Tape Length-Dependence of Critical Current and $n$-Value in Coated Conductor with a Local Crack

Shojiro Ochiai$^1$, Hiroshi Okuda$^2$ and Noriyuki Fujii$^2$

$^1$Elements Strategy Initiative for Structural Materials, Kyoto University, Kyoto 606-8501, Japan
$^2$Department of Materials Science and Engineering, Kyoto University, Kyoto 606-8501, Japan

Tape length-dependence of voltage-current curve, critical current and $n$-value in a coated conductor with a local crack were studied by modeling analysis. In calculation, the specimen length was varied in the range of 1.5 to 18 cm where the influence of cracking of superconducting layer is sharply reflected in the voltage–current curve. The following results were obtained, which can be utilized for analysis and interpretation of the experimental results under applied tensile stress in laboratory scale of specimen length. The existence of a crack changed the critical current and $n$-value through the decrease in current-transportable cross-sectional area of the superconducting layer and the current shunting in the cracked cross-section for any specimen length and any crack size, while the extent of the change was dependent on specimen length and crack size. When the crack was large, critical current increased slightly and $n$-value decreased significantly with increasing specimen length due to the enhanced shunting current. On the other hand, when the crack was small, critical current increased slightly with specimen length due to the enhanced shunting current similarly to the case of large crack, but $n$-value decreased with increasing specimen length due to the enhanced shunting current but then it increased due to the enhanced voltage-development at higher voltage in the non-cracked part. Also, the features of the dependence of the relation of $n$-value to critical current on specimen length were revealed; the decrease in $n$-value with decreasing critical current became sharper for longer specimen. [doi:10.2320/matertrans.MAW201401]

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1. Introduction

RE(Y, Dy, Sm, Gd, . . . )Ba$_2$Cu$_3$O$_7$-$\delta$ coated conductors (hereafter noted as REBCO conductors) are attractive for applications such as power cables, fault current limiters and current leads. In fabrication and service, they are subjected to thermal, mechanical and electromagnetic stresses. When the superconducting layer is cracked by such stresses, the critical current $I_c$ is reduced (YBCO, DyBCO, SmBCO, GdBBCO). The same situation occurs also in the filamentary Bi$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10+x}$ (Bi2223), Nb$_3$Sn, Nb$_3$Al and MgB$_2$ conductors due to the formation of collective cracks (cracks composed of successively cracked filaments in a transverse cross-section). It is required to clarify the cracking behavior and its influence on superconducting properties for safe and reliability design both for coated and filamentary conductors.

In general, cracking takes place heterogeneously not only in coated conductors but also in filamentary conductors, and, accordingly, the critical current $I_c$ and $n$-value in cracked specimens are different from location to location. In our preceding work using specimens composed of three sections, there arose two cases in the cracking behavior. In one case, three sections were cracked almost similarly, and in another case, one section was far more severely cracked than the other sections. In the former case, the voltages developed at the sections were summed up for the voltage of the specimen. As a result, the critical current and $n$-value of the specimen were affected by all sections. On the contrary, in the latter case where one section was far more severely cracked than the other sections, the $V$–$I$ curves of the most severely cracked section and overall specimen were almost overlapped. In such a case, the critical current and $n$-value of overall specimen decreased with decreasing critical current more sharply than that of sections. In this case, the local section which was cracked most severely than the other sections acted a deterministic role for determination of $I_c$ and $n$-value of the specimen.

