Determination of True Stress–Strain Curves of Sheet Metals in Post-Uniform Elongation Range*¹

Kenichi Hasegawa*², Zhong-Chun Chen*³, Keiou Nishimura*⁴ and Keisuke Ikeda

Department of Metallurgy, School of Engineering, Tohoku University, Sendai 980-8579, Japan

A novel approach to determination of the true stress–strain relation in a post-uniform elongation range by finite element modeling has been proposed, on the basis of the variation of strain-hardening exponent (n) with strain during uniaxial tensile tests. The strain dependence of n -value can be expressed by a linear or simple quadratic function. The proposed method has been applied to several sheet metals with large differences in strain-hardening behavior. The finite element analytical results, obtained by using constitutive equations derived from a variation of n -value with strain, are in good agreement with the measured load–elongation curves. As the number of parameters is sufficient by two at the most, the optimization by an inverse analysis becomes very easy. It has been verified that the proposed method is effective to estimate the stress–strain curves of sheet metals in the post-uniform elongation range. [doi:10.2320/matertrans.P-MRA2008838]

(Received April 22, 2008; Accepted September 29, 2008; Published November 19, 2008)

Keywords: tensile test, post-uniform elongation, strain dependence of n -value, constitutive equations, finite element modeling

1. Introduction

In finite element (FE) simulations of sheet metal forming with large strain deformation, such as press forming and deep drawing, it is necessary to establish appropriate constitutive equations that are applicable to both uniform and post-uniform elongation ranges. A stress–strain relation can be easily obtained by uniaxial tensile tests. A correct true stress–strain curve, however, is achieved only in the uniform elongation range. It is difficult to directly obtain a true stress–strain relation in the post-uniform elongation range from the results of tensile tests, due to occurrence of plastic instability and flow localization¹⁾ beyond the onset of necking after a maximum load.

So far, a lot of research^{2–9)} has concerned with the necking phenomenon during tensile tests. The well known Bridgman equation^{2,3)} can be used for necking correction for round tensile specimens, which is based on a dimensional change in the necking section after diffuse necking starts. This correction method, however, requires continuous measurements of the diameter and curvature radius of the neck during the tests. Needleman⁶⁾ and Le Roy *et al.*⁷⁾ investigated the necking of axisymmetric specimens by FE analysis. Recently, Zhang *et al.*,^{10,11)} Scheider *et al.*,¹²⁾ and Bacha *et al.*¹³⁾ proposed several methods to determine true stress–strain curves from tensile specimens with a rectangular cross section. Nevertheless, practically, a simple extrapolation method is often used to describe the stress–strain relation in the post-uniform elongation range, where the stress–strain curve prior to necking is expressed by the power-hardening law or multinomial. Obviously, the resultant stress–strain curve is not guaranteed to be correct, and there may be a large discrepancy between the experimental and analytical results. Furthermore, in some case, it is difficult to estimate the

stress–strain relation by the extrapolation method. For example, two tempered Cu alloys, Cu-Ni-Si and Cu-Sn-P, show highly different deformation behavior¹⁴⁾ during tensile tests. The former exhibits large uniform and small post-uniform elongations, while the latter (phosphor bronze) has a relatively large post-uniform elongation. Although both have almost the same total elongation values, in the case of Cu-Sn-P alloy, the majority of the total elongation consists of the post-uniform elongation. This difference in deformation behavior cannot be estimated through the extrapolation by the power-hardening law. To improve the accuracy in analyzing sheet forming, it is essential to correctly describe a true stress–strain relation in the post-uniform elongation range.

With the help of FEM, it is possible to determine constitutive equations through an inverse analysis which makes a load–elongation or load–nominal strain curve to agree with measured data. Some researchers^{15,16)} have attempted to give a unified expression for stress–strain relations of various materials under different loading conditions. It is not easy to reflect the influence of material factors, even in the case of uniaxial tensile tests at room temperature. In order to express constitutive equations accurately, one has to increase the number of parameters and, of course, take a long time for optimization. From the viewpoint of practical use, it is expected to establish constitutive equations with less optimization parameters.

