Fracture Toughness for Brittle Fracture of Elastic and Plastic Materials

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The crack energy density causes a new expression of fracture toughness for brittle fracture, together with the concept of Barenblatt's characteristic distance d_c of cohesive zone at the edge of the crack. The Griffith's formula for fracture toughness is modified by the ratio of σ_T/Σ_T , which σ_T is the true fracture stress and Σ_T the maximum strength of the material obtainable by ordinary processing. The fracture toughness of elastic-plastic material, K_{Ic}^* for brittle fracture is described by the product of σ_T and the root of the observed absorption energy by Charpy impact test corrected for instrumental effect, $\sqrt{I_{cp}^{corrected}}$. The characteristic distance d_c depends linearly on the released elastic energy, which is the physical basis of above-mentioned estimation formula. [doi:10.2320/matertrans.M2012348]

(Received October 18, 2012; Accepted December 10, 2012; Published January 25, 2013)

Keywords: fracture toughness, crack energy density, true fracture stress, corrected Charpy impact value, Barenblatt's characteristic distance

1. Introduction

For steels showing the unstable fracture (brittle fracture), Oda, Nakamura and Kawakami have reported before ca. 40 years^{1,2)} that the fracture toughness K_{Ic}^* is related to the two observed values, the true fracture stress σ_T and the Charpy impact value I_{cp}^{obs} as

$$K_{Ic}^* = k\sigma_T \sqrt{\frac{I_{cp}^{obs}}{E}} \quad (1)$$

where E is the Young's modulus and k a nondimensional constant.¹⁻³⁾ Equation (1) is important at two points, 1) that the fracture toughness K_{Ic}^* is described by the true fracture stress σ_T and Charpy impact value I_{cp}^{obs} which are easily measured without any restriction of specimen size, and 2) that this equation can be used for the brittle fracture of materials with wide range of plastic deformation such as perfectly elastic, small scale yielding, large scale yielding and general yielding. T. Oda *et al.* have explained eq. (1) based upon the dimensional analysis. Trying to make clear the physical meaning of eq. (1) needs careful consideration because the well-known fracture mechanics is essentially based on elasticity not only in linear fracture mechanics but also in J-integral for larger scale yielding. In J-integral, the outside of integral contour must be elastic state. J-integral can be used in perfectly elastic state, small scale yielding and large scale yielding states but not in general yielding state. The method of analysis including general yielding is not known. This paper starts from the concept of crack energy density (CED) proposed by K. Watanabe.⁴⁻⁷⁾ The CED can be defined in any state of plastic deformation including general yielding.

The crack energy density is defined to be the area density of (strain) energy representing the work (per unit area) given at the point of crack edge from the outside of the crack in the plane including its point of the crack edge. The CED can be experimentally evaluated from the load-displacement curves as $\varepsilon = -(\partial U/\partial a)$, where total strain energy is $U(a, u) = \int P du$, the load P , the displacement u and the crack length $2a$.

In elastic-plastic materials, CED is denoted by $\varepsilon = \varepsilon^e + \varepsilon^p$ where ε^e and ε^p are the parts of CED preserved elastically and plastically, respectively, during plastic deformation.⁶⁾ The

elastic CED, ε^e is released after unloading of the external force P . In the case of linear elastic materials, $\varepsilon = \varepsilon^e = ((1 - \nu^2)/E)K_I^2 = G$ where ν is the Poisson's ratio, K_I the stress intensity factor, G the energy release rate. Only the mode I (opening mode) of crack surface displacement is considered and discussion is made based on the two-dimensional elasticity and the condition of plane strain, hereafter.

2. Brittle Fracture of Perfectly Elastic Materials

The problem of singularity at each crack tip in stress field is overcome by introducing the cohesive forces (or attraction) acting in small edge regions of crack, which is previously discussed by Barenblatt⁸⁾ and Goodier⁹⁾ in detail.