An extreme situation in the latter case occurs when one section is cracked locally but not the other sections. Such a case has been observed actually. Analogous situation may happen when the tape is damaged locally in fabrication and winding process. The present work was carried out to investigate the influence of specimen length on the $V$–$I$ (current) curve, critical current $I_c$ and $n$-value in coated conductor with a local crack by modeling analysis. In calculation, the specimen length was varied in the range of 1.5 to 18 cm with an interval 1.5 cm due to the followings reasons. (i) The cracking of superconducting layer is reflected sharply in $V$–$I$ curve of short specimen (short resistance probe-distance). Therefore, from the viewpoint of materials characterization, it is important to obtain primary features in short specimen length range. (ii) The experimental gage length of specimen for measurement of $I_c$ and $n$-value under applied tensile stress is, in usual, limited to less than a few tens of centimeters. Due to the reasons (i) and (ii) above, the length of specimen was taken to be less than 20 cm. (iii) When cracking occurs under tensile stress, part of the imposed current shunts into stabilizer. For calculation of $V$–$I$ curve, the electric resistance of the shunting circuit is needed. In the present work, the resistance value in the specimens with a length 1.5 cm, obtained by analysis of the measured $V$–$I$ curves, was taken from our preceding work. Due to this reason, the shortest length (and interval) was taken to be 1.5 cm.

Longitudinal inhomogeneity of current transport properties and the relation of the local homogeneity to overall properties have been studied by application of the statistics of local current distribution. Also the length-dependence of critical current has been accounted for by nonlinear current flow at defects by application of the model of Friesen and Gurevich. They treated with long tapes for practical design and application, using as-fabricated tape containing fabrica-
tion process-induced defects. The dependence of $n$-value on specimen length has not been shown explicitly. In the present work, the influence of applied stress-induced crack on the specimen length-dependence of critical current and $n$-value was studied for wide range of crack size. The specimen length of the present work was far small in comparison with the other works. However, as stated above, from the viewpoint of materials characterization, it is important to obtain primary features in short specimen length range where the influence of cracking of superconducting layer on $V-I$ curve is detected sharply. The technical merit of this work is to clarify the features in such a specimen length range. Also, as the specimen length range studied in this work was in a usual experimental range of specimen length in laboratory, the features of specimen length dependence of $V-I$ curve, critical current and $n$-value obtained in the present work can be utilized for analysis and interpretation of the experimental results under applied tensile stress in usual laboratory scale of specimen length.

2. Model for Analysis

For analysis, the model of current shunting at the cracked part proposed by Fang et al. was employed in a modified form, whose details are shown below.

The configuration of the model specimen is shown in Fig. 1(a), where the specimen constitutes of $N$ sections with a length $L_0 = 1.5$ cm. Among the $N$ sections, one section has a crack in the coated superconducting layer and the other sections have no crack, as schematically shown in Fig. 1(b). In the calculation, the length $L$ of the specimen was taken to be 1.5, 3.0, 4.5, and 18 cm, in step of 1.5 cm, corresponding to $N = 1, 2, 3, \ldots, 12$, respectively.

Figure 2 shows a schematic representation of (a) current path and (b) simplified electric circuit in a cracked section with a length $L_0$, based on the partial crack-current shunting model where partial crack means the crack that exists in a part of transverse cross-section of the superconducting REBCO layer. In the transverse cross-section in which a partial crack exists (Fig. 2(a)), the crack that has lost superconductivity and the ligament that keeps superconductivity constitute of a parallel electric circuit. We define the ratio of cross-sectional area of the crack to the total cross-sectional area of the tape as $f$. The ligament of the REBCO layer, having an area ratio $1 - f$, transports current $I_{RE}$. At the crack having an area ratio $f$, current $I_s (= I - I_{RE})$ shunts into the stabilizer (Ag, Cu). In Fig. 2(b), the resistance in the shunting circuit is noted as $R_s$. The voltage, developed in the ligament that transports current $I_{RE}$, is noted as $V_{RE}$. The voltage $V_s = I_s R_s$, developed at the crack by shunting current $I_s$, is equal to $V_{RE}$ since the ligament- and crack constitute of a parallel circuit.