For annealed metals, the strain-hardening behavior is commonly approximated by a power-law relationship with a constant strain-hardening exponent (n):

$$\sigma = K\varepsilon^n \quad (1)$$

where σ is the true stress, ε is the true strain, and K is the strength coefficient. If n -value is considered as a variable depending on strain, various stress–strain relations may be described. In fact, the strain-hardening exponent of a metal is of strain dependence to some extent. A lot of metals show continuously varying values of strain-hardening exponent during plastic deformation.^{17–19)} It has been found that some metals exhibit monotonous variations of n -value with strain, such as linear or parabolic shape.²⁰⁾ The variation of n -value

*¹This Paper was Originally Published in Japanese in J. JSTP **49** (2008) 143–147.

*²Graduate Student, Tohoku University. Present address: Dentsu Inc., Tokyo, Japan

*³Corresponding author, E-mail: czc@material.tohoku.ac.jp

*⁴Present address: Visual Technology, Inc., Tokyo, Japan

can be easily calculated from measured tensile load–elongation curves. In addition, a relation of $\varepsilon = n$ is valid at the maximum load, even if n -value varies with deformation. In this way, uniform and post-uniform elongation ranges can be clearly distinguished from each other. Accordingly, if the variation of n -value with strain is approximated by simple functions, various constitutive equations may be expressed with less parameters and, hence, the optimization becomes easy.

In the current work, we propose a method to determine the true stress–strain relations in the post-uniform elongation range through an inverse analysis by FEM, on the basis of measured change of n -value with strain in the uniform elongation range. Besides, the validity of the proposed approach is verified by applying it to several representative sheet metals.

2. Strain Dependence of n -Value and Its Relation with Constitutive Equations

Assuming that n -value varies with strain, the relation between strain-hardening coefficient ($d\sigma/d\varepsilon$) and n -value can be expressed as

$$\frac{1}{\sigma} \frac{d\sigma}{d\varepsilon} = \frac{n(\varepsilon)}{\varepsilon} \quad (2)$$

Equation (2) is originally derived from the power-hardening law (eq. (1)) in which n -value remains unchanged during the deformation. The variation of n -value with strain is generally calculated by considering both strength coefficient K and n -value as constants in an infinitesimal strain range. Hence, the same relation is effective even in the case of a variation of n -value.

If $n(\varepsilon)$ varies with strain in a simple proportional relation given by

$$n(\varepsilon) = \varepsilon_h + a_1(\varepsilon - \varepsilon_h) \quad (3a)$$

where ε_h is the uniform strain and a_1 is a parameter. Substituting eq. (3a) into eq. (2) and then integrating it, the stress–strain relation can be written as

$$\sigma = \sigma_h \cdot \exp[a_1(\varepsilon - \varepsilon_h)] \cdot \left(\frac{\varepsilon}{\varepsilon_h}\right)^{(1-a_1)\varepsilon_h} \quad (3b)$$

where σ_h is the stress at the uniform strain, *i.e.*, stress corresponding to the maximum load. If $a_1 = 0$, the above equations are equivalent to the power-hardening law, whereas if $a_1 = 1$, $n(\varepsilon) = \varepsilon$ continues, no necking growth occurs and the deformation takes place under a constant tensile load.

