Influence of Frequency on High-Temperature Fatigue Behavior of 17-4 PH Stainless Steels

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The effect of frequency (2 and 20 Hz) on the high-cycle fatigue and fracture behavior was investigated at 573–773 K for 17-4 PH stainless steels in three conditions: Condition A (unaged), H900 (peak-aged) and H1150 (overaged). S–N results indicated that at 573 and 673 K, there was generally no difference in fatigue strength between 2 and 20 Hz, except for H900 at 673 K where the fatigue strength at 2 Hz was lower than that at 20 Hz. At 773 K, the fatigue strength of each condition at 2 Hz was lower than that at 20 Hz due to the occurrence of creep mechanism at this low frequency. At 773 K and 2 Hz, the fatigue fracture mode exhibited a mixed mode involving transgranular and intergranular cracking and the grain boundary cavities were also observed. At a given temperature and frequency, the fatigue strength for the three conditions generally took the following order: H900 > Condition A > H1150, except for Condition A at 773 K in the long life regime where the fatigue strength was close to that of H1150 due to a precipitate-coarsening effect. With the exception of Condition A tested at 673 K, the fatigue strength of each condition was decreased with increasing temperature as a result of a reduction in yield strength. At both frequencies, the fatigue strength of Condition A at 673 K was greater than that at 573 K as a result of an *in-situ* precipitation-hardening effect. Fractography observations indicated that the fatigue crack initiation site, crack propagation path and fracture surface morphology were functions of testing temperature, loading frequency and applied cyclic stress level.

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1. Introduction

Precipitation-hardening stainless steels have been extensively used as structural components in various applications, such as nuclear, chemical, aircraft, and naval industries due to their excellent mechanical properties, good fabrication characteristics and excellent corrosion resistance. Of the former, 17-4 PH stainless steel is presently one of the most commonly used alloys.¹⁾ In general, the maximum strength and hardness values can be obtained by aging at 723–783 K, during which the precipitation of coherent copper-rich clusters occurs.^{1,2)} Aging at higher temperatures (above 813 K) would result in the precipitation of incoherent fcc copper-rich phases, lower strength and hardness, and enhancement in toughness.^{1,2)}

Most of the previous studies on 17-4 PH steels were focused on the analyses of microstructure, mechanical and fatigue properties at room temperature;^{2–4)} little work⁵⁾ has been done on high-temperature mechanical and fatigue properties of this alloy and there is still lack of consideration of frequency effect on high-temperature fatigue of this alloy. As some applications of 17-4 PH stainless steels are subjected to long-term cyclic loading with different frequencies at high temperatures and the deformation might be a combination of creep and fatigue, it is important to characterize the frequency effect on the high-temperature fatigue and fracture behavior of this alloy so as to better predict the fatigue life for components made from this material.

In this study, systematic experiments were conducted to investigate the influence of frequency (2 and 20 Hz) on the high-cycle fatigue (HCF) and fracture mode of 17-4 PH stainless steels in three heat-treated conditions, *i.e.* solution-

annealed (Condition A), peak-aged (H900) and overaged (H1150) at 573, 673 and 773 K.

2. Experimental Procedures

The commercially available 17-4 PH stainless steels used in the current study were supplied by the vendor in the form of hot-rolled, solution-annealed bars. The chemical composition of this alloy (mass%) is 15.18 Cr, 4.47 Ni, 3.47 Cu, 0.65 Mn, 0.38 Si, 0.2 (Nb+Ti), 0.15 Mo, 0.03 S, 0.02 C, 0.016 P and Fe (balance). Three types of heat treatments were applied to the specimens, *i.e.* as-received “Condition A”, peak-aged “Condition H900” and overaged “Condition H1150.” For Condition A, specimens were heated to 1311 K (1900°F), held 0.5 h at heat and cooled in air. For Condition H900 and H1150, specimens were first heat treated by solution annealing and then, respectively, aged at 755 K (900°F) for 1 h and 894 K (1150°F) for 4 h, followed by air cooling. The mechanical properties at room temperature for each condition are listed in Table 1.

