Temperature Distribution Estimated by Optimization and Near-Field Acoustical Holography

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We propose a method to estimate temperature distribution non-destructively using combination of near-field acoustical holography (NAH) method and optimization method. NAH method is valid to calculate forward and back propagations of ultrasonic wave in a uniform medium. To estimate temperature distribution, we proposed a modified method of NAH, called sectional NAH (SNAH), for using NAH in an inhomogeneous medium. In SNAH, a calculation space is discretized into a number of sections, so that the temperature in each section becomes nearly uniform, and NAH calculation is performed in each section. Calculation results using SNAH in the inhomogeneous medium well agreed with the calculation result using finite element method (FEM). By using SNAH, the temperature distribution is estimated as bellows. Firstly, sound fields, which are complex amplitude of harmonic oscillation, in three planes are measured. Next, sound field at one of measured plane is calculated from the other measured sound fields, which are calculated from other measurement planes using SNAH, with an initial sound velocity distribution. Then, difference between calculated and measured sound fields is minimized by optimizing the sound velocity distribution using multi-start downhill simplex method. In this method, a temperature distribution is obtained as the sound velocity distribution. In simulations, temperature distributions given as linear and Gaussian distributions were estimated. Validity of our proposal methods is confirmed by simulations. [doi:10.2320/matertrans.I-M2011850]

(Received November 12, 2009; Accepted March 11, 2011; Published January 18, 2012)

Keywords: non-destructive measurement, optimization, near-field acoustical holography, multi-start downhill simplex method

1. Introduction

Temperature is an important parameter in various fields. Currently, a major method for measuring temperature is a direct measurement using contact type sensors, such as a thermocouple. However, the physical probe needs to be put in the temperature distribution, and the physical probe affects the temperature distribution. In contrast, non-contact measurement methods, which are called acoustical tomography method¹ or ultrasonic diffraction tomography method,⁵ can obtain the temperature non-destructively from a sound velocity distribution which can be obtained without any physical contacts. The sound velocity distribution is reconstructed on the basis of propagation times on a number of sound paths. However, these tomography methods have problems. When the acoustical tomography method is used, affection of diffraction is ignored. Thus a sound wave assumed to go straight. When the ultrasonic diffraction tomography method is used, many measurement points are required to reconstruct the temperature distribution. In addition, an ideal sound source, which radiates a plane sound wave, is required.

To solve these problems, we propose a method for estimating a temperature distribution by combining two methods; calculation of wave propagation method and optimization method. The calculation method of wave propagation is applied to obtain a propagation function, which contains information on a medium. In the calculation method of wave propagation, near-field acoustical holography (NAH)³ is used. Advantages of using NAH are; any sound source can be used for NAH and number of measurement point is decreased compared with ultrasonic diffraction tomography method. Here, if the medium is uniform, the temperature is calculated by a theoretical propagation function of NAH.⁴ In the medium has temperature distribution, it is difficult to obtain theoretical propagation function. To overcome this problem, applied NAH is proposed. For estimating temperature distribution, NAH needs to be applied to calculate ultrasonic wave propagation in the inhomogeneous medium.

In this paper, we propose a method to estimate a water temperature distribution by measuring two-dimensional sound fields, which are complex amplitude of sound pressures eliminated time component of harmonic oscillation. The temperature distribution is estimated by a combination of applied NAH method and optimization method, such as multi-start downhill simplex method.⁵ The temperature distribution is confined to vary only one-axis in this fundamental research.

2. Principles of Temperature Estimation

We consider a coordinate system, as shown in Fig. 1. The origin is located at the center of an ultrasonic transducer, and it radiates ultrasonic wave in direction of z-axis. Estimation of a temperature distribution, which varied only z-axis, is conducted as following steps.

Firstly, sound fields at $z = z_1$ and $z_2$ are measured, for example, using optical computerized tomography.⁵ Then, sound field at $z = z_2$ is calculated from the measured sound field at $z = z_1$ using applied NAH with an initially set sound velocity distribution. Difference between calculated and measured sound fields at $z = z_2$ is minimized by optimizing the sound velocity distribution. The optimal solution is assumed to be an actual sound velocity distribution. Finally, a temperature distribution is calculated from the estimated sound velocity distribution.
2.1 Calculation of sound field