Noting the critical current and $n$-value of all sections in the original non-cracked state as $I_{c0}$ and $n_0$, respectively, and the critical electric field for determination of critical current as $E_c$ ($= 1 \mu V/cm$ in this work), the relation of voltage $V$ to current $I$ in each non-cracked section is expressed as

$$V = E_c L_0 \left( \frac{I}{I_{c0}} \right)^{n_0}$$

The voltage $V$ of the cracked section is the sum of the voltage $E_c L_0 (I/I_{c0})^{n_0}$ developed at the non-cracked part and the voltage $V_{RE}$ developed at the cracked part, where the cracked part has a length $s$ in the current transport direction, which is far smaller than the specimen length $L_0 (s \ll L_0)$ in the model of Fang et al. Using this model and modifying the expression of formulations, we can obtain the $V-I$ relation of the cracked section in the form:

$$V = E_c L_0 \left( \frac{I}{I_{c0}} \right)^{n_0} + V_{RE}$$

$$I = I_{RE} + I_s = L_0 (1 - f) \left[ \frac{V_{RE}}{E_c L_0} \right]^{1/n_0} + \frac{V_{RE}}{R_s}$$
While the length of the cracked part was taken as $s$ in Fang’s model, it can be replaced by the current transfer length, and the constitutive equations for $V$–$I$ curve can be expressed by eqs. (2) and (3) in a similar manner under a condition of $s \ll L_0$. The term $(1 - f)(L_0/s)^{1/n_0}$ in eq. (3) has a following physical meaning. When the $(1 - f)(L_0/s)^{1/n_0}$ is small, namely when the crack is large (large $f$), the voltage $E_cL_0(1/I_0)^{n_0}$ developed in the non-cracked part is almost zero and accordingly the voltage $V_{RE}$ developed at the cracked part is almost equal to the voltage $V$ of the cracked section (and also $V$ of whole specimen, as shown later in Subsection 3.1). In such a case, setting $E_cL_0(1/I_0)^{n_0} = 0$ and $V_{RE} = V$ in eqs. (2) and (3), we have $V$–$I$ curve in one equation:

$$I = I_{RE} + I_0(1 - f)\left(\frac{L_0}{s}\right)^{1/n_0} \left[\frac{V}{E_cL_0} + \frac{V}{R_t}\right]^{1/n_0} + \frac{V}{R_t} \tag{4}$$

If we take a virtual case of high $R_t$, the term $V/R_t$ is negligible in comparison with the term $I_0(1 - f)(L_0/s)^{1/n_0}$ in eq. (4). In such a case, $I_c$ of the cracked section is expressed as

$$I_c = I_0(1 - f)(L_0/s)^{1/n_0} \text{ by substituting } V = V_c = E_cL_0 \text{ and } V/R_t = 0 \text{ into eq. (4). Thus the ratio of } I_c \text{ (current under an existent crack) to } I_0 \text{ (critical current under no crack) is given by } I_c/I_0 = (1 - f)(L_0/s)^{1/n_0}. \text{ In this way, the term (1 - f)(L_0/s)^{1/n_0} has a physical meaning of the critical current } I_c \text{ in the cracked state normalized with respect to the critical current } I_0 \text{ in the non-cracked state under the condition of large crack and high } R_t. \text{ Hereafter, (1 - f)(L_0/s)^{1/n_0}, which refers to the area ratio } 1 - f \text{ of the ligament, is simply called as “ligament parameter”.

The ligament parameter $(1 - f)(L_0/s)^{1/n_0}$ gives $I_c/I_0$ only under the conditions stated above. In practice, the crack size varies with increasing applied stress and also current shunting takes place more or less, and accordingly $(1 - f)(L_0/s)^{1/n_0}$ is not equal to $I_c/I_0$ in practical tapes. However, it is noted that, as the shunting current $I_c$ is low at low voltage, the ligament parameter can give fairly good approximation for critical current of short specimen (details are shown later in Fig. 9(a) in Subsection 3.4). The values of $(1 - f)(L_0/s)^{1/n_0}$ and $R_t$ can be estimated by fitting eqs. (2) and (3) to the measured $V$–$I$ curve without preliminary estimation of the values of $1 - f$ and $s$, and once they are estimated, the REBCO ligament-transported current $I_{RE}$ and shunting current $I_c$ can be obtained with high accuracy, as has been shown in our preceding works.\cite{6,7,9,11,15} This point is a feature of the present approach.