Similarly, supposing that $n(\varepsilon)$ varies in a gradual parabolic mode, $n(\varepsilon)$ and σ can be expressed as

$$n(\varepsilon) = \frac{a_p(\varepsilon - \varepsilon_h) + \varepsilon_h}{1 - \varepsilon_h + \varepsilon} \quad (4a)$$

$$\sigma = \sigma_h \cdot (1 - \varepsilon_h + \varepsilon)^{\frac{a_p - \varepsilon_h}{1 - \varepsilon_h}} \cdot \left(\frac{\varepsilon}{\varepsilon_h}\right)^{\frac{(1-a_p)\varepsilon_h}{1 - \varepsilon_h}} \quad (4b)$$

where a_p is a parameter. Equations (4a) and (4b) are consistent with the power-hardening law when $a_p = \varepsilon_h$. If $a_p = 1$, linear hardening behavior is shown, that is, the stress is proportional to the strain.

In fact, $n(\varepsilon)$ does not always follow eq. (3a) or eq. (4a) which contains only one parameter. If $n(\varepsilon)$ is expressed as a quadratic equation of the strain, the following equations can be written.

$$n(\varepsilon) = \alpha - \beta(\gamma - \varepsilon)^2 \quad (5a)$$

$$\sigma = \sigma_h \cdot \exp[2\beta\gamma(\varepsilon - \varepsilon_h)] \times \exp\left[\frac{\beta}{2}(\varepsilon_h^2 - \varepsilon^2)\right] \cdot \left(\frac{\varepsilon}{\varepsilon_h}\right)^{(1-2\beta\gamma+\beta\varepsilon_h)\varepsilon_h} \quad (5b)$$

In eq. (5a), there are three parameters, α , β , and γ . Since $\varepsilon_h = n(\varepsilon_h)$ at the maximum load, α in eq. (5a) can be removed. Thus, two parameters, β and γ , remain, which can be used to represent various variations of $n(\varepsilon)$ with strain.

If $n(\varepsilon)$ in a uniform elongation range is known, to a first approximation, it is supposed that the same variation trend continues in the subsequent post-uniform elongation range. As a consequence, the initial value of the parameter a_1 or a_p in eq. (3) or eq. (4) can be determined immediately. Because $n(\varepsilon)$ does not always keep the same relation with strain during the deformation up to fracture, it is expected that the accuracy of a load–elongation curve, predicted by FEM, may turn worse from some point within the post-uniform elongation range. In that case, the parameters can be optimized by replacing eq. (3) or eq. (4) by eq. (5) completely, or changing to eq. (5) around the strain at which the accuracy starts to become worse. Moreover, there exist some materials whose uniform strain is extremely small and the variation tendency of $n(\varepsilon)$ is unknown. Some examples are given in Section 4.

3. Experimental and Analytical Methods

Four kinds of sheet metals with large differences in strain-hardening behavior, annealed high-tensile strength steel (HTSS), pre-strained HTSS, annealed α -brass (C2600), and tempered phosphor bronze (C5191), were used in this investigation. The dimensions and tensile elongation values of the specimens used in the current work are summarized in Table 1. A 4% elongation pre-strain was added to some annealed HTSS sheets along the rolling direction (RD) by uniaxial tensile tests. The tensile tests were conducted with a constant crosshead speed of 10 mm/min. An extensometer with a gage length of 50 mm was used to measure the displacement during tensile tests.

The stress–strain curves were derived from load–elongation curves which were corrected to take into account the elastic deformation of the testing machine. The strain-hardening exponent, n , was calculated by using the strain, stress, and strain-hardening coefficient at each strain via eq. (2). The strain-hardening coefficient was obtained by approximating the practical stress–strain curves in the uniform elongation range using a multinomial expression and then making a least-squares treatment.

A commercially available FE code MARC was used for analysis in the current work. It was known from the preliminary analysis that there was almost no difference in solutions between solid and plane elements. Consequently, plane elements were used for saving calculation time, and two-dimensional rigid-plastic analysis was carried out. Considering the symmetry of sheet specimens, 1/4 of

Table 1 Dimensions and tensile elongation values of specimens.

