Generation of Compression Waves by Simultaneously Imposing a Static Magnetic Field and an Alternating Current and Its Use for Refinement of Solidified Structure

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A new generating method of compression waves in a liquid metal has been proposed in which a static magnetic field and an alternating current are simultaneously imposed. The theoretical expressions of intensities and distributions of pressure and velocity accompanied with the compression waves have been derived. The pressure change in liquid gallium excited by the method proposed here was measured under different intensities of the magnetic field and the alternating current. The measured pressures approximately agreed with the theoretical evaluation. The structure of a Sn-Pb alloy that was solidified under the imposition of the compression waves, was completely refined.

(Received August 31, 2000; Accepted December 18, 2000)

Keywords: compression waves, magneto-acoustic waves, crystal refinement, solidification, electromagnetic processing of materials, static magnetic field, steel-making

1. Introduction

Compression waves are an attractive tool for use in materials processes because they have a lot of useful functions such as the refinement of solidified structure,¹⁻⁴⁾ the promotion of reaction rates^{1-3,5)} and so on. However, when compression waves are introduced into industrial scale processes, the conventional approach using an electrostrictive or magnetstrictive vibrator presents several problems such as the destruction of the transmitter due to the high temperature environment⁶⁾ or the power limitation of the vibrator.⁷⁾ Therefore, direct generation of compression waves in a liquid metal for the refinement of solidified metal structures with a high temperature melting point, such as aluminum or iron, is highly desirable. Amano *et al.*⁸⁾ directly excited compression waves in a liquid metal by applying an alternating magnetic field though the intensity of the compression waves was weak. To obtain intense compression waves, another method⁹⁾ has been proposed in which a static and an alternating magnetic field were simultaneously imposed on a liquid metal and a pressure change of more than one atmosphere was achieved. Thus, the utilization of electromagnetic fields has developed into a promising way of introducing compression waves into materials processing.

On the other hand, refinement of solidified structures can be achieved by applying the vibration excited by either mechanical or electromagnetic methods. One example is the refinement of aluminum alloy structures by the simultaneous imposition of a static magnetic field and an alternating current on the whole of a sample during solidification.¹⁰⁻¹²⁾ However, these methods are not suitable for industrial applications because the vibration has to be imposed on the whole product, especially for large size products, and an intense electric current density and/or a high magnetic field are also required for the vibration to be sufficient enough to refine solidified structures.¹³⁾ In contrast, compression waves can propagate vibration to the whole solidifying area from a local generat-

ing source of vibration. Therefore, direct excitation of compression waves in a metal by using electromagnetic fields for the refinement of solidified structures on an industrial scale is anticipated.

In this paper, a new method of generating compression waves is proposed, in which a static magnetic field and an alternating current are simultaneously imposed on a liquid metal. Theoretical expressions for the velocity and pressure accompanying the compression waves have been derived and confirmed in experimental works. Also the effect of compression waves excited by this method on the refinement of solidified structures has been studied.

2. Theoretical Analysis

When a sinusoidal force acts on a fluid with density ρ_0 , magnetic permeability μ and electrical conductivity σ , the density and the pressure vary depending on the compressibility of the fluid. If the force has only a x-component that is uniform in the x-plane, the variable quantities of density ρ' , pressure p' and the velocity of a fluid element v_x are related to the equilibrium quantities ρ_0 , p_0 and $v_0 (=0)$ and the perturbed quantities $\rho(x, t)$, $p(x, t)$ and $v_x(x, t)$.

$$\rho' = \rho_0 + \rho(x, t) \quad (1)$$

$$p' = p_0 + p(x, t) \quad (2)$$

$$v_x = v_0 + v_x(x, t) = 0 + v_x(x, t) \quad (3)$$

The continuity equation and Euler's equation for a one-dimensional compressible fluid are written as:

$$\frac{\partial \rho'}{\partial t} + \frac{\partial (\rho' v_x)}{\partial x} = 0 \quad (4)$$

$$\rho' \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} \right) = -\frac{\partial p'}{\partial x} + F_x \quad (5)$$

where F_x is the force acting on the fluid. For small amplitude perturbations, the wave equations for the velocity and the pressure are derived from eqs. (1)–(5).