HCF tests were carried out on a commercial closed-loop servo-hydraulic test machine at 573, 673, and 773 K in ambient air. The axial smooth-surface fatigue specimens had a uniform cylindrical gage section of 6 mm in diameter and 18 mm in length. The HCF tests were performed under a sinusoidal loading waveform with a load ratio of $R = 0.1$ and frequencies $f = 2$ and 20 Hz until failure or 2×10^6 cycles where specimen was considered to be a runout. Each fatigue specimen was held in the furnace at a given temperature for 15 min to reach thermal equilibrium prior to the start of fatigue loading.

Scanning electron microscopy (SEM) was used for characterization of fracture surface morphology and microstructural variation. Microstructures in fatigue specimens were analyzed with transmission electron microscopy (TEM) to observe the morphology of precipitates and substructure of

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Table 1 Room-temperature mechanical properties of 17-4 PH stainless steels in different aged conditions.

Condition	Ultimate Tensile Strength (MPa)	Yield Strength (MPa)	Elastic Modulus (GPa)	Elongation (in 25 mm) (%)	Hardness (HRc)	V-notch Impact Toughness (J)
Condition A	1018	992	199	13.4	32.0	67
H900	1414	1387	223	12.5	44.5	21
H1150	966	880	196	18.4	31.5	75

dislocations. The thin foils for TEM analysis were prepared by cutting out 3-mm-diameter disks from some of the fatigue specimens followed by standard grinding and twin-jet electropolishing procedures.

3. Results and Discussion

3.1 Effect of frequency on the fatigue behavior

Figures 1, 2 and 3 show comparisons of the S–N curves under two different frequencies (20 and 2 Hz) for variously heat-treated 17-4 PH stainless steels at 573, 673, and 773 K, respectively. It can be seen in Figs. 1 and 2 that at 573 and 673 K, the S–N curves under these two frequencies for a given heat-treated condition were almost merged together indicating no difference in fatigue strength between 20 and 2 Hz except for H900 condition tested at 673 K where the fatigue strength was slightly reduced with decreasing frequency. However, at 773 K, the significant reduction in fatigue resistance of each condition with a decrease in frequency was observed, as shown in Fig. 3. In other words, at 773 K, the fatigue strengths of each condition under 20 Hz were greater than those under 2 Hz. The occurrence of time-dependent creep mechanism in high-temperature fatigue has been known to be a function of temperature and loading frequency.^{6,7)} At higher temperatures and lower loading frequencies, creep damage would readily take place and cause premature fatigue failure as compared with time-independent fatigue failure mechanisms.^{6,7)} That is to say, the high-temperature fatigue resistance of alloys was generally reduced with increasing temperature and decreasing frequency due to an interaction between creep and fatigue.^{6,7)} For this reason, it was suggested that, in the present work, the testing temperatures, 573 and 673 K, were still too low or the loading frequencies, 20 and 2 Hz, were too high to generate any detrimental creep damage during cyclic loading. Hence, time-dependent mechanisms such as creep or others did not significantly contribute to the fatigue failures for the given alloys at 573 and 673 K except for H900 condition tested at 673 K.

Note that no grain-boundary cavities were observed for H900 specimens tested at 673 K and 2 Hz implying that creep damage was not responsible for the slightly decreased fatigue strength at this testing condition. The reduction of fatigue strength in H900 condition with decreasing frequency at 673 K was considered related to the activity of climbing mechanism of dislocations. Generally, a decrease in cyclic loading frequency at high temperatures, the climbing mechanism of dislocations would become more active and the resistance to dislocation motion would be reduced.⁸⁾ The Cu-rich phase in the peak-aged H900 condition was a very

Fig. 1 Comparisons of S–N curves under two different frequencies at 573 K for 17-4 PH stainless steel in three heat-treated conditions: (a) Condition A, (b) H900, and (c) H1150. (Arrows designate runout tests.)