Specimen	Dimensions L × W × T (mm)	Tensile direction	Uniform elongation (%)	Post-uniform elongation (%)	Total elongation (%)
Annealed HTSS	120 × 12.5 × 1.6	TD	19.6	10.0	29.6
Pre-strained HTSS	120 × 12.5 × 1.6	TD	14.5	11.8	26.3
Annealed α -brass	190 × 25 × 0.29	RD	36.3	8.1	44.4
Tempered phosphor bronze	190 × 25 × 0.2	RD	5.7	9.4	15.1
		TD	2.2	11.1	13.3

Fig. 1 Tensile load–nominal strain curves for annealed high-tensile strength steel sheets.

parallel region was used as an analysis model. In addition, an initial imperfection was given by making the width in the central section to be 0.5% smaller than that in the parallel region to ensure occurrence of necking at the central section. The parameter values estimated from the variation of $n(\epsilon)$ in the uniform elongation range were regarded as the initial values. The optimal constitutive equations were determined by modifying the parameter values until the differences between the FE analytical results and measured load–nominal strain data were less than 0.5%.

4. Results and Discussion

The representative results by applying the proposed method to annealed and pre-strained HTSS, annealed α -brass, and tempered phosphor-bronze sheets are described as follows.

4.1 Annealed HTSS sheets

The steel sheet used in this work was one of the HTSS sheets with a uniform elongation of 19.6% and total elongation of 29.6% (Table 1). The $n(\epsilon)$ in the uniform elongation range showed a slight increase trend except for the initial period of deformation, and it was well approximated using a parabolic mode with $a_p = 0.20$ (eq. (4a)). Since the change of $n(\epsilon)$ was very small, the power-hardening law with $n = 0.18$ exhibited a similar stress–strain curve. Figure 1

Fig. 2 Stress–strain curves and variation of n -value with strain for annealed high-tensile strength steel sheets. The arrow indicates the position of the maximum load.

shows a comparison of the tensile load–nominal strain curves between the FE analytical results and experimental data. The result based on the power-hardening law is also given in the figure. The beginning of the curve by eq. (4) was in good agreement with the experimental curve, but the analytical values were higher than the measured ones from the position where the load rapidly decreased. By using eq. (5) and making an optimization treatment, the analytical and experimental results were extremely consistent with each other within the whole strain range as shown in Fig. 1. The values of the optimized parameters were $\beta = 2.51$ and $\gamma = 0.22$.

Figure 2 shows the true stress–strain curves and variation of $n(\epsilon)$ with strain. The values in the uniform elongation range, plotted in the figure, were achieved directly from the measured load–nominal strain data. After the diffuse necking starts, at first almost no change of $n(\epsilon)$ was observed. Subsequently, because of the decrease in $n(\epsilon)$ with increasing the strain, the stress–strain curve, obtained from the power-hardening law, was overestimated. However, the difference was not so large, and a quite good approximation could be achieved even using the power-hardening law. In addition to HTSS sheets, other annealed metals with large n -values also exhibit good approximations of stress–strain curves when the power-hardening law is used (see Section 4.3). On the other hand, it was observed from the stress–strain curves shown in Fig. 2 that the stress increase due to strain-hardening became gentle in the post-uniform elongation range, although a large

Fig. 3 Tensile load–nominal strain curves for pre-strained (4%) high-tensile strength steel sheets.

strain-hardening feature was shown in the uniform elongation range. This is attributed to the decrease of n -value with strain after necking.

4.2 Pre-strained HTSS sheets

The annealed HTSS sheets described above were uniaxially pulled in rolling direction (RD) to incorporate a 4% pre-strain, followed by tensile test in transverse direction (TD). Because of the incorporation of the pre-strain, the uniform and total elongation values were reduced to 14.5 and 26.3%, respectively, but the post-uniform elongation was somewhat increased (Table 1). The $n(\epsilon)$ showed a minimum value in the initial period of deformation, and then increased with strain parallel to the straight line of $n = \epsilon$ (see Fig. 4). Nevertheless, similar to the annealed HTSS sheets, the pre-strained specimens can be approximated by a parabolic change of $n(\epsilon)$ with $a_p = 0.65$, except for the early period of deformation. Figure 3 shows the tensile load–nominal strain curves obtained from FE analysis and experiments. The analytical results by a parabolic approximation (eq. (4)) were over-estimated in the latter part of the curve. When the variation of $n(\epsilon)$ was expressed by a quadratic equation (eq. (5)), the analytical results were in good agreement with the experimental data.