As shown in Fig. 1, the specimen constitutes of $N$ sections with a length $L_0 = 1.5 \text{ cm}$, among which one section is cracked and other sections are not cracked. By using eqs. (1) and (2), the voltage of the specimen, constituting of a series circuit of $N$ sections and having length $L = NL_0$, is expressed as

$$V = E_cL \left(\frac{1}{I_0}\right)^{n_0} + V_{RE} \tag{5}$$

The $V$–$I$ curve of the specimen with various length can be calculated by eqs. (3) and (5). In calculation, as a monitor of the crack size, the ligament parameter $(1 - f)(L_0/s)^{1/n_0}$ was used (the larger the crack size, the smaller becomes the ligament parameter) and was varied from 0.01 to 1 in calculation. $R_t$ was taken to be $2 \mu \Omega$, which is an average value obtained by fitting eqs. (2) and (3) to the measured $V$–$I$ curves of DyBCO coated conductor with a length $L_0 = 1.5 \text{ cm}.\cite{12}$ The critical current and $n$-value in the non-cracked state were given by $I_0 = 200 \text{ A}$ and $n_0 = 40$, respectively. Substituting $(1 - f)(L_0/s)^{1/n_0} = 0.01 - 1$, $R_t = 2 \mu \Omega$, $I_0 = 200 \text{ A}$, $n_0 = 40$ and $L = 1.5$ to 18 cm in step of 1.5 cm into eqs. (3) and (5), we calculated $V$–$I$ curve, voltage $V_{RE} = V_c$ developed at the cracked part, voltage $E_cL(1/I_0)^{n_0}$ developed in the non-cracked part, REBCO-transported current $I_{RE}$ in the ligament and shunting current $I_c$ in the shunting circuit at the cracked part. From the calculated $V$–$I$ curves, the critical current $I_c$ of the specimen with a length $L$ was obtained as a value of $I$ at $V = V_c = E_cL \mu V$. The $n$-value of the specimen with a length $L$ was obtained by fitting the $V$–$I$ curve to the form of $V \propto I^n$ for the electric field range of $E = 0.1 - 10 \mu \text{V/cm}$, corresponding to $V = 0.1L - 10L \mu V$. From these calculation results, the influence of specimen length on the superconducting property values in the specimens with a crack were revealed. Also the influence of the specimen length on the relation of $n$-value to critical current $I_c$ was elucidated, as shown below.

### 3. Results and Discussion

#### 3.1 Influence of specimen length on the $V$–$I$ curve

The voltage $V$ developed in the specimen, containing a crack, is given by the sum of the voltage developed at the cracked part ($V_{RE}$) and non-cracked part ($E_cL(1/I_0)^{n_0}$) (eq. (5)). The contribution of the voltages developed at cracked and non-cracked parts to the total voltage $V$ is dependent on the crack size (hence on the ligament parameter $(1 - f)(L_0/s)^{1/n_0}$) and on the specimen length $L$. Figure 3 shows the influence of the ligament parameter $(1 - f)(L_0/s)^{1/n_0}$ on (a) the relation of $V$ (total voltage developed in the specimen) to $I$ (current) and the relation of $V_{RE} (= V_c$; voltage developed at the cracked part) to $I$, and on (b) the relation of the $E_cL(1/I_0)^{n_0}$ (= $V - V_{RE}$; voltage developed in non-cracked part) to $V$, for an example of $L = 6 \text{ cm}$. The calculation results in Fig. 3 show the following features for the influence of the ligament parameter on the voltage development under a given specimen length.

1. When the ligament parameter is small (0.2), namely when the crack is large, $V$–$I$ curve and $V_{RE}$–$I$ curve are the same (Fig. 3(a)) and $V - V_{RE}$ is zero (Fig. 3(b)). This means that, when crack is large, voltage developed in the non-cracked part is negligible $E_cL(1/I_0)^{n_0} \approx 0 \mu \text{V}$ and the voltage $V_{RE}$ developed at the cracked part is equal to the total voltage $V$ of the specimen.

