Mixing Time in a Bath Agitated Simultaneously by Bottom Gas Injection and Side Liquid Injection

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Effective mixing of a cylindrical bath is of crucial for wastewater treatment. Measurements were carried out on the mixing time in a cylindrical bath agitated simultaneously by bottom gas injection and side liquid injection. Air was injected into the bath through a centered single-hole bottom nozzle. Water was injected into the bath through a single-hole nozzle attached to the side wall of the vessel, drained from a pipe installed at the opposite bottom corner, and injected again with a small pump. The mixing time due to simultaneous gas and water injection, T_{m3} , was satisfactorily correlated by the following empirical equation.

$$1/T_{m3} = 1/T_{m1} + 1/T_{m2}$$

where T_{m1} and T_{m2} are the mixing time values for water injection alone and gas injection alone, respectively. This relationship means that mixing by gas injection and that of liquid injection proceed in parallel with each other.

(Received February 16, 2004; Accepted May 14, 2004)

Keywords: wastewater treatment, gas injection, two-phase flow, liquid jet, mixing time

1. Introduction

Concerning the mixing time of batch-type metals refining processes agitated by gas injection, remarkable progress was achieved by pioneering work of Nakanishi *et al.*¹⁾ Since then numerous investigations on the mixing time for various types of reactors agitated by gas injection have been carried out.²⁻⁹⁾

Gas injection techniques find wide applications in another fields of engineering. The present authors have applied the gas injection method to snow melting¹⁰⁾ and wastewater treatment.¹¹⁾ The melting rate of snow and the efficiency of wastewater treatment were highly enhanced in the presence of the swirl motion.^{8,9)}

In this study, particular attention was paid to the effect of liquid injection on the mixing time in a cylindrical water bath subjected to centered bottom gas injection. Water was injected into the bath through a circular nozzle installed flush-mount to the side wall of the vessel, drained through a pipe attached to the opposite bottom corner, and injected again to keep the bath surface constant. The water therefore was circulated. The mixing time was found to be independent of the tracer and sensor positions. An empirical equation was proposed for the mixing time in a bath agitated simultaneously by gas and water injection as a function of the mixing times for gas injection alone and water injection alone.

This agitation system can be applied to a batch type wastewater treatment. As mentioned above, the authors reported previously that refractory organic wastewater contained in a batch type vessel can be effectively processed by bottom gas injection.¹¹⁾ The gas was a mixture of air and ozone. The efficiency of the treatment would be highly enhanced by simultaneous gas and liquid injection. This is the main reason why mixing time was chosen to quantitatively evaluate the intensity of mixing in the bath. If we use vessels of more than two, we can develop a continuous

process for wastewater treatment. In this case, however, the residence time must be taken into consideration in place of mixing time.

2. Experimental Apparatus and Procedure

2.1 Experimental apparatus

Figures 1 and 2 show schematic diagrams of the experimental apparatus and test vessel, respectively. The mixing time was defined as a period for an instantaneous tracer concentration to settle within $\pm 5\%$ deviation around the final constant tracer concentration in the bath (called 95% criterion), as shown in Fig. 3.

Aqueous KCl solution with a concentration of 1 kmol/m^3 was used as tracer. The volumetric concentration of the tracer

Fig. 1 Schematic diagram of experimental apparatus.

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Fig. 2 Schematic of test vessel.

Fig. 3 Definition of mixing time.

in the bath was adjusted to be 0.1%. The cylindrical vessel made of transparent acrylic resin had an inner diameter, D , of 20 cm and a height, H , of 44 cm. Air was injected through a centered bottom nozzle with an inner diameter, d_n , of 0.2 cm. Water with the same temperature as the bath as well as the air was injected at a prescribed flow rate through a circular nozzle with an inner diameter, d_{nw} , of 1.3 cm. The nozzle exit was flush-mount to the side wall of the vessel. The height of the nozzle, H_{ni} , for water injection was changed in the vertical direction. The water was drained through another pipe with the same inner diameter of 1.3 cm, being located at the opposite bottom corner, and circulated by a small pump through a very short circulation pipe line.