Figure 4 illustrates the stress–strain curves and variation of $n(\epsilon)$, when modified constitutive equations with a good approximation were used. The $n(\epsilon)$ increased with increasing the strain and gradually closed to the value of the annealed specimen. After passing through a peak, the $n(\epsilon)$ showed a variation trend similar to the annealed specimen (Fig. 2). Corresponding to the change of $n(\epsilon)$, the stress rose linearly as the strain increased, and gradually closed to the stress value of the annealed specimen. The strain at which $n(\epsilon)$ reached a maximum value ($\epsilon = 0.295$) was larger than that of the annealed specimen, thus leading to a larger post-uniform elongation. Due to a large change of $n(\epsilon)$, it is clear that the power-hardening law is not applicable to express the stress–strain relation of pre-strained HTSS sheets in the post-uniform elongation range. If the n -value in the uniform elongation range is used, the flow stress is necessarily underestimated, as shown in the dotted curve in Fig. 4.

Fig. 4 Stress–strain curve and variation of n -value with strain for pre-strained high-tensile strength steel sheets. The arrow indicates the position of the maximum load.

Fig. 5 Tensile load–nominal strain curves for annealed α -brass (C2600) sheets. Tensile test was carried out in rolling direction.

4.3 Annealed α -brass sheets

As another example with large n -values, annealed α -brass sheets were examined. With regard to the materials with large n -values, it is necessary to describe the variation slope of $n(\epsilon)$ up to the maximum load. α -brass showed serrations in the stress–strain curve until the maximum load, and thus there existed large scatters of $n(\epsilon)$ values (see Fig. 6). On the whole, however, it seemed that $n(\epsilon)$ showed a decrease trend (represented by a thin line in Fig. 6) up to the maximum load as the strain increased. The constitutive equations can be derived by using the variation slope of $n(\epsilon)$, and the FE analytical results are shown in Fig. 5. When $a_l = -0.45$ in eq. (3), the analytical results coincided with the experimental values up to around 2/3 of the post-uniform elongation. Subsequently, the difference between the analytical and experimental results became large with increasing the strain. Similarly, no good agreement was found under the conditions of $a_p = -0.45$ by eq. (4). To improve the analysis accuracy, it is assumed that $n(\epsilon)$ varies in a quadratic curve. When the resulting constitutive equations were used, the analytical results were in good agreement with the measured values as shown in Fig. 5. In this case, the corresponding values of β

Fig. 6 Stress-strain curves and variation of n -value with strain for annealed α -brass (C2600) sheets. Tensile test was carried out in rolling direction. The arrow indicates the position of the maximum load.

and γ in eq. (5) were 5.12 and 0.27, respectively. Besides, the tensile load–nominal strain curve with a constant n -value was also shown in the figure, and the load values were over-estimated.

Figure 6 shows the stress–strain curves and variation of $n(\varepsilon)$ of annealed α -brass sheets, when modified constitutive equations showing a good approximation of load–nominal strain curves were used. The $n(\varepsilon)$ values in the uniform elongation range were relatively large. It is believed that a smaller post-uniform elongation in annealed α -brass sheets is the result of a large reduction in $n(\varepsilon)$ with strain. In comparison with two stress–strain curves obtained from the power-hardening law and modified equation using eq. (5), the difference was very small. In other words, the power-hardening law can give a moderate approximation for those materials with large n -values. This is similar to the results of the annealed HTSS sheets described in Section 4.1.