fine structure such that when the loading frequency was decreased to 2 Hz at 673 K, the dislocation was easier to climb over a fine precipitate to continue its motion leading to a reduction of fatigue life. As shown in Fig. 4(a), planar dislocations were the typical dislocation morphology observed in H900 condition. On the other hand, for the

Fig. 2 Comparisons of S-N curves under two different frequencies at 673 K for 17-4 PH stainless steel in three heat-treated conditions: (a) Condition A, (b) H900, and (c) H1150. (Right arrows designate runout tests; left arrow designates a premature failure.)

overaged H1150 condition, as the Cu-rich precipitates became larger and incoherent with the matrix, dislocation climbing would be more difficult than that in the H900 condition. Moreover, the precipitate particles in the H1150 condition have grown large enough for dislocation segments to bend and pass between adjacent particles and leave expanding loops around the precipitates as shown in a representative TEM micrograph, Fig. 4(b). The formation of dislocation loops around precipitates would add resistance to the motion of the next dislocation. Consequently, the fatigue strength of the H1150 condition tested at 673 K was barely affected by the frequency effect. For Condition A tested at 673 K, as a result of an *in-situ* precipitation-hardening effect, the chance for occurrence of creep mechanism during cyclic loading would be lower such that no significant difference in fatigue strength between the two given frequencies was

Fig. 3 Comparisons of S-N curves under two different frequencies at 773 K for 17-4 PH stainless steel in three heat-treated conditions: (a) Condition A, (b) H900, and (c) H1150. (Arrows designate runout tests.)

observed. A previous study⁸⁾ similarly indicated that it was possible for an alloy with second phase precipitates under deformation to have a reaction that lowered the creep rate and even prohibited the creep mechanism from occurring.

It was found that, the decrease in fatigue strength for each condition tested at 773 K and 2 Hz, compared with 20 Hz, was associated with the presence of creep damages, as described below. SEM analyses indicated that no substantial difference in cracking behavior between 20 and 2 Hz was observed at 573 and 673 K where transgranular cracking was a common crack propagation mode. No evidence of creep cavities in any fatigue specimens was noticed at these two temperatures, either. These observations were consistent with the S-N curve results in which, at 573 and 673 K, the fatigue strengths under 20 and 2 Hz for each condition were comparable, except for H900 condition tested at 673 K. All of the fatigue specimens failed at 773 K and 20 Hz also

Fig. 4 (a) Planar dislocations formed in the H900 fatigue specimen tested at 673 K with 2 Hz and (b) dislocation loops around Cu-rich precipitates in the H1150 condition tested at 673 K with 2 Hz.

showed a fracture mode dominated by transgranular cracking, as exemplified in Fig. 5(a). However, at 773 K and 2 Hz, the fracture morphology for all three heat-treated conditions was apparently different from that at 20 Hz and numerous grain boundaries were observed on the fracture surfaces, as exemplified in Fig. 5(b). This implied that as the loading frequency was decreased from 20 to 2 Hz at 773 K, the crack propagation mode changed from a purely transgranular to a mixed mode of transgranular and intergranular cracking as a result of a creep-assisted damaging mechanism. Figure 6 showed the presence of grain-boundary cavities in a fatigue specimen tested at 773 K and 2 Hz as indicated by arrows in the micrograph. On the contrary, no evidence of cavities at grain boundaries was observed at 773 K and 20 Hz. Note that the SEM micrograph in Fig. 6 was an axially sectioned, etched view of the gage section in the fractured specimen, which was longitudinally cut along the loading axis.

For metals and alloys tested under a higher frequency in high-temperature fatigue, the fracture mechanism would tend

Fig. 5 SEM fractographs of 17-4 PH stainless steels tested at 773 K: (a) Condition A, 20 Hz, $N_f = 739892$ cycles and (b) H900, 2 Hz, $N_f = 926603$ cycles.