Tensile stress σ_y normal to the surface of a crack and the associated vertical displacement v of the crack surface (y direction, see Fig. 1), are given by

$$\sigma_y = \frac{K_I + K_0}{\sqrt{2\pi s_1}} + g(0) + O\left(s_1^{-\frac{1}{2}}\right) \quad (2)$$

$$v = \pm 4 \frac{1 - \nu^2}{E} (K_I + K_0) \sqrt{\frac{s_2}{2\pi}} + O\left(s_2^{\frac{3}{2}}\right) \quad (3)$$

where $\pm$ refers to upper and lower faces of displacement. The stress intensity factors K_I and K_0 are

$$K_I = 2\sqrt{\frac{a}{\pi}} \int_0^a \frac{T_0(t)}{\sqrt{a^2 - t^2}} dt \quad (4)$$

$$K_0 = -\sqrt{\frac{2}{\pi}} \int_0^d \frac{g(\xi)}{\sqrt{\xi}} d\xi \quad (5)$$

and $T_0(t)$ is the distribution of applied loads, $g(\xi)$ the distribution of cohesive forces near the crack tip, $2a$ the length of crack, $x = a - t$ where t is a running coordinate on the x axis, $s_1 = x - a$, $s_2 = a - x$ and d is the distance for acting cohesive forces (see Fig. 1). The stress singularity occurs from the term $s_1^{-\frac{1}{2}}$ in eq. (2), and the postulate $K_I + K_0 = 0$ by Barenblatt⁸⁾ and Goodier⁹⁾ removes its singularity. The remaining main terms in eq. (3) are of order $s_2^{\frac{3}{2}}$, which is demonstrated in Fig. 1.

When the cohesive force distribution exists near the crack edge (tip), the crack energy density ε at one end of crack is the work done near the crack edge ($\mathcal{W}$) during an infinitesimal extension of the crack.

where $\sigma_T/\Sigma_T = \sqrt{G_c^{\text{low}}/G_c^{\text{high}}}$. Equations (12) and (17) give

$$\sigma = \frac{\sigma_T}{\Sigma_T} \sqrt{\frac{EG_c^{\text{high}}}{\pi(1-\nu^2)a}} \quad (18)$$

which is the breaking stress with a crack of $2a$. This equation is different in the term of σ_T/Σ_T from the usual Griffith's formula.^{10–13} The factor $\sigma_T/\Sigma_T = \sqrt{G_c^{\text{low}}/G_c^{\text{high}}}$ causes the difference of fracture toughness between well-processed extreme state and less processed ordinary one. The factor σ_T/Σ_T describes the difference of the stored elastic energy at the break in perfectly elastic materials.

3. Fracture Toughness K_{Ic}^* of Elastic-Plastic Solids

The CED of elastic-plastic materials for the mode I of crack surface displacement is

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{1-\nu^2}{E} K_I^2 + \varepsilon^p \quad (19)$$

where ε^e and ε^p are the elastic and plastic parts of CED, respectively, and $\varepsilon^e = ((1-\nu^2)/E)K_I^2 = G$ where G is the energy release rate. The nonlinear part of ε^e is neglected. If the critical value of ε in the right-hand side of eq. (19) is denoted by $\varepsilon_c = ((1-\nu^2)/E)K_{Ic}^{*2}$, K_{Ic}^* is a kind of extended fracture toughness since ε_c is the energy per unit area for breaking(rupture) of the crack edge. The parameter K_{Ic}^* is the redefined fracture toughness containing the effect of plastic deformation. In perfectly elastic materials, K_{Ic}^* is the same as the fracture toughness K_{Ic} and related to the energy release rate G . The redefined fracture toughness K_{Ic}^* is given as

$$K_{Ic}^{*2} = K_{Ic}^2 + \left(\frac{E}{1-\nu^2}\right)2\gamma_p \quad (20)$$

where $\varepsilon_c^p = 2\gamma_p$ as described in various publications.^{11–13} Using eq. (17),

$$K_{Ic}^* = \frac{\sigma_T}{\Sigma_T} \sqrt{\left(\frac{E}{1-\nu^2}\right) \left[G_c^{\text{high}} + \left(\frac{\Sigma_T}{\sigma_T}\right)^2 2\gamma_p \right]} \quad (21)$$

where G_c^{high} is the stored elastic energy in the material after plastic deformation, and is also the released energy. In eq. (21), $K_{Ic} = (\sigma_T/\Sigma_T)\sqrt{(EG_c^{\text{high}}/(1-\nu^2))}$ is related to the state of materials just before breaking(rupture) after plastic deformation finishes (see eq. (17)). Equation (21) is applicable for all elastic-plastic materials independent of degree of plasticity.