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$$\frac{\partial^2 v_x}{\partial t^2} - c^2 \frac{\partial^2 v_x}{\partial x^2} = \frac{1}{\rho_0} \frac{\partial F_x}{\partial t} \quad (6)$$

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial x^2} = -c^2 \frac{\partial F_x}{\partial x} \quad (7)$$

where $c \equiv (\sqrt{\partial p / \partial \rho})_{\text{isotropic}}$ is the propagation velocity of the compression waves.

The right hand sides of eqs. (6) and (7) are the generation terms of the compression waves by the force F_x , which is a function of time t and position x . When a static magnetic flux density of $B_{DC} \mathbf{i}_z$ is applied to a fluid of length l ($0 \leq x \leq l$) and an alternating electric current $\sqrt{2} J_{AC} \cos(\omega t) \mathbf{i}_y$ is supplied to one side of the fluid as shown in Fig. 1, the electromagnetic force F_{AC} generated under the condition of $\delta \ll l$, is as in eq. (8),

$$F_{AC} = \sqrt{2} J_{AC} B_{DC} \exp\left(-\frac{x}{\delta}\right) \cos\left(-\frac{x}{\delta} + \omega t\right) \quad (8)$$

where $\delta (\equiv \sqrt{2/\sigma \mu \omega})$ is the electromagnetic skin layer.

In the derivation of eq. (8), it was assumed that the intensity of the static magnetic field was much larger than that of the alternating magnetic field induced by the electric current. The force F_{AC} directly excites compression waves in the fluid and the fluid elements on which this force acts play the roles of a vibrators. The fluid motion perpendicular to the imposed magnetic flux density $B_{DC} \mathbf{i}_z$ induces an eddy current $J_m \mathbf{i}_y$ that is accompanied by perturbations of the electric field $E_m \mathbf{i}_y$ and the magnetic field $b_z \mathbf{i}_z$. Assuming that the fluid is perfectly conducting, these variables are related to each other by Ohm's law, Faraday's law and Ampere's law, as follows.

$$E_m - v_x B_{DC} = \frac{J_m}{\sigma} = 0 \quad (9)$$

$$\frac{\partial b_z}{\partial t} = -\frac{\partial E_m}{\partial x} = -B_{DC} \frac{\partial v_x}{\partial x} \quad (10)$$

$$J_m = -\frac{1}{\mu} \frac{\partial b_z}{\partial x} \quad (11)$$

Then, an electromagnetic force F_m in the x -direction, which is excited by the interaction between the induced current and the imposed magnetic field, is given by eq. (12) under the assumption of small perturbation ($|B_{DC}| \gg |b_z|$).

$$F_m \mathbf{i}_x = J_m \mathbf{i}_y \times (B_{DC} + b_z) \mathbf{i}_z = -\frac{B_{DC}}{\mu} \frac{\partial b_z}{\partial x} \mathbf{i}_x \quad (12)$$

This force acts on the whole of the fluid while the electromagnetic force $F_{AC} \mathbf{i}_x$ acts only on the fluid elements in the electromagnetic skin layer. The propagation velocity a of the compression waves deviates from the ordinary sound velocity

Fig. 1 Analytical system.

c when the force F_m acts on a fluid. The compression waves propagating perpendicularly to the magnetic field are called magneto-acoustic waves. When these forces act on a fluid, the wave equations for velocity and pressure are as follows:

$$\begin{aligned} \frac{\partial^2 v_x}{\partial t^2} - a^2 \frac{\partial^2 v_x}{\partial x^2} &= \frac{1}{\rho_0} \frac{\partial F_{AC}}{\partial t} \\ &= -\frac{\sqrt{2} J_{AC} B_{DC} \omega}{\rho_0} \\ &\quad \times \exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t\right) \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} - a^2 \frac{\partial^2 p}{\partial x^2} &= -c^2 \frac{\partial F_{AC}}{\partial x} \\ &= -2c^2 J_{AC} B_{DC} \frac{1}{\delta} \\ &\quad \times \exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t - \frac{\pi}{4}\right) \end{aligned} \quad (14)$$

where $a^2 = c^2 + \frac{B_{DC}^2}{\rho_0 \mu}$.