The volume of water contained in the pump and circulation pipe line is less than 100 cm^3 , being very small compared with the volume of water of 1570 to 9400 cm^3 in the cylindrical bath, and hence, almost all amount of tracer remains in the bath. Furthermore, the concentration of the tracer in the circulation pipe line is almost equal to that in the bath because the passing time of liquid through the line is much shorter than the measured mixing time. The mixing time value thus obtained, therefore, can be regarded as a good approximation of the real mixing time and would be useful to

Fig. 4 Sensor of electric conductivity meter.

predict the mixing characteristics of practical reactors agitated by liquid injection.

The electrical conductivity of liquid in the bath was measured using a sensor shown in Fig. 4. The time constant of the present system composed of the electric conductivity meter and a pen recorder was 0.25 s. The mixing time measurements were repeated ten times at each measurement position.

2.2 Effects of sensor and tracer positions on mixing time

Full understanding of the effects of sensor and tracer positions on the mixing time has not been obtained.^{2-9,12)} We therefore investigated at first the effects of sensor and tracer positions on the mixing time.

2.2.1 Vertical sensor position

The radial sensor position was fixed at the position A ($r_s = 9 \text{ cm}$) in Fig. 2 and the sensor was placed at a vertical position of $H_s = 1, 5, 10, 15, 20$, or 25 cm , where H_s was measured from the bottom of the bath. Tracer was added onto the bath surface at the radial position D of $r_t = 9 \text{ cm}$, as shown in Fig. 2. The air and water flow rates, Q_g and Q_w , were $120 \text{ cm}^3/\text{s}$ and $150 \text{ cm}^3/\text{s}$, respectively.

2.2.2 Radial sensor and tracer positions

The radial tracer position was fixed on the bath surface at the position D and the sensor was placed in turn at the position A, B, C or D. Later, the sensor was placed at the position D and the radial tracer position was shifted to the position A, B or C to examine the symmetry of mixing time with respect to the line connecting the inlet and exit of the forced circulation pipe line. The other parameters were chosen as follows: $H_s = 1 \text{ cm}$, $Q_g = 120 \text{ cm}^3/\text{s}$ and $Q_w = 150 \text{ cm}^3/\text{s}$.

2.3 Effect of water flow rate, Q_w , on mixing time

The sensor was placed at $H_s = 1 \text{ cm}$ and $r_s = 9 \text{ cm}$ (Position A), and tracer was added onto the bath surface at the position D. The water flow rate, Q_w , was set at 0, 50, 100, or $150 \text{ cm}^3/\text{s}$ for a gas flow rate of $Q_g = 120 \text{ cm}^3/\text{s}$.

2.4 Effect of gas flow rate, Q_g , on mixing time

The sensor was placed at $H_s = 1 \text{ cm}$ and $r_s = 9 \text{ cm}$ (Position A), and the position D was chosen as the tracer position. The water flow rate was kept constant ($Q_w = 100 \text{ cm}^3/\text{s}$), and the air flow rate, Q_g , was changed from 0 to $240 \text{ cm}^3/\text{s}$ at equal intervals of $60 \text{ cm}^3/\text{s}$.

Table 1 Mixing time values for different sensor positions.