Fig. 6 SEM micrograph of an axially sectioned view for Condition H900 specimen tested at 773 K and 2 Hz with $N_f = 529927$ cycles. (Arrows point to the cavities.)

towards the pure mechanical fatigue and the crack usually propagated transgranularly.⁹⁾ However, as the frequency was lowered to a certain level, the fatigue fracture mode would change into a mixed mode of transgranular and intergranular cracking due to a creep-fatigue interaction with the possible occurrence of grain boundary sliding and/or cavities.⁹⁾ It can thus be seen that, in the current study, a creep-assisted fatigue

mechanism took place in the specimens cyclically loaded at 773 K with 2 Hz to cause a change in cracking mode and formation of creep cavitation, and hence reduce the fatigue strengths from the 20 Hz values.

3.2 Effect of heat treatment on the fatigue resistance

Comparisons of the S-N curves of variously heat-treated 17-4 PH steels at different temperatures with 2 Hz are shown in Fig. 7. It can be seen in Fig. 7 that at a given temperature the peak-aged H900 condition exhibited the highest fatigue strength among the given three conditions, while the over-aged H1150 condition showed the lowest one, except in the long life regime at 773 K where the fatigue strength of Condition A was close to that of the H1150 condition. Similar trends in the relative rank of fatigue strength were also found for the given three heat-treated conditions tested at 20 Hz, as

Fig. 7 S-N curves for variously heat-treated 17-4 PH stainless steels tested at different temperatures under 2 Hz: (a) 573 K, (b) 673 K, and (c) 773 K. (Right arrows designate runout tests; left arrow designates a premature failure.)

Table 2 Yield strengths of 17-4 PH stainless steels at various high-temperature exposure conditions.

High-Temperature Exposure Condition	0.2%-offset Yield Strength (MPa)		
	Condition A	H900	H1150
573 K-0.25 h	895	1175	825
573 K-32 h	900	1160	830
673 K-0.25 h	860	1060	750
673 K-32 h	1060	1060	770
773 K-0.25 h	725	875	637
773 K-32 h	650	710	640

reported in our earlier work.⁵⁾ Therefore, frequency effect had no influence on the relative rank of fatigue strength in the given three heat-treated conditions.

In general, fatigue failure can be divided into two steps: crack initiation and crack propagation.¹⁰⁾ For steels, the fatigue life of smooth surfaces was usually predominated by the crack initiation stage which took up most of the lifetime, in particular at low stress levels.¹⁰⁾ It is generally found that fatigue strength of steel is increased with tensile strength.¹⁰⁾ For an alloy with a higher yield strength, dislocations in the matrix are more difficult to move and thus the resistance to crack initiation is enhanced and the fatigue life will be increased.¹¹⁾ Table 2 showed the 0.2%-offset yield strengths of the given three heat-treated conditions tested at various high-temperature exposure conditions. It can be seen in Table 2 that at a given temperature the yield strength in the three conditions was ranked as: H900 > Condition A > H1150. Accordingly, the fatigue strength for the three conditions took the following sequence: H900 > Condition A > H1150. This also indicated that the initial, coherent precipitates in the matrix of peak-aged H900 condition provided more resistance to crack initiation in 17-4 PH steel as compared with the unaged Condition A and overaged H1150 condition. As for Condition A tested at 773 K, an *in-situ* coarsening effect of Cu-rich precipitates occurred during cyclic loading, the fatigue resistance was gradually reduced to a level close to that in Condition H1150, as evidenced by the TEM micrographs in Fig. 8. The precipitate size in a Condition A specimen failed at a longer life (Fig. 8 (b)) was apparently larger than that in a short-life one (Fig. 8(a)). Table 2 also indicated that after exposure at 773 K for 32 h, the yield strength of Condition A (650 MPa) was very close to that (640 MPa) of Condition H1150. On the other hand, in the H900 condition, microstructural analyses indicated that the coarsening effect of precipitates was not so effective at 773 K as compared with that in Condition A. Therefore, the peak-aged H900 condition still exhibited the greatest fatigue strength among the given three conditions in long life regime at 773 K.