When uniform plane waves propagating in a medium 1 are incident perpendicularly on a medium 2 and the specific acoustic resistance, $\rho_0 c$ of medium 1 is much larger than that of medium 2, the interface becomes a node for the pressure and a loop for the motion of fluids. This condition pertains when a liquid metal is surrounded with gases because the specific acoustic resistance of a liquid metal ($\sim 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$) is much larger than that of a gas ($\sim 10^2 \text{ kg m}^{-2} \text{ s}^{-1}$). Furthermore, this is valid even when the thickness of the wall between the metal and the gas is much less than the wave-length of the compression waves. Then the following boundary conditions can be adopted:

$$\left. \frac{\partial v_x}{\partial x} \right|_{x=0} = 0 \quad (15)$$

$$p|_{x=0} = 0 \quad (16)$$

$$\left. \frac{\partial v_x}{\partial x} \right|_{x=l} = 0 \quad (17)$$

$$p|_{x=l} = 0 \quad (18)$$

The distributions of the velocity and the pressure in the fluid under a periodical steady state condition are derived as:

$$\begin{aligned} v_x &= V_m \frac{N \gamma}{\sqrt{1 + N^2 \gamma^4}} \\ &\quad \times \left[\frac{1}{\gamma \sqrt{N}} \exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t + \frac{\pi}{4} - \phi\right) \right. \\ &\quad \left. + \frac{\cos(kx - kl)}{\sin(kl)} \cos(\omega t - \phi) \right] \end{aligned} \quad (19)$$

$$\begin{aligned} p &= P_m \frac{N}{\sqrt{1 + N^2 \gamma^4}} \\ &\quad \times \left[\exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t - \phi\right) \right. \\ &\quad \left. + \frac{\sin(kx - kl)}{\sin(kl)} \sin(\omega t - \phi) \right] \end{aligned} \quad (20)$$

where $V_m \equiv \frac{\delta B_{DC} J_{AC}}{\rho_0 c}$, $\gamma \equiv \frac{a}{c}$, $N \equiv \frac{2c^2}{\delta^2 \omega^2}$, $\phi \equiv \tan^{-1}(N \gamma^2) + \frac{\pi}{4}$, $k \equiv \frac{\omega}{a}$ and $P_m \equiv \delta B_{DC} J_{AC}$.

Fig. 2 Predicted pressure and velocity distributions in liquid gallium.

On the other hand, when an alternating current is imposed on both sides of a fluid of length $2l$, because of the geometrical symmetry, the boundary conditions at the center of the fluid are as follows:

$$v_x|_{x=l} = 0 \quad (21)$$

$$\left. \frac{\partial p}{\partial x} \right|_{x=l} = 0 \quad (22)$$

The distributions of the velocity and the pressure in the half region ($0 \leq x \leq l$) are derived and shown in eqs. (23) and (24) under the boundary conditions of eqs. (15), (16), (21) and (22).

$$v_x = V_m \frac{N\gamma}{\sqrt{1 + N^2\gamma^4}} \times \left[\frac{1}{\gamma\sqrt{N}} \exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t + \frac{\pi}{4} - \phi\right) + \frac{\sin(kx - kl)}{\cos(kl)} \cos(\omega t - \phi) \right] \quad (23)$$

$$p = P_m \frac{N}{\sqrt{1 + N^2\gamma^4}} \times \left[\exp\left(-\frac{x}{\delta}\right) \sin\left(-\frac{x}{\delta} + \omega t - \phi\right) - \frac{\cos(kx - kl)}{\cos(kl)} \sin(\omega t - \phi) \right] \quad (24)$$

