. The exponents of the three simulations without consideration of mobility are more than 0.3. Consequently then, the exponent in the GBE + Mobility model provided with low mobility in HAGBs is reduced from 0.287 to 0.128 with an increase in the interaction factor μ_{HAGB} from 0.2 to 0.5, respectively. The grain growth exponent n of 0.128 in the GBE + Mobility model with $\mu_{\text{HAGB}} = 0.5$ is in the best agreement with the experimental value.

Here, Fig. 16 shows the correspondence of the simulation time to the real time for the experimental results and the best simulation for the grain growth exponent. It is easily derived from the mean grain size, and this is one way of disclosing the correspondence of the real and simulation times. This is important for the prediction models. Upper and lower axes in the figure indicate simulation and real times for the logarithm. The mean grain sizes of the experiment shown by the open circles increase linearly with some scatters on log-log plots, and the grain sizes of the simulation results shown by the solid

markers increase to represent a somewhat accelerating curve. As a result of fitting the experiment, the GBE + Mobility model where $\mu_{\text{HAGB}} = 0.5$, a simulation time of 1MCS can be estimated to be about 0.716 s of annealing time. However, further study is needed for the correspondence of the simulation time to the real time on the basis of the relationship between the frequency of atomic vibration and the grain boundary migration rate, and the correlation of cross sections of 3-dimensional microstructure and 2-dimensional simulation. It is found that the simulated result is in good agreement with the experimental results as shown in Fig. 16, though it is a simple trial. It is, therefore, expected to be able to predict the microstructural evolutions based on the conversion of simulation time to real time.

4. Conclusions

In the present study, the Potts model MC simulation has been performed applying the actual orientation data for a pure aluminum. The SEM/EBSP analysis reveals that the sample of pure aluminum has a residual rolling texture after annealing. The simulation based on the actual orientation data has successfully reproduced microstructural features and textural evolution while considering the dependence of grain-boundary energy on a misorientation. It is, therefore, confirmed that the misorientation dependence of grain-boundary energy plays the most important role in the evolution of the actual microstructure and texture. The long CSL boundaries were observed in the simulated microstructure for the LAGB/HAGB & CSL model. The microstructural evolution was retarded in the GBE + Mobility model as expected. The textural evolution of the model was independent of the mobility with low mobility for the present HAGB simulation models. The best fit in the grain growth exponent with the experimental value has been achieved in the GBE + Mobility model. Conversion of the simulation time to real time was attempted and 1MCS was estimated to be about 0.716 s.

Consequently, it has been found that the MC simulation of grain growth by the Potts model is applicable to the actual orientation data obtained by SEM/EBSP analysis. The Potts model simulation is also able to reproduce the microstructural evolutions such as grain boundary features, grain size distributions, and textural evolution. Accurate and practical prediction of the microstructural evolutions is expected through determination of more appropriate values for each parameter.

REFERENCES

- 1) B. L. Adams, S. I. Wright and K. Kunze: *Metall. Trans. A* **24** (1993) 819–831.
- 2) M. P. Anderson, D. J. Srolovitz, G. S. Grest and P. S. Sahni: *Acta Metall.* **32** (1984) 783–791.
- 3) D. J. Srolovitz, M. P. Anderson, P. S. Sahni and G. S. Grest: *Acta Metall.* **32** (1984) 793–802.
- 4) M. P. Anderson, G. S. Grest and D. J. Srolovitz: *Philos. Mag.* **59** (1989) 293–329.
- 5) T. Shibayanagi, H. Takatani and S. Hori: *J. Japan Inst. Metals* **54** (1990) 650–656.
- 6) Y. Saito and M. Enomoto: *ISIJ Int.* **32** (1992) 267–274.
- 7) J. Gao and R. G. Thompson: *Acta Mater.* **44** (1996) 4565–4570.
- 8) D. J. Srolovitz, G. S. Grest and M. P. Anderson: *Acta Mater.* **34** (1986) 833–845.
- 9) P. Peczak: *Acta Metall. Mater.* **43** (1995) 1279–1291.

- 10) M. Kobayashi, Y. Takayama, T. Tozawa and H. Kato: J. Japan Inst. Metals **62** (1998) 834–843.
- 11) M. Kobayashi, Y. Takayama, H. Kato, H. Nakajima: J. Japan Inst. Metals **65** (2001) 85–92.
- 12) M. Kobayashi, Y. Takayama and H. Kato: J. Japan Inst. Metals **65** (2001) in press.
- 13) T. Baudin, P. Paillard and R. Penelle: Scripta Materialia **36** (1997) 789–794.
- 14) T. Baudin and R. Penelle: ICOTOM-12, Proceeding of the twelfth International Conference on textures of Materials, JA Szpunar, Ed., (NRC Research Press 1999) pp. 1967–1701.
- 15) D. G. Brandon: Acta Metall. **14** (1966) 1479–1484.
- 16) K. T. Aust and J. W. Rutter: Trans. AIME **215** (1959) 119–127.
- 17) K. T. Aust and J. W. Rutter: Trans. AIME **215** (1959) 820–831.
- 18) H. Sautter, H. Gletter and G. Bälo: Acta Metall. **25** (1977) 467–473.
- 19) K. Ito: J. Japan Inst. Light Metals **43** (1993) 285–293.
- 20) S. Nagashima: *Texture*, (Maruzen, Tokyo, 1994) pp. 133–140.
- 21) H. V. Atkinson : Acta Metall. **36** (1988), 469–491.
- 22) F. J. Humphreys and M. Hatherly: Recrystallization and Related Annealing Phenomena, (Pergamon, 1995) p. 59.
- 23) G. C. Hasson and C. Goux: Scr. Metall. **5** (1971) 889–849.