4.4 Tempered phosphor-bronze alloy sheets

Tempered phosphor-bronze alloy sheets have a feature of small uniform elongation and large post-uniform elongation as mentioned previously. The values of the uniform and total elongations in RD were 5.7 and 15.1%, respectively, while those in TD were 2.2 and 13.3%, respectively (Table 1). During the tensile tests in TD, the $n(\varepsilon)$ was rapidly reduced to an extremely low value in the uniform elongation range (see Fig. 9). Due to a large post-uniform elongation, it is expected that $n(\varepsilon)$ turns to increase beyond the onset of necking, but the variation trend of $n(\varepsilon)$ is unclear. As a consequence, the variation of $n(\varepsilon)$ of tempered phosphor-bronze alloy sheets after the maximum load cannot be described by the variation slope of $n(\varepsilon)$ in the uniform elongation range.

On the basis of a strain distribution in the tensile direction when fractured, the authors²¹⁾ have derived simplified equations for estimating the true stress and true strain in a post-uniform elongation range as follows:

$$\sigma_i = P \left[\frac{1}{A_h} + \frac{E_i}{A_0} \frac{\alpha}{1 - \exp(-\alpha)} \right] \quad (6a)$$

$$\varepsilon_i = \ln \left[\frac{A_0}{A_h} + \frac{E_i \alpha}{1 - \exp(-\alpha)} \right] \quad (6b)$$

Fig. 7 Stress-strain curves for tempered phosphor-bronze alloy (C5191) sheets.

Fig. 8 Tensile load–nominal strain curves for tempered phosphor-bronze alloy (C5191) sheets. Tensile test was carried out in transverse direction.

where σ_i is the true stress, ε_i the true strain, P the tensile load, A_0 the initial cross-sectional area of a specimen, A_h the minimum cross-sectional area in a uniform elongation range, i.e., the area corresponding to the maximum load, E_i the nominal post-uniform strain, and α the parameter representing the degree of strain localization. Equations (6a) and (6b) are obtained by calculating the minimum cross-sectional area after necking. Because the shape effect of a necking section is not taken into consideration, eq. (6) causes an overestimation of the stress. Nevertheless, a reasonable stress–strain curve can be achieved in a short period after the necking starts. To a first approximation, the parameter a_1 in eq. (3) can be determined to make the results obtained by eq. (6) to agree with eq. (3). If the estimated results are not consistent with the measured load–nominal strain data, they can be modified by eq. (4) or eq. (5). A comparison of some stress–strain curves is given in Fig. 7. The dotted line in Fig. 7 is the stress–strain curve predicted by eq. (6), whereas the solid line corresponds to the results from eq. (3), and $a_1 = 0.55$ gives a good approximation. It is seen from Fig. 8 that the tensile load–nominal strain data obtained by FEM are in good accordance with the experimental results. The determined stress–strain curve and variation of $n(\varepsilon)$ are shown in Fig. 9.

Fig. 9 Stress–strain curves and variation of n -value with strain for tempered phosphor-bronze alloy (C5191) sheets. Tensile test was carried out in transverse direction. The arrow indicates the position of the maximum load.

Fig. 11 Stress–strain curve and variation of n -value with strain for tempered phosphor-bronze alloy (C5191) sheets. Tensile test was carried out in rolling direction. The arrow indicates the position of the maximum load.

Fig. 10 Tensile load–nominal strain curves for tempered phosphor-bronze alloy (C5191) sheets. Tensile test was carried out in rolling direction.

As a result of the increase in $n(\varepsilon)$ immediately after the uniform elongation, the stress increased with increasing the strain almost linearly. The tensile stress values, estimated by the power-hardening law using the n -value at the maximum load, are much lower than the experimental ones. This is one of the typical examples indicating that the stress–strain relation in a post-uniform elongation range cannot be approximated by the power-hardening law.