2. When the ligament parameter is intermediate (0.5), namely when the crack is not large but not small, $V$ is equal to $V_{RE}$ and the voltage $E_cL(1/I_0)^{n_0}$ (= $V - V_{RE}$) developed at the non-cracked part is zero for low $V$ (< around 150 $\mu \text{V}$ in this example). However, at high voltage ($V > 150 \mu \text{V}$), voltage is developed also at the non-cracked part and increases with current $I$ (Fig. 3(a)) and $V$ (Fig. 3(b)). In this way, only the voltage $V_{RE}$ developed at the cracked part contributes to total voltage $V$ in low $V$ range, but both voltages ($V_{RE}$ developed at cracked part and $E_cL(1/I_0)^{n_0}$ developed at non-cracked part) contribute to $V$ in high $V$ range.
(3) When the ligament parameter is large (0.9), namely when the crack is small, voltage is developed both at cracked and non-cracked parts even at low $V$ as shown in Fig. 3(b). In this example, the voltage $E_cL(1/L_0)^{1/n_0}$ developed at the non-cracked part is 70 $\mu$V at $V = 100$ $\mu$V and is 160 $\mu$V at $V = 200$ $\mu$V. The voltage developed at the non-cracked part contributes largely to the total voltage in contrast to that under existence of large crack.

Figure 4 shows the calculated voltage ($V$)–current ($I$) curves of the specimens with length $L = 1.5$ to 18 cm for the ligament parameters $(1 - f)(L_0/s)^{1/n_0} = (a) 0.2$, (b) 0.5 and (c) 0.9. The following features are found.

(i) In the case of $(1 - f)(L_0/s)^{1/n_0} = 0.20$ (Fig. 4(a)), corresponding to large crack, the $V$–$I$ curve is common in all specimens, even though the specimen length $L$ is different to each other. This result shows that, when a crack is large, voltage is developed only at the cracked part and no voltage is developed in the non-cracked part for wide range of the specimen length $L$ (1.5–18.0 cm).

(ii) In the case of $(1 - f)(L_0/s)^{1/n_0} = 0.50$ (Fig. 4(b)), corresponding to intermediate-size crack, the $V$–$I$ curve is common in all specimens with different lengths up to around $V = 150$ $\mu$V, but it becomes different at higher voltage ($V > 150$ $\mu$V). This result means that, in the voltage range of $V < 150$ $\mu$V, the voltage $V_{RE}$ developed at the cracked part is equal to the voltage $V$ of the specimen, as in the case of $(1 - f)(L_0/s)^{1/n_0} = 0.2$. In the higher voltage range ($V > 150$ $\mu$V), the voltage $E_cL(1/L_0)^{1/n_0}$ is also developed at the non-cracked part, and the voltage of the specimen is given by the sum of $V_{RE}$ and $E_cL(1/L_0)^{1/n_0}$. Under this situation, the voltage $V$ of the specimen increases with increasing specimen length $L$.

(iii) In the case of $(1 - f)(L_0/s)^{1/n_0} = 0.90$ (Fig. 4(c)), corresponding to small crack, voltage is developed at both cracked and non-cracked parts. The $V$–$I$ curves can be distinguished by specimen length. The increase in voltage $V$ of the specimen with increasing current is enhanced for longer specimen.

### 3.2 Influence of specimen length on the $E$–$I$ curves

Figure 5 shows the electric field ($E$)–current ($I$) curves of the specimens with length $L = 1.5$ to 18 cm for the ligament parameters $(1 - f)(L_0/s)^{1/n_0} = (a) 0.2$, (b) 0.5 and (c) 0.9, which were converted from the $V$–$I$ curves shown in Fig. 4. The $E$–$I$ curve for each specimen length is distinguished, while the $V$–$I$ curves for small ligament parameter (0.2) were common for all specimen lengths calculated (Fig. 4(a)).