3.3 Effect of environmental temperature on the fatigue behavior

Figure 9 shows the S-N curves of each condition tested at various temperatures under 2 Hz. Similar trends in variation of fatigue strength with testing temperature were also found for each heat-treated condition tested at 20 Hz, as reported in our previous work.⁵⁾ It can be seen in Fig. 9 that the fatigue

Fig. 8 TEM micrographs of microstructure in Condition A failed at 773 K and 2 Hz with (a) $N_f = 85007$ cycles and (b) $N_f = 1283340$ cycles.

strengths of a given condition were generally decreased with increasing temperature except for Condition A tested at 673 K where the fatigue strengths were greater than the corresponding ones at 573 K. The variation of fatigue strength with testing temperature for each condition exhibited a similar trend to that of yield strength listed in Table 2. Apparently, an increase in temperature generated a similar effect on the fatigue strength as it did on the yield strength such that the fatigue resistance was correlated to the yield strength. In other words, at a given frequency, the fatigue strength of each condition was decreased with increasing temperature due to a reduction of yield strength except Condition A at 673 K. For Condition A tested at 673 K, the greater fatigue strengths over the 573 K values can be attributed to an *in-situ* precipitation-hardening effect. Our previous investigation⁵⁾ on high-temperature tensile behavior of this steel indicated that when Condition A tested at 673 K, Cu-rich phases would gradually precipitate in the matrix and then increase the yield strength to a value equivalent to that of

Fig. 9 S-N curves under 2 Hz at various temperatures for 17-4 PH stainless steel in three conditions: (a) Condition A, (b) H900, and (c) H1150. (Right arrows designate runout tests; left arrow designates a premature failure.)

the H900 condition after a long enough time at heat. This precipitation-hardening effect could also enhance the fatigue resistance of Condition A at 673 K to extend the fatigue lifetimes over those at 573 K.

3.4 Fractography analyses

Based on fractography observations, it can be summarized that for each condition tested at 573 and 673 K in the short life regime, fatigue cracks were generally initiated from slip bands at the surface followed by a flat river-pattern region of stable crack growth, as exemplified in Fig. 10. However, at 773 K in the short life regime, the fatigue fracture surface generally exhibited a cup-and-cone pattern because of a higher applied stress level and a decreasing tensile strength due to an *in-situ* precipitate-coarsening effect during cyclic loading, as shown in Fig. 11. On the other hand, at all given temperatures in the long life regime the fatigue specimens of

Fig. 10 SEM fractographs of Condition A failed at 573 K and 2 Hz with a fatigue life of 19966 cycles: (a) slow crack growth region and (b) fracture origin. (i: crack initiation site.)

Fig. 12 SEM fractographs of Condition A failed at 673 K and 2 Hz with $N_f = 577729$ cycles: (a) overview and (b) high magnification of area m in (a). (i: crack initiation site; m: faceted cracking region; f: final fast fracture zone.)

Fig. 11 SEM fractograph of Condition A failed at 773 K and 2 Hz with a fatigue life of 21746 cycles.

the three conditions revealed internal crack initiation and evidence for this was shown in Fig. 12. The internal fatigue cracks usually originated from a weak structure within the specimens and formed a flat area at the initiation site due to the localized deformation (Fig. 13(a)), but sometimes originated from inclusions (probably impurities) (Fig. 13(b)). An earlier study by Nagai *et al.*¹²⁾ indicated

that a flat initiation facet in an internally initiated crack was mainly caused by the heterogeneous microplasticity due to planar or limited slip systems in weak structures.

The reason for the change in crack initiation site was considered related to the pronounced surface oxidation observed for high temperature specimens. Our earlier work⁵⁾ indicated that when tested at a temperature above 573 K, uniform surface oxide layers would be formed on the fatigue specimens to strengthen the surface resistance to fatigue crack initiation such that crack initiation occurred inside the specimen. Previous studies^{5,11,13,14)} also showed that internal fatigue crack initiation in high-temperature ambient air was attributed to a reduction in the surface slip intensity, as a thin oxide layer on slip steps would lead to greater surface work hardening and prevent surface fatigue crack initiation. Moreover, in the long life regime, the oxide layer would not be severely damaged by a smaller deformation due to a lower fatigue load such that it could effectively enhance the resistance to surface crack initiation.^{5,11)} In the present work, at a given testing temperature, frequency effect did not change crack initiation mode, *i.e.* surface or internal initiation, for all the given three heat-treated conditions.