With regard to the tensile tests in RD, the load–nominal strain and stress–strain curves are shown in Figs. 10 and 11, respectively. Although the uniform elongation value was small, the $n(\varepsilon)$ revealed a change tendency from decrease to increase with strain immediately over a straight line of $n = \varepsilon$. The load–nominal strain curve, predicted from a parabolic approximation with $a_p = 0.60$, was consistent with the experimental curve. In comparison with Fig. 8 and Fig. 10, the latter looks like somewhat larger differences between the predicted and experimental results. This is because the load axis in Fig. 10 was magnified with respect to that in Fig. 8. The error was actually less than 0.3%. Moreover, in the case of the tensile tests in RD, the variation behavior of n -value was similar to that in TD, except that the uniform elongation

in RD was slightly larger because of the reversal of n -value over the straight line of $n = \varepsilon$.

When the tempered phosphor-bronze alloy sheets were pulled in RD and TD, although there existed some differences in variation of $n(\varepsilon)$ (either parabolic or straight line), a similar change trend of $n(\varepsilon)$ from decrease to increase has been observed. However, the strain amount where $n(\varepsilon)$ reached the minimum value ($n_{\min}$) was different, $n_{\min} > \varepsilon$ in RD and $n_{\min} < \varepsilon$ in TD. This may result in the difference in the uniform elongation. It is well known^{22,23} that the post-uniform elongation also depends on strain-rate sensitivity (m) in addition to n -value. The m -value of the tempered phosphor-bronze alloy sheets at room temperature is so small that it can be ignored. In fact, no difference in necking growth has been found even though the influence of m -value was taken into consideration in the FE analysis. Large values of the post-uniform elongation in both RD and TD are likely to result from a continuous increase of $n(\varepsilon)$, which restrains the growth of necking.

From the above results, if a sheet specimen has a larger uniform elongation, just as annealed α -brass, annealed and pre-strained HTSS specimens as mentioned above, the constitutive equations with an initial parameter value of a_1 or a_p in eq. (3) or eq. (4) can be obtained by using the variation slope of $n(\varepsilon)$ in the uniform elongation range. If necessary, the analysis accuracy can be further improved by using a quadratic mode (eq. (5)). As for the materials with a small uniform strain or their variation of $n(\varepsilon)$ is unknown, for example, tempered phosphor-bronze alloy sheets described above, the variation of $n(\varepsilon)$ immediately after the start of necking can be estimated from the stress–strain relation calculated by eq. (6). In this way, the constitutive equations are determined to make them to agree with the estimated variation of $n(\varepsilon)$. Because almost no necking growth occurs immediately after the maximum load, the estimated slope from calculated stress–strain relation is believed to be reliable. The subsequent optimization can be done in a way similar to the materials with a larger uniform elongation. These suggest that the proposed approach in the current work can be applied to various materials with different strain-hardening behavior.

The above results indicate that the differences in tensile deformation behavior in annealed, tempered, and pre-strained materials are reflected by variations of $n(\varepsilon)$ distinctly. It should be pointed out that the effects of m and r (a plastic anisotropy parameter) values on a post-uniform elongation are not considered in this FE analysis. The influences of m -value, r -value, and even microstructure of materials, however, if there exist, must be reflected in measured tensile load–nominal strain curves. Accordingly, it is reasonable to consider that the effects of various factors on a stress–strain relation are included in an optimized $n(\varepsilon)$. It is important to quantitatively describe the effects of above-mentioned factors on both magnitude of flow stress and difference in strain-hardening behavior in the post-uniform elongation range.