When $G_c^{\text{low}} \gg 2\gamma_p$ (or $G_c^{\text{high}} \gg (\Sigma_T/\sigma_T)^2 2\gamma_p$), in which brittle fracture occurs,

$$K_{Ic}^* \cong \frac{\sigma_T}{\Sigma_T} \sqrt{\left(\frac{E}{1-\nu^2}\right) G_c^{\text{high}}} \quad (22)$$

and when $G_c^{\text{low}} \ll 2\gamma_p$ (or $G_c^{\text{high}} \ll (\Sigma_T/\sigma_T)^2 2\gamma_p$), in which ductile fracture occurs,

$$K_{Ic}^* = \sqrt{\left(\frac{E}{1-\nu^2}\right) (G_c^{\text{low}} + 2\gamma_p)} \cong \sqrt{\left(\frac{E}{1-\nu^2}\right) 2\gamma_p} \quad (23)$$

Equation (22) shows that the structure of the material changes through plastic deformation although the plastic energy is very small, and the elastic energy is stored as G_c^{high} which is larger than G_c^{low} . This is experimentally verified by

the result that the larger elastic energy state appears by unloading just before the break(rupture).

The fracture toughness K_{Ic} appearing in eq. (20), is related to the linear part of preserved elastic CED just before breaking(rupture). Here, it is implicitly assumed that the breaking of materials such as metals occurs only at the condition of perfectly elastic state. In other words, metals undertake the plastic deformation until the perfectly elastic state appears. The appearance of perfectly elastic state after the plastic deformation, causes the Griffith's breaking. The reason why the perfectly elastic state appears after plastic deformation, is that the plastic deformation continues until the moving units in materials vanish. The state that the moving units vanish, is the perfectly elastic one. Breaking occurs only in the perfectly elastic state.

Equation (23) is directly derived from eq. (19). This is well-known result as Griffith¹⁰–Irwin¹⁴–Orowan¹⁵ formula. Compared with eq. (22), eq. (23) has no effect of the factor σ_T/Σ_T . The difference of stored elastic energy causes the factor σ_T/Σ_T and does not affect the plastic part of $2\gamma_p$.

4. Estimation of Absorbed Energy by Experimentally Observable Parameter

The exact method to observe the absorbed energy relating to the static fracture toughness is not yet known. The surface energy $2\gamma_s$ discussed by Griffith and many authors is effective concept only in single crystals and amorphous materials. The absorbed energy can be approximately observed by impact tests under some conditions. The observed Charpy impact value I_{cp}^{obs} contains $G_c^{\text{high}} + 2\gamma_p$ together with the stored energy in the instrument and the toss energy of broken specimen. The absorption energy is related to Charpy impact value as $G_c^{\text{high}} + 2\gamma_p = \alpha I_{cp}^{\text{obs}} = I_{cp}^{\text{corrected}}$ where the correction parameter α is $0 < \alpha < 1$ and α must be determined from experimental system. If the observed impact value I_{cp}^{obs} contains only the absorption energy by deformation and fracture, $\alpha = 1$. Usually, α includes other effects such as elastic deformation of testing machine and toss energy of broken specimen. Kobayashi *et al.*^{16,17} have shown the method to measure the correction parameter α by the instrumented impact testing method, and report that α is about 0.8 for steel and 0.5 for cast iron.¹⁶ Brittle materials like ceramics reveal rather lower values of α as 0.4–0.6.¹⁷

It must be noted that the observed Charpy impact value generally reflects the mechanism of dynamic fracture toughness and is not always related to the mechanism of static fracture toughness discussed in the above-mentioned sections. This means that the absorbed energy for static fracture toughness would be different from the dynamic case. In the low-velocity (low blow) impact tests in the instrumented Charpy impact testing system, the dynamic fracture toughness is nearly equal to the static one for steel within the error of measurement, which suggests α is correctly estimated.¹⁸ In the high-velocity impact test, other discussion must be needed including the effect of loss of contact and is beyond the reach of this paper.