(a)															
H_s (cm)	Q_w		50 cm ³ /s		Q_w		100 cm ³ /s		Q_w		150 cm ³ /s				
	Q_g		0 cm ³ /s		Q_g		0 cm ³ /s		Q_g		0 cm ³ /s				
	mean	max	min	mean	max	min	mean	max	min						
1	52.1	58.0	47.4	22.7	28.7	17.8	15.1	16.8	13.8						
5	49.1	53.4	47.2	22.9	25.0	19.0	15.2	17.7	12.6						
10	46.1	51.5	41.9	21.4	27.9	14.6	14.6	17.2	11.7						
15	48.9	54.0	42.4	21.9	26.3	16.0	13.5	15.5	10.0						
20	50.6	57.6	41.6	26.4	29.4	21.2	14.9	17.2	12.0						
25	51.9	60.0	44.8	26.3	29.0	21.6	14.4	16.8	12.0						
(unit: s)															
(b)															
Q_w		0 cm ³ /s		Q_w		50 cm ³ /s		Q_w		100 cm ³ /s		Q_w		150 cm ³ /s	
Q_g		120 cm ³ /s		Q_g		120 cm ³ /s		Q_g		120 cm ³ /s		Q_g		120 cm ³ /s	
mean	max	min	mean	max	min	mean	max	min	mean	max	min	mean	max	min	
21.2	24.9	19.0	16.5	21.0	14.1	9.6	12.3	6.0	9.6	12.2	8.6				
22.3	28.1	18.9	19.0	21.3	17.0	10.1	11.8	6.5	9.1	10.0	8.4				
22.8	27.9	18.4	15.2	19.3	12.6	11.4	13.1	7.7	12.0	13.0	11.3				
20.1	23.8	15.6	12.0	14.2	10.8	9.3	11.1	8.6	9.0	9.7	7.2				
20.5	25.0	18.9	14.6	17.9	10.8	10.1	11.2	7.2	8.3	10.6	6.1				
20.2	23.8	16.7	13.3	15.3	11.8	9.4	12.3	6.8	6.8	7.2	6.3				
(unit: s)															

2.5 Effect of bath depth, H_L , on mixing time

According to previous investigations,^{13–15)} a bottom blown bubbling jet rotates around the vessel axis when the ratio of the bath depth, H_L , to the bath diameter, D , satisfies a certain condition. This is called the swirl motion of the bubbling jet. Two kinds of swirl motions are known. One is observed for $0.3 < H_L/D < 1$ and closely related to so-called rotary sloshing¹⁶⁾ appearing in a cylindrical bath subjected to external forced oscillation. The swirl radius, r_c , of the bubbling jet on the bath surface is relatively small and expressed by $r_c = 0.37R$, where R is the vessel radius.^{8,9)} The other is induced by a hydrodynamic instability of a large scale ring vortex enclosing the bubbling jet in a bath with $H_L/D > 2$. This instability is closely associated with the Coanda effect¹⁷⁾ acting on a confined jet near a wall. The bubbling jet approaches the side wall of the vessel and ascends along it while swirling. These two kinds of swirl motions are called, for convenience, the first and second kinds of swirl motions, respectively.^{13,14)}

In this study the bath depth, H_L , was changed from 5 cm to 30 cm. The first kind of swirl motion appeared for $H_L = 10$ cm and 15 cm, but the second kind did not occur under the present experimental conditions. The sensor and tracer positions are the same as described in Section 2.4.

2.6 Effect of water injection height, H_{ni} , on mixing time

Measurements were made for $Q_w = 100$ cm³/s, $Q_g = 120$ cm³/s, $H_L = 30$ cm, and three water injection heights of $H_{ni} = 10, 20$, and 30 cm. The same sensor and tracer positions as described in Section 2.4 were chosen here.

3. Experimental Results and Discussion

For the sake of simplicity, we denote mixing time values

for water injection alone, air injection alone, and simultaneous air and water injection by T_{m1} , T_{m2} , and T_{m3} , respectively.

3.1 Effects of sensor and tracer positions on mixing time

The measured values of the mixing time for different vertical sensor positions are listed in Table 1. They are also plotted in Figs. 5 through 7 for individual water flow rate. The scatter of data points at each measurement position was indicated by an error bar. A dashed line for each experimental run denotes a mean value averaged in the axial, *i.e.*, vertical direction. The measured T_{m3} values were scattered around the

Fig. 5 Relation between mixing time and sensor position for $Q_w = 50$ cm³/s.

Fig. 6 Relation between mixing time and sensor position for $Q_w = 100 \text{ cm}^3/\text{s}$.

Fig. 8 Mixing time as a function of sensor and tracer positions for $Q_w = 150 \text{ cm}^3/\text{s}$ and $Q_g = 120 \text{ cm}^3/\text{s}$.