Here, two-dimensional sound fields are measured using a microphone or an optical probe which obtains sound pressure distribution non-destructively using an interferometer and computerized tomography method.6) Then, we consider that sound wave propagation in an inhomogeneous medium is calculated. As shown in Fig. 2, the origin is located at a center of an ultrasonic transducer, and an ultrasonic wave is radiated in the direction of z-axis. In this paper, the transducer is circular, and a temperature distribution is uniform in x-y plane. Thus, the coordinate system is z-axis. Here, two-dimensional sound field at \( z = z_1 \) is expressed as \( p(x, y, z_1) \), and the sound field in wave number domain is expressed as \( P(k_x, k_y, z_1) \). A relationship between the sound fields at \( z = z_1 \) and \( z = z_2 \) is given by

\[
p(x, y, z_2) = p(x, y, z_1) * h(x, y, z_2 - z_1),
\]
\[
P(k_x, k_y, z_2) = P(k_x, k_y, z_1)H(k_x, k_y, z_2 - z_1),
\]

where \( * \), \( h \), and \( H \) denote convolution integral, a propagation function, and the propagation function in wavenumber domain, respectively. The sound field at \( z = z_2 \) is calculated by the theoretical propagation function using NAH.3) Here, \( H \) is expressed as

\[
H(k_x, k_y, z_2 - z_1) = \exp\{j k_x(z_2 - z_1)\},
\]

\[
k_z = \sqrt{k_x^2 - k_y^2 - k_y^2}.
\]

where \( k \) is wavenumber, which must be constant in ordinary NAH theory.

We assume that the temperature only varies in z-axis. Then, a space between \( z = z_1 \) and \( z = z_2 \) is discretized into a number of sections, as shown in Fig. 2, so that the temperature in each section can be approximately assumed as uniform. We call this method as sectional NAH (SNAH). A propagation function in SNAH is expressed as

\[
H(k_x, k_y, z_2 - z_1) = \prod_n \exp\{j k_z n d_z\},
\]

\[
k_{zn} = \sqrt{k_x^2 - k_y^2 - k_y^2}/C_0.
\]

where, \( k_{zn} \) and \( d_z \) are a wavenumber in the n-th section and a step size of discretization in z axis, respectively. The propagation function is given by a product of propagation functions in all sections.

In this paper, we consider the case that the variation of temperature is small, so influence of reflection and refraction caused by acoustic impedance is ignored. Also, influence of attenuation caused by a density distribution is ignored, because variation of density caused by a temperature distribution in a liquid is small.

2.2 Estimation of temperature by optimization

Heretofore, NAH method is used for calculation of wave propagation. On the other hand, we proposed that inverse analysis of NAH is applied for determination of a propagation function and a sound velocity.5) However, only constant sound velocity in calculation area is obtained by inverse analysis of NAH.

Here, we propose a method using an optimization for estimating a temperature distribution. A coordinate system is shown in Fig. 1. Firstly, two-dimensional sound fields at \( z = z_1 \) and \( z = z_2 \), \( p_{\text{meas}}(z_1) \) and \( p_{\text{meas}}(z_2) \), are measured for estimation of the temperature distribution in a range from \( z = z_1 \) to \( z = z_2 \). Then, initial sound velocities of some representative points in the measurement area are set randomly, and the initial sound velocities in all section are obtained by an interpolation. Based on the initial sound velocity distribution, sound field at \( z = z_2 \), \( p_s(z_2) \), is calculated from the measured sound field \( p_{\text{meas}}(z_1) \) using SNAH. If the sound velocity distribution is equal to an actual sound velocity distribution, the sound field \( p_s(z_2) \), which is calculated using SNAH, is identical to the measured sound field \( p_{\text{meas}}(z_2) \). Thus, the actual sound velocity distribution is optimized to minimize an evaluation function, which is difference of the calculated and measured sound fields. Multi-start downhill simplex method,5) which is a kind of global search, is employed to optimize without falling into local solutions.
In fact, it is necessary to measure the sound fields at \( z = z_1, z_2 \) and \( z_3 \), because an optimized solution do not converge with two measured sound fields. The propagation function in SNAH is a linear function and carried by commutation rules. So, the actual sound velocity distribution and its symmetric distribution are the same propagation function. To solve the problem, the sound fields at \( z = z_1 \) are calculated from \( z = z_1 \) and \( z_2 \), and the sound field at \( z = z_2 \) is calculated from \( z = z_3 \). An evaluation function for the optimization is sum of difference of sound fields in real part. Optimization coefficients are representative values of sound velocity distribution, and the sound velocity distribution is interpolated for calculating the sound fields using SNAH.

Finally, the temperature is obtained by the estimated sound velocity distribution. We use a relationship between a water temperature \( T \) and sound velocity \( c \), given by

\[
c = 1557 - 0.0245(74 - T)^2.
\]

This equation is good agreement with experimental data in large range of water temperature.\(^7\)

The temperature distribution is estimated by the radiating ultrasonic wave and using proposed method. However, a pulse ultrasonic wave is not adequate for proposed method because of that the temperature distribution is obtained by inverse analysis of an analysis of harmonic vibration. In addition, for an experimental measurement of three-dimensional temperature distribution, a method using three-dimensional spatial discretization is required to be considered.