In both cases mentioned above, the frequency of the compression waves is the same as that of the imposed current while its intensity is proportional to the product of the imposed magnetic field and the alternating current in a periodical steady state condition. The compression waves form standing waves with wave number k , except in the electromagnetic skin layer on which the electromagnetic force F_{AC} acts. The distributions of the effective values of the pressure and the velocity in liquid gallium with a length of 40 mm are calculated from eqs. (19) and (20) under the conditions of $B_{DC} = 10\text{ T}$ and $J_{AC} = 5.5 \times 10^5\text{ A/m}^2$ and shown in Fig. 2. The density, electrical conductivity and propagation velocity of ordinary compression waves used in the calculation are 6090 kg/m^3 , $3.8 \times 10^6\text{ S/m}$, and 2740 m/s , respectively. Since the vessel length of 40 mm is much smaller than the wave-length of 1.4 m, the sinusoidal distributions of the pressure and velocity are not clearly observed under this condition.

3. Detection of Pressure Change

A schematic view of the experimental apparatus is shown in Fig. 3. A rectangular glass vessel with a width of 25 mm and a length of 40 mm was filled with liquid gallium and set in the bore of a super-conducting magnet. A static magnetic field was applied to the metal in the vertical direction. An alternating current with a frequency of 2 kHz was imposed on the liquid metal through a pair of 2 mm thick, 5 mm wide copper electrodes placed near the short wall of the vessel. Pressure changes were detected by a pressure sensor set at the opposite side of the electrodes as shown in Fig. 3. The distance between the electrodes and the sensor, 17 mm, was about three times the depth of the electromagnetic skin layer of 5.8 mm for liquid gallium at a frequency of 2 kHz, so that the signal detected by the sensor was not the vibrating force caused by the interaction between the static magnetic field and the alternating current, but the compression waves that were excited in the electromagnetic skin layer around the electrodes by this force and propagated to the sensor. The effective values of the detected pressure are shown in Fig. 4 as a function of the product of the imposed static magnetic flux density and the alternating current. The theoretical expression for pressure is proportional to this product as seen in eq. (20), which is useful in this experimental condition because the glass thickness of 1.5 mm is much less than the wave length of 1.4 m. The solid line in Fig. 4 is the theoretically

Fig. 3 Schematic view of experimental apparatus.

Fig. 4 Detected pressures as a function of the product of imposed magnetic field and electrical current and the theoretical line.

calculated pressure. The detected pressure agrees approximately with the theoretical pressure though the former is a little smaller than the latter. A reason for this discrepancy is that the evaluated current density in the theoretical calculation could be different from the actual current density because of the complicated distribution of the electrical current in the metal. From the experimental and theoretical results, it can be concluded that the compression waves can be directly generated in a liquid metal by the simultaneous imposition of a static magnetic field and an alternating current.

4. Refinement of Solidified Structure

A mother alloy of Sn–10 mass%Pb was prepared by alloying pure Sn(99.9%) and pure Pb(99.9%). The schematic view of the experimental apparatus is shown in Fig. 3. When the temperature of the sample set in the bore of the superconducting magnet reached 573 K, it was allowed to cool. An alternating current was applied when the temperature dropped to 523 K, which is higher than the liquidus temperature (492 K), and it was removed when the temperature reached 443 K, which is just below the solidus temperature of 456 K. A static magnetic field was applied during the whole of the solidifying period as shown in Fig. 5. The temperature was measured by a thermocouple set in the middle of the long wall at a height of 20 mm from the bottom. The measured histories of the temperature under the imposition of a 10 T static magnetic field are shown in Fig. 6. Recalescence was not observed whilst the static magnetic field and the electrical current were simultaneously applied, whereas it was observed when only the static magnetic field was imposed. The sample with the alternating current solidified slightly slower compared to that without the alternating current because of the Joule heating. The temperature distribution measured at three points in the sample is shown in Fig. 7 when both the static magnetic field and the alternating current were applied. These three points were along the long wall at a height of 20 mm from the bottom. The first one was at the same point mentioned above and the second was in the neighborhood of the electrodes, and the last

was on the opposite side to the electrodes. The temperature difference in the sample was within a few degrees Kelvin.