Fig. 7 Relation between mixing time and sensor position for $Q_w = 150 \text{ cm}^3/\text{s}$.

Fig. 9 Relation between mixing time and water flow rate for $Q_g = 0$ and $120 \text{ cm}^3/\text{s}$.

dashed line within a deviation of $\pm 25\%$, being much smaller than the scatter of data points at each measurement position. Accordingly it can be said that there exists no definite dependence of T_{m3} on H_s under the present experimental conditions.

Figure 8 shows the measured values of the mixing time for different radial sensor and tracer positions. The mixing time was independent of the radial sensor position as well as the tracer position.

3.2 Effect of water flow rate on mixing time

Measured values of the mixing time for different water flow rates are shown in Fig. 9 and Table 2. The following relation was found among T_{m1} , T_{m2} and T_{m3} .

$$1/T_{m3} = 1/T_{m2} + 1/T_{m1} \quad (1)$$

The dashed line in Fig. 9 denotes the value calculated from

eq. (1). A good agreement between the measured and calculated values can be seen. Equation (1) has the same functional relationship as that for the combined electrical resistance in the case that two electrical resistances are connected in parallel. The mixing time is regarded as a measure for the flow resistance. A long mixing time value is realized when the flow resistance is high. Therefore, eq. (1) means that the mixing by gas injection proceeds in parallel with the mixing by liquid injection. Taking pictures of the flow field in the bath was difficult because many nozzles were attached to the side wall of the vessel. According to eye inspection of the flow field, however, the flow pattern produced by bottom gas injection in the bath seems to be hardly affected by water injection from the side wall of the vessel when eq. (1) is valid.

Measured mixing time values for water injection alone decreased in inversely proportion to the water flow rate Q_w ,

Table 2 Mixing time values for different water flow rates.

Q_w (cm ³ /s)	Q_g		0 cm ³ /s	Q_g		120 cm ³ /s	Esti- mated
	mean	max	min	mean	max	min	
0	—	—	—	21.2	24.9	19.0	—
50	49.9	52.1	46.8	15.4	19.0	12.0	14.9
100	23.6	26.4	21.4	9.9	11.4	9.3	11.2
150	14.6	15.2	13.5	9.1	12.0	6.8	8.7

(unit: s)

Fig. 10 Comparison of the measured values of mixing time with eq. (2).

as shown in Fig. 10. We derived the following empirical equation considering the dependence of the bath depth, H_L , on T_{m1} as well.

$$T_{m1} = 430 H_L^{0.5} Q_w^{-1} \quad (2)$$

(5 cm $\leq H_L \leq$ 30 cm,
50 cm³/s $\leq Q_w \leq$ 150 cm³/s)

The adequacy of the relation between H_L and T_{m1} will be demonstrated in a later section.

Torii and Yang¹⁸⁾ carried out model experiments on the mixing characteristics of gas-powder stirred ladle systems with through flow to determine optimum conditions for solid-liquid mixing. Gas was injected together with solid particles into a vessel with a rectangular cross section from a nozzle attached to one of four side walls of the vessel at a constant flow rate. Water was injected and subsequently ejected in a similar manner as described in this study but it was not circulated. The mixing time was defined as a period required

for achievement of a uniform particle concentration in the bath. They determined the mixing time using an image processing technique and concluded that the contribution of liquid flow to the mixing time is very small. This result is appreciably different from the present experimental results. Such a difference might be caused by the difference between the experimental facilities used by Torii and Yang¹⁸⁾ and the present authors.

3.3 Effect of gas flow rate on mixing time

The relationship between the mixing time and air flow rate Q_g is shown in Fig. 11 and Table 3. The measured T_{m3} values decreased with an increase in Q_g . The dependence of T_{m3} on Q_g is relatively weak compared with its dependence on the water flow rate Q_w . Equation (1) can approximate the measured T_{m3} values satisfactorily.

Fig. 11 Relation between mixing time and air flow rate for $Q_w = 0$ and 100 cm³/s.

Table 3 Mixing time values for different air flow rates.