3. Simulation

3.1 Confirmation of sectional NAH

We validate these methods by simulations. Validity of SNAH is confirmed by finite element method (FEM).\(^8\) In the simulation, a continuous ultrasonic wave is radiated into water, whose density is 1000 kg/m\(^3\). A transducer is circular and driven at 5.0 MHz. Diameter of the transducer is 5.0 mm. Temperature has a Gaussian distribution curve in \( z \)-axis. Calculation space is modeled using FEM as an analysis of harmonic vibration in two-dimensional \( z \)-axial symmetry. Size of calculation area is \( x \times z = 30 \times 20 \text{mm}^2 \). The calculation space is discretized into 60,000 quadrilateral elements. Input signal into the transducer is given by an acceleration, whose amplitude is 1 m/s\(^2\). Setting of temperature distribution is shown in Fig. 3(a). A sound velocity distribution shown in Fig. 3(b) is obtained by the temperature distribution.

Calculation result is shown in Fig. 4. In this result, the two-dimensional sound field at \( z = 5 \text{ mm} \) in a range of \( 20 \times 20 \text{ mm}^2 \) shown in Fig. 5 is used for confirmation of SNAH. In Fig. 5, the amplitude of the sound field at \( z = 5 \text{ mm} \) is shown in Fig. 5(a), and the phase is shown in Fig. 5(b). Then, sound fields at \( z = 10 \text{ mm} \) and \( 15 \text{ mm} \) are calculated from the sound field at \( z = 5 \text{ mm} \) with the setting temperature distribution using SNAH. Step size of discretization in \( z \) axis, which is used for SNAH, is 0.1 mm.

Figure 6 shows a comparison result between SNAH and FEM. These are profiles of sound fields at \( z = 10 \text{ mm} \) and \( 15 \text{ mm} \) on \( x \)-axis. Figures 6(a) and 6(b) show the amplitude of sound fields, and Figs. 6(c) and 6(d) show the phase, respectively. Figures 6(a) and 6(c) show the sound field at \( z = 10 \text{ mm} \), and Figs. 6(b) and 6(d) show the sound field at \( z = 15 \text{ mm} \), respectively. The solid lines in Fig. 6 show the calculation result using SNAH, and the dashed lines show the calculation result using FEM. The amplitude of sound fields, calculated by SNAH, is accord with the amplitude calculated by FEM, and the phase calculated by SNAH is well agreed with that by FEM in the central area. In Figs. 6(c) and 6(d), the comparison result outside the center of transducer has errors. However, the amplitude of the area is low, so the errors have little influence on optimization for estimation of a temperature distribution.

3.2 Estimation of temperature distribution

We validate the method of estimation of temperature
distribution by a simulation. Settings of calculation of sound wave propagation are the same as preceding section without frequency of ultrasonic wave and a temperature distribution. The frequency of ultrasonic wave is 1 MHz. Then, two-dimensional sound fields at $z = 5$, 10, and 15 (mm) are calculated as measured sound fields using SNAH in range of $20 \times 20 \text{mm}^2$. The space between $z = 5$ and 15 mm is discretized into 1,000 sections. These sound fields are used for estimation of the sound velocity distribution, and those sound fields are added white noise so that signal-to-noise ratio becomes 10 dB. Then two kinds of temperature distributions are estimated: linear and Gaussian distributions. The number of optimization coefficients is determined by points required to interpolate the sound velocity distribution. In these simulations, the number of optimization coefficients is three points. Sound velocity distribution is interpolated to 100 points with cubic spline function for SNAH.

Figure 7 shows an estimated result using SNAH and multi-start downhill simplex method. In Figs. 7(a) and 7(c), solid line is estimated result of sound velocity, and dashed line is calculated sound velocity from exact temperature distribution. In Figs. 7(b) and 7(d), solid line is calculated temperature from the estimated sound velocity, and dashed line is exact temperature distribution. Exact temperature distributions in Figs. 7(b) and 7(d) are linear and Gaussian distributions, respectively. Then, root mean square error (RMSE) of the temperature distribution is calculated. In Fig. 7(b), the RMSE is 0.037°C, and the RMSE in Fig. 7(c) is 0.15°C. The estimated result well agrees with the setting value. However, the estimated result is worse if accuracy of interpolation is worse, which comes from the low number of optimization coefficients.

4. Conclusion

We have proposed methods for calculation of sound field in an inhomogeneous medium; we call SNAH, and estimation of a temperature distribution using SNAH and optimization method. Validity of our proposal methods were confirmed by the following simulations. The calculation result using SNAH was agreement with the calculation result using FEM. The estimated result of temperature distribution well agreed with the exact value. Our technique is expected to be applied to be non-destructive measurement of three-dimensional temperature distribution with scanning transducer or transducer array.
Acknowledgement

This work is supported by the Japan Society for the Promotion of Science for a grant for young scientists (20-581).

REFERENCES