After solidification of the samples, their vertical cross-sections cut parallel to the long wall were examined. The macrostructures and microstructures of the samples under the different experimental conditions are shown in Fig. 8. In cases (a,b), which were without the electric current, coarse grains composed of dendrites were obtained in the whole cross-section since an electromagnetic force was not excited in the alloy. In case (c), which was with the electric current but without the magnetic field, the grain sizes slightly decreased. This may be due to compression waves excited by the alternating current. In case (d), in which the static magnetic field and the alternating current were simultaneously imposed, all of

Fig. 6 Cooling curves of samples under the imposition of 10T static magnetic field.

Fig. 5 Imposing periods of static magnetic field and alternating current.

Fig. 7 Temperature distribution in the sample under the simultaneous imposition of static magnetic field and alternating electrical current.

Fig. 8 Macro- and micro-structures of samples under different electric and magnetic conditions.

the observed area was refined, although the region on which the electromagnetic force was acting was limited to the electromagnetic skin layer only, calculated to be 8.1 mm. One of the mechanisms for the refinement is believed to be compression waves propagating in the alloy cutting off dendrites during solidification, since the macrostructure changed drastically while the grain sizes did not show a marked change. Moreover, compression waves may affect nucleation since recalcification was not observed under the imposition of compression waves as shown in Fig. 6. Therefore the compression waves must provide the function of refinement of the solidified structure.

5. Conclusion

A new method of generating compression waves is proposed in which a static magnetic field and an alternating current are simultaneously imposed on a liquid metal. The distributions of the pressure and the velocity in the compression waves have been theoretically derived. The intensities of the pressure were measured under different conditions and compared with the theoretical values. Then, the compression waves were imposed on a Sn-Pb alloy confirming the refining function for solidified structures. The main results obtained in the research are as follows.

- (1) The amplitudes of pressure and velocity accompanying the compression waves are in proportion to the product of the intensities of the static magnetic field and the alternating current.
- (2) The compression waves generated by the simultaneous imposition of a static magnetic field and an alternating current can exist as a standing wave in a liquid metal except in the electromagnetic skin layer.
- (3) The compression waves have a refining function on the solidified structures, and this was confirmed by imposing compression waves on a Sn-Pb alloy during solidification.

Acknowledgment

The authors gratefully acknowledge the financial support of the Ministry of Education, Science and Culture, Japan through a Grant-in-Aid for scientific research (No. 10211204).

Nomenclature

- a : propagation velocity of magneto-acoustic waves (m/s)
- B_{DC} : static magnetic flux density (T)
- b_z : perturbation of magnetic flux density (T)
- c : propagation velocity of ordinary compression waves (m/s)
- E_m : perturbation of electric field (V/m)
- F_{AC} : force density caused by imposed electromagnetic fields (N/m³)
- F_m : force density caused by motion of fluid element in a magnetic field (N/m³)
- F_x : force density in x-direction (N/m³)
- f : frequency (Hz)

$\mathbf{i}_x$: unit vector in x-direction
 $\mathbf{i}_y$: unit vector in y-direction
 $\mathbf{i}_z$: unit vector in z-direction
 J_{AC} : effective value of imposed alternating current density (A/m²)
 J_m : perturbation of electrical current density (A/m²)
 k : wave number (1/m)
 l : length of a liquid metal (m)
 N : constant (—)
 P_m : constant (Pa)
 p : pressure (Pa)
 p_e : effective value of pressure (Pa)
 t : time (s)
 V_m : constant (m/s)
 v_e : effective value of velocity (m/s)
 v_x : velocity in x-direction (m/s)
 x, y, z : distance in x, y, z -direction (m)
 γ : propagation velocity ratio (—)
 δ : electromagnetic skin layer (m)
 μ : magnetic permeability (H/m)
 ρ : density (kg/m³)
 σ : electrical conductivity (S/m)
 ϕ : constant (rad)

ω : angular frequency (rad/s)
 superscript '': sum of perturbation and equilibrium quantities
 subscript 0: equilibrium quantity

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