Q_g (cm ³ /s)	Q_w			Q_w			Esti- mated
	mean	max	mix	mean	max	mix	
0	—	—	—	23.6	26.4	21.4	—
60	21.3	21.8	20.8	13.3	15.3	10.9	11.2
120	21.2	23.5	18.0	10.0	11.4	9.3	11.2
180	14.4	15.0	13.8	9.9	12.4	7.6	8.9
240	13.3	14.9	12.0	8.6	9.5	7.9	8.5

(unit: s)

Fig. 12 Comparison of the measured values of mixing time with eq. (4).

Previous investigators¹⁻⁵⁾ found that the mixing time, T_{m2} , in a bath stirred by bottom gas injection alone can be correlated as a function of the specific mixing power, ε , defined by

$$\varepsilon = \rho_L g Q_g H_L / (\pi \rho_L H_L D^2 / 4) = 4gQ_g / (\pi D^2) \quad (3)$$

Nakanishi *et al.*¹⁾ investigated systematically the mixing time in a bath stirred by bottom gas injection and reported that T_{m2} is proportional to -0.4 powers of ε . Nowadays it is generally accepted that the power of ε is dependent on the mode of energy input and the vessel geometry.^{5,12,19-22)}

The same functional relationship as derived by Nakanishi *et al.*¹⁾ was valid for the presently measured values, as shown in Fig. 12. The solid line denotes an empirical correlation of T_{m2} expressed by

$$T_{m2} = 190\varepsilon^{-0.4} \quad (10 \text{ cm} \leq H_L \leq 30 \text{ cm}, \quad 10^2 \text{ cm}^2/\text{s}^3 < \varepsilon < 10^3 \text{ cm}^2/\text{s}^3) \quad (4)$$

Nakanishi *et al.*¹⁾ gave a coefficient of 240.

Torii and Yang¹⁸⁾ derived two empirical correlations having different dependence on ε , just like Asai *et al.*^{21,22)}

3.4 Effect of bath depth on mixing time

Measured values of the mixing time for different bath depths are shown in Fig. 13 and Table 4. In the case of water injection alone, T_{m1} increased in proportion to $H_L^{0.5}$. On the

Fig. 13 Relation between mixing time and bath height.

other hand, for gas injection alone, T_{m2} remained almost unchanged for $H_L \geq 10$ cm but increased significantly as H_L became smaller than 10 cm. This is because injected gas leaves out of the bath without transferring most of its energy to water when the bath is shallow. Such a phenomenon is referred here to as “blow out”. The mixing time, T_{m3} , also increased as H_L became smaller than 10 cm. According to a previous study,^{23,24)} a bath contained in a cylindrical vessel can be classified into two types: shallow and deep baths. A motion of a shallow bath is influenced by the bottom wall of the vessel in addition to the side wall of the vessel. On the other hand, a motion of a deep bath is influenced only by the side wall. Therefore, a liquid in a deep bath is much more easier to move than that in a shallow bath. In other words, agitation of a shallow bath is not easy compared to a deep bath. The aspect ratio, H_L/D , for the boundary between the two types of baths is 0.3. This critical aspect ratio is 6 cm in the present experiment as the vessel diameter, D , is 20 cm. This may be another reason why the mixing time increases for $H_L < 10$ cm.

The first kind of swirl motion took place for $H_L = 10$ cm and 15 cm when the bath was agitated by gas injection alone. The same kind of swirl motion occurred in the bath accompanied by simultaneous gas and water injection but it did not affect the mixing time just like the case of gas injection alone.^{8,9)} Figure 13 indicates that eq. (1) can

Table 4 Mixing time values for different bath depths.