In eq. (21), G_c^{high} appears. Even small plastic deformation causes the change of structure of the material and the elastic energy G_c^{low} increases to G_c^{high} . This G_c^{high} is observed by

low-velocity impact tests in the case of brittle fracture. Impact tests include all energy as elastic and inelastic energies until the break occurs, and also reflect the increase of elastic energy through structural change caused by plastic deformation. If inelastic energy $2\gamma_p$ is very small, G_c^{high} is approximated by $I_{\text{cp}}^{\text{corrected}}$.

When $G_c^{\text{low}} \gg 2\gamma_p$, in which brittle fracture occurs,

$$K_{\text{Ic}}^* \cong \frac{\sigma_T}{\Sigma_T} \sqrt{\left(\frac{E}{1-\nu^2}\right) I_{\text{cp}}^{\text{corrected}}} \quad (24)$$

and when $G_c^{\text{low}} \ll 2\gamma_p$, in which ductile fracture occurs,

$$K_{\text{Ic}}^* \cong \sqrt{\left(\frac{E}{1-\nu^2}\right) I_{\text{cp}}^{\text{corrected}}} \quad (25)$$

These equations (eqs. (24) and (25)) are the estimation formula of fracture toughness by using easily observable parameters σ_T and $I_{\text{cp}}^{\text{corrected}}$. The parameter Σ_T is the maximum strength of materials achieved by processing plastically. It is well known that the ordinary deformation process leads a kind of upper limit of mechanical strength which is about ten % of ideal mechanical strength. This upper limit is Σ_T and steel shows Σ_T as ca. 2 GPa (for example, piano wire). Ordinary processing techniques such as forging, rolling, pressing and heat treatment cannot reveal the ideal mechanical strength (ideal cleavage strength) because those techniques cannot control the fine structure of materials below the size of a few hundreds of nanometer. In other words, Σ_T is the maximum mechanical strength of the specimen in case that its fine structure above a few hundreds of nanometer is precisely controlled.^{11,12,16)}

The fracture toughness K_{Ic}^* in eq. (24) includes the two effects, one is the effect of absorption energy $I_{\text{cp}}^{\text{corrected}}$ through $\sqrt{(EI_{\text{cp}}^{\text{corrected}}/(1-\nu^2))}$ with Young's modulus E and the other is the effect of the true fracture stress σ_T . If $\sigma_T = \Sigma_T$ (in the case of perfectly elastic and extremely well-processed materials), eq. (24) is equal to the Griffith's formula (eq. (11)) because of $I_{\text{cp}}^{\text{corrected}} = G_c = 2\gamma_s$.

Equation (24) is derived based on the CED consideration and the assumption that the break(rupture) of materials occurs at the perfectly elastic state even after the plastic deformations. Equation (24) can be used not only in perfectly elastic materials but in general yielding because of no restriction of degree of plasticity, although the energy condition $G_c^{\text{low}} \gg 2\gamma_p$ must be maintained. The estimation formula of fracture toughness, eq. (24) has physical base and can be used to estimate the fracture toughness from easily measurable parameters, absorption energy $I_{\text{cp}}^{\text{corrected}}$ and true tensile strength σ_T .

5. Discussion

From eqs. (15) and (16),

$$d_c = \frac{1}{\pi} \left(\frac{K_{\text{Ic}}^{\text{low}}}{\sigma_T} \right)^2 = \frac{E}{\pi(1-\nu^2)} \frac{1}{\Sigma_T^2} G_c^{\text{high}} \quad (26)$$

This equation means that the characteristic distance d_c is proportional to G_c^{high} for brittle materials. The eq. (26) is the same as eq. (22). As described before, G_c^{high} is measured by $I_{\text{cp}}^{\text{corrected}}$, and d_c is calculated from $K_{\text{Ic}}^{\text{low}}$ and σ_T . Figure 2 is the replotting of data by Oda *et al.*,^{1,2)} although they have not

Fig. 2 The relationship between the Barenblatt's characteristic distance d_c and observed Charpy impact value $I_{\text{cp}}^{\text{obs}}$, in which data used are by Oda *et al.*^{1,2)}

corrected the instrumental effect of impact test but its effect $\sqrt{\alpha}$ is rather small in steels. It is clear that d_c depends on $I_{\text{cp}}^{\text{obs}}$ linearly. And the obtained slope 12.5 (GPa)^{-1} gives ca. 2 GPa for Σ_T , which is reasonable one. This result denotes that G_c^{high} is described as $G_c^{\text{high}} \approx (\Sigma_T^2/2E)d_c$. The strain energy $\Sigma_T^2/2E$ per unit volume gathered in the range of d_c , gives G_c^{high} , which is the elastic energy released by breaking. It is not clear whether the value of G_c^{high} would coincide with the surface energy or not. In addition, the result that Σ_T is obtained from the slope in Fig. 2, suggests that the elastic state just before breaking is the one with the tensile strength Σ_T at least in the range of d_c . Moreover, the larger d_c , the larger the fracture toughness. The eq. (24) is based on that d_c depends linearly on G_c^{high} .