In some fatigue specimens with internal crack initiation, the cracks, after initiation from weak structures or inclusions, would grow along a stage II-like planar path normal to the

Fig. 13 SEM micrographs for typical internal crack initiation sites: (a) a flat area, H900 failed at 773 K and 2 Hz with $N_f = 636083$ cycles and (b) an inclusion, H1150 failed at 773 K and 20 Hz with $N_f = 1042762$ cycles.

loading direction (label i in Fig. 12(a)) and shifted into a plane at $40\text{--}50^\circ$ to the loading axis with a faceted profile on the fracture surfaces (label m in Fig. 12(a) and enlarged view in Fig. 12(b)) followed by the final fast fracture (label f in Fig. 12(a)) where the fracture area often appeared as a shear lip with elongated dimples. Note that the fracture surface morphology in the faceted cracking region (label m) was different from that in the final fast fracture zone (label f). In other words, the crack propagation mode would be changed from stage II planar cracking to faceted cracking along a plane at about 45° to the direction of the applied stress for some internally initiated fatigue cracks. The reason for the transition from stage II to faceted cracking along a plane oriented 45° from the direction of the applied stress in an internal fatigue crack was likely to be related to a change in the state of stress at the crack tip. Generally, when the crack propagated under a state of plane strain at the crack tip, the crack growth path favored a stage II cracking and the fracture surface was roughly perpendicular to the loading axis.¹⁵⁾ Conversely, under a condition of plane stress, the fracture surface would tend to shift into a plane at 45° to the loading direction.¹⁵⁾ In the current study, when fatigue cracks were initiated within the specimen body, they were initially facing a plane strain condition at the crack tip. Once these cracks propagated toward the specimen surfaces, the plastic zone

ahead of the crack tip would become larger and the ligaments would also become thinner such that the stress state at the crack tip would change into a plane stress condition. Hence, the transition from stage II planar cracking to faceted cracking along a plane oriented 45° from the loading axis for the internal crack growth was likely to be attributed to the change from plane strain to plane stress condition at the crack tip.

4. Conclusions

- (1) No frequency effect between 20 and 2 Hz was observed on the fatigue strength of each heat-treated condition of 17-4 PH stainless steel at 573 and 673 K except for H900 condition tested at 673 K where the fatigue strength was slightly reduced with decreasing frequency.
- (2) At 773 K, the fatigue strength in each heat-treated condition at 2 Hz was reduced from the 20 Hz value as a result of an additional creep damage to the pure fatigue damage. The fracture mode was changed from a transgranular cracking to a mixed mode of transgranular and intergranular cracking accompanied with grain-boundary creep cavities when the frequency was reduced to 2 Hz at 773 K.
- (3) For variously heat-treated 17-4 PH stainless steels tested at a given frequency and temperature, the fatigue strength was generally ranked as: H900 > Condition A > H1150, except for Condition A at 773 K in the long life regime where the fatigue strength was close to that of H1150 due to a coarsening effect of Cu-rich phases.
- (4) Regardless of the testing frequency, the fatigue strength of each heat-treated condition was decreased with an increase in temperature due to a reduction in yield strength, except for Condition A tested at 673 K where the fatigue strengths were superior to those at 573 K as a result of an *in-situ* precipitation-hardening effect.
- (5) For the given three conditions of 17-4 PH alloy, fatigue cracks were initiated at the surface in the short life regime at 573 and 673 K. At 773 K in the short life regime, the fracture surface exhibited a tensile fracture mode consisting of dimples and shear lips. However, in the long life regime, crack initiation occurred inside the specimen for all the given testing conditions.

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