5. Conclusions

An approach to estimation of the true stress–strain relation in a post-uniform elongation range through an inverse analysis by FEM has been proposed. The analysis is based on the dependence of n -value on strain during uniaxial tensile tests. The proposed method has been applied to several sheet metals with different strain-hardening behavior. The results show that the strain dependence of n -value can be expressed by a linear or simple quadratic function for various materials (such as annealed, tempered, and pre-strained metals) with large differences in strain-hardening behavior. If there is a larger uniform elongation, the constitutive equations with a good approximation can be obtained by using the variation slope of $n(\varepsilon)$ in the uniform elongation range. With regard to the materials with a small uniform strain or their variation of $n(\varepsilon)$ is unknown, the variation of $n(\varepsilon)$ immediately after the start of necking can be estimated from the stress–strain relation calculated by the derived equations.

FE solutions can be obtained by using constitutive equations derived from a variation of n -value with strain. It has been shown that the FE analytical results are in good agreement with the measured load–nominal strain curves. The proposed method is simple with only one or two parameters, and can be applied to various metals with

different strain-hardening behavior. Moreover, the power-hardening law with a constant n -value gives a moderate approximation for annealed metals with large n -values, but large errors occur when it is used to deal with pre-strained and tempered metals.

REFERENCES

- 1) S. L. Semiatin and J. J. Jonas: *Formability and Workability of Metals*, (ASM, Ohio, 1984).
- 2) P. W. Bridgman: *Trans. Am. Soc. Metals* **32** (1944) 553–573.
- 3) P. W. Bridgman: *Studies in Large Plastic Flow and Fracture*, (McGraw-Hill, New York, 1952).
- 4) N. N. Davidenkov and N. I. Spiridonova: *Proc. ASTM* **46** (1946) 1147–1158.
- 5) E. R. Marshall and M. C. Shaw: *Trans. Am. Soc. Metals* **44** (1952) 705–721.
- 6) A. Needleman: *J. Mech. Phys. Solids* **20** (1972) 111–127.
- 7) G. Le Roy, J. D. Embury, G. Edwards and M. F. Ashby: *Acta Metall.* **29** (1981) 1509–1522.
- 8) V. Tvergaard and A. Needleman: *Acta Metall.* **32** (1984) 157–169.
- 9) C. García-Garino, F. Gabaldón and J. M. Goicolea: *Finite Elem. Anal. Des.* **42** (2006) 1187–1197.
- 10) Z. L. Zhang, M. Hauge, J. Ødegård and C. Thaulow: *Int. J. Solids Struct.* **36** (1999) 3497–3516.
- 11) Z. L. Zhang, J. Ødegård, O. P. Sjøvik and C. Thaulow: *Int. J. Solids Struct.* **38** (2001) 4489–4505.
- 12) I. Scheider, W. Brocks and A. Cornec: *J. Eng. Mater. Technol.* **126** (2004) 70–76.
- 13) A. Bacha, D. Daniel and H. Klocker: *J. Mater. Process. Technol.* **184** (2007) 272–287.
- 14) K. Hatakeyama: Ph.D. thesis (Tohoku University, 2003).
- 15) M. Gotoh: *J. Japan Soc. Technol. Plast.* **43** (2002) 270–277.
- 16) M. Kuroda and K. Shizawa: *J. Japan Soc. Technol. Plast.* **43** (2002) 299–309.
- 17) K. Saka, M. J. Painter and R. Pearce: *J. Mech. Work. Technol.* **3** (1979) 17–30.
- 18) M. De Meyer, D. Vanderschueren and B. C. De Cooman: *ISIJ Int.* **39** (1999) 813–822.
- 19) L. Gu, A. Tanaka, K. Tanaka and K. Nakajima: *Tetsu-to-Hagane* **88** (2002) 88–94.
- 20) K. Hatakeyama, Y. Yamashita and K. Ikeda: *J. JRICu* **41** (2002) 241–245.
- 21) K. Hasegawa, K. Nishimura, Z. Chen and K. Ikeda: *Proc. 55th Jpn. Conf. Technol. Plast.* (2004) 275–276.
- 22) E. W. Hart: *Acta Met.* **15** (1967) 351–355.
- 23) A. K. Ghosh: *J. Eng. Mater. Technol.* **99** (1977) 264–274.