Equation (24) is important because the fracture toughness K_{Ic}^* can be estimated from the observed physical values as true fracture stress σ_T , Young's modulus E and the absorption energy $I_{\text{cp}}^{\text{corrected}}$ which can be measured easily and without using excessively large specimen.

The apparent maximum strength of specimen, Σ_T seems to be a kind of physical parameters because bulk materials such as metals and ceramics can be controlled only over the region of several hundreds of nanometer by ordinary processing methods.¹⁹⁾ The value of Σ_T can be observed on the bulk materials with precisely controlled fine structure over the range of a few hundreds of nanometer.

The previously proposed estimation formula, eq. (1) has apparently a different form from eq. (24). Compared with eqs. (1) and (24), the nondimensional constant k is given by

$$k = \frac{E}{\sqrt{1-\nu^2}} \frac{\sqrt{\alpha}}{\Sigma_T} \quad (27)$$

The constant k is represented by the ratio of Young's modulus E and the maximum strength of materials Σ_T reachable by plastic processing. In the case of steel, E is ca. 200 GPa, ν is ca. 0.3, Σ_T ca. 2 GPa and α ca. 0.8,¹⁵⁾ and k is ca. 94 which is a similar value as shown by Oda *et al.*^{1,2)} and Nakamura *et al.*³⁾ It must be noted that Oda *et al.* have not corrected the observed Charpy impact values but the effect of $\sqrt{\alpha}$ is rather small. Equation (24) is better for use than eq. (1) because it has clear physical meaning. And eq. (24) is important in the brittle fracture and cannot be used in the case of ductile fracture, as seen from the difference between eqs. (24) and (25).

6. Conclusions

The concept of crack energy density (CED) is the base of the discussion of fracture phenomena of elastic-plastic solids such as metallic materials. The fracture toughness is redefined as $\varepsilon_c = ((1 - \nu^2)/E)K_{Ic}^{*2}$, where ε_c is the critical value of CED and the undertaking energy per unit area at the crack edge to break.

In brittle fracture of perfectly elastic materials, modified Griffith's formula of fracture toughness (eq. (17)) and breaking stress with a crack $2a$ (eq. (18)) is obtained by taking into account of the Barenblatt's characteristic distance d_c at the edge of crack (tip).

The general expression of fracture toughness of elastic-plastic solids is given by eq. (21). Equation (21) is applicable to all materials independent of degree of plasticity because its derivation is based on the energy consideration without expecting detailed mechanisms of elastic and/or plastic deformations and fracture.

For the brittle fracture of elastic-plastic materials, eq. (22) is derived and this equation causes the new estimation formula (eq. (24)) which can be used for brittle fracture regardless of the deformation such as perfectly elastic, small scale yielding, large scale yielding and general yielding under the condition of larger elastic energy than plastic energy.

The points are that the break(rupture) of bulk materials occurs at the perfectly elastic state after the plastic deformation, which is described by Griffith's mechanism, and the elastic energy stored in the material after plastic deformation is different from the initial one because of structural change. This elastic energy G_c^{high} can be observed by the low-velocity impact test of instrumented Charpy impact test, although some correction is needed for instrumental contribution.

The new estimation formula of fracture toughness, eq. (24) is of practical use since it is possible to estimate with easily measurable parameters such as $I_{cp}^{\text{corrected}}$ and σ_T , without using the remarkably large size specimen.

Acknowledgments

Sincere thanks are due to Mr. Makoto Nakamura, (formerly) Mitsubishi Heavy Industries, Ltd. for the valuable discussions during the course of this research and also due to Mr. Seiji Fukuda, (formerly) Mitsubishi Heavy Industries, Ltd. for offering a lot of information.

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