H_L (cm)	Q_w 100 cm ³ /s		Q_w 0 cm ³ /s		Q_w 100 cm ³ /s		Q_w 100 cm ³ /s		Esti- mated	
	Q_g 0 cm ³ /s		Q_g 120 cm ³ /s		Q_g 120 cm ³ /s		Q_g 120 cm ³ /s			
	mean	max	min	mean	max	min	mean	max		min
5	9.2	9.7	8.8	32.3	34.8	30.4	22.7	24.1	21.9	7.2
10	13.3	15.3	11.7	15.1	17.3	13.2	9.9	10.8	8.6	7.1
15	15.6	16.4	15.2	16.6	20.0	13.2	10.7	13.8	8.7	8.0
20	16.8	17.9	16.0	17.5	18.5	17.0	10.7	11.4	10.3	8.6
30	23.6	26.4	21.4	21.2	24.9	19.0	10.0	11.4	9.3	11.2

(unit: s)

Table 5 Mixing time values for different water injection heights.

H_{ni} (cm)	Q_w		100 cm ³ /s		Q_w		0 cm ³ /s		Q_w		100 cm ³ /s		Esti- mated
	Q_g		0 cm ³ /s		Q_g		120 cm ³ /s		Q_g		120 cm ³ /s		
	mean	max	min	mean	max	min	mean	max	min				
10	26.7	27.6	26.2	21.2	24.9	19.0	10.1	10.5	9.6			11.8	
20	25.7	28.8	21.2	21.2	24.9	19.0	12.6	14.9	10.6			11.6	
30	23.6	26.4	21.4	21.2	24.9	19.0	10.0	11.4	9.3			11.2	

(unit: s)

Fig. 14 Relation between mixing time and water injection height.

reasonably approximate the measured T_{m3} values in the absence of the blow out phenomenon for $H_L > 10$ cm.

3.5 Effect of injection height of water, H_{ni} , on mixing time

Experimental results of the mixing time are shown against the nozzle height, H_{ni} , in Fig. 14 and Table 5. The measured values are in good agreement with eq. (1) for H_{ni} from 10 cm to 30 cm.

3.6 Effects of vessel diameter, D , ejection height of water, H_{ne} , and other parameters on mixing time

Effects of influential parameters such as D and H_{ne} on the mixing time, T_{m3} , will be investigated in a future study.

4. Conclusions

A fundamental investigation was carried out on the mixing time in a cylindrical bath agitated simultaneously by bottom gas injection and side liquid injection. For example, information on the mixing time is useful for developing a new batch type process for wastewater treatment. The results obtained in this study are summarized as follows:

(1) The mixing time in a bath with simultaneous air and water injection, T_{m3} , is nearly constant in the whole bath irrespective of the sensor and tracer positions.

(2) In the absence of the blow out phenomenon, the mixing time in a bath with simultaneous gas and water injection, T_{m3} ,

can be correlated by the following empirical equation.

$$1/T_{m3} = 1/T_{m1} + 1/T_{m2} \quad (1)$$

where T_{m1} and T_{m2} denote the mixing time values for water injection alone and air injection alone, respectively. This equation has the same functional relationship as that for the combined electrical resistance in the case that two electrical resistances are connected in parallel. Therefore, mixing by gas injection and that by water injection proceed in parallel with each other.

(3) The measured values of T_{m1} and T_{m2} are approximated by the following empirical equations under the present experimental conditions.

$$T_{m1} = 430H_L^{0.5}Q_w^{-1} \quad (2)$$

$$T_{m2} = 190\varepsilon^{-0.4} \quad (4)$$

where H_L is the bath depth, Q_w is the water flow rate, and ε is the specific mixing power.

Nomenclature

- D : inner diameter of vessel (cm)
- d_n : inner diameter of bottom nozzle for gas injection (cm)
- d_{nw} : inner diameter of side nozzle for water injection (cm)
- H_L : bath depth (cm)
- H_{ne} : nozzle height for water drainage (cm)
- H_{ni} : nozzle height for water injection (cm)
- H_s : height of electric conductivity sensor (cm)
- Q_g : gas flow rate (cm³/s)
- Q_w : water flow rate (cm³/s)
- r_s : radial sensor position (cm)
- r_t : radial tracer addition position (cm)
- T_m : mixing time (s)
- T_{m1} : mixing time for water injection alone (s)
- T_{m2} : mixing time for air injection alone (s)
- T_{m3} : mixing time for simultaneous water and gas injection (s)
- ε : specific mixing power (cm²/s³)

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