In Fig. 5, for a given length, the $E$–$I$ curve shifts to higher current region with increasing ligament parameter $(1 - f)(L_0/s)^{1/n_0}$, namely with decreasing crack size. This
result reflects the increase in cross-sectional area of the current-transportable superconducting layer. The current

\[
I_c \propto \frac{1}{L}
\]

derivative to 10 µV/cm increases with increasing ligament parameter, for any specimen length L. For a given ligament parameter \((1 - f)(L_0/s)^{1/n_0}\) (for a given crack size), critical current \(I_c\) at \(E = E_c = 1 \mu V/cm\) increases with increasing specimen length L. The slope of the \(E-I\) curves in the range of \(E = 0.1\) to 10 µV/cm for \((1 - f)(L_0/s)^{1/n_0} = 0.2\) and 0.5 decreases with increasing specimen length, indicating that n-value decreases with specimen length when crack size is not small. The change in slope of the \(E-I\) curves for \((1 - f)(L_0/s)^{1/n_0} = 0.9\) (small crack) is not clear in the form of Fig. 5(c). For estimation of n-value, numerical analysis is needed. In the next subsection, it will be shown that n-value decreases and then increases with increasing specimen length when the crack size is small.

3.3 Change in critical current and n-value with specimen length

3.3.1 Calculation results of change in critical current and n-value with specimen length

The \(V-I\) (and \(E-I\)) curves were calculated for wide variety of the ligament parameter and specimen length \((L = 1.5\) to 18 cm in step of 1.5 cm), from which the critical current \(I_c\) and n-value were estimated. Figure 6 shows (a) critical current \(I_c\) and (b) n-value for \((1 - f)(L_0/s)^{1/n_0}\) (namely with increasing crack size).

(1) For any length L, \(I_c\) and n-value decrease with decreasing ligament parameter \((1 - f)(L_0/s)^{1/n_0}\) (namely with increasing crack size).

(2) For any ligament parameter (crack size), \(I_c\) increases slightly with increasing specimen length L.

(3) The change in n-value with increasing specimen length was dependent on the value of the ligament parameter \((1 - f)(L_0/s)^{1/n_0}\) In the range of \((1 - f)(L_0/s)^{1/n_0} < 0.8\), n-value decreases monotonically with increasing L. However, when the \((1 - f)(L_0/s)^{1/n_0}\) is large such as 0.9 and 0.95, n-value decreases and then increases with increasing L. For instance, in the case of \((1 - f)(L_0/s)^{1/n_0} = 0.9\), n-value decreases from 31.3 to 26.2 when L increases from 1.5 to 6.0 cm but then it increases to 28.9 when L increases to 18 cm.

3.3.2 Influences of the voltage developed at the non-cracked part and shunting current at cracked part in the specimen length-dependence of critical current and n-value

In this sub-section, the changes in critical current and n-value with specimen length were explored. Figure 6 shows (a) critical current \(I_c\) and (b) n-value with increasing specimen length L for \((1 - f)(L_0/s)^{1/n_0} = 0.05\) to 0.95.
value with specimen length (Fig. 6) are discussed from the viewpoint of the specimen length-dependence of the change in contribution of the voltages developed at cracked and non-cracked parts to total voltage $V$ and the extent of the shunting current at the cracked part.

As has been shown in Figs. 3 and 4, when the ligament parameter $(1 - f)(L_0/s)^{1/n_0}$ is small (0.2), voltage is developed only at the cracked part and $V = I_{RE}$ is hold; when the ligament parameter is intermediate (0.5), voltage is developed only at the cracked part and no voltage is developed in the non-cracked part in the range of low $V$ ($V = V_{RE}$), but it is developed both at cracked and non-cracked parts in the range of high $V$ ($V = V_{RE} + LE_c(I/I_{c0})^{n_0}$). Namely, due to the development of the voltage $LE_c(I/I_{c0})^{n_0}$ at the non-cracked part in addition to the voltage $V_{RE}$ at the cracked part, $V$ increases largely. When the ligament parameter is large (0.9), voltage is developed both at the cracked and non-cracked parts in wide range of $V$ ($V = V_{RE} + LE_c(I/I_{c0})^{n_0}$). The voltage $LE_c(I/I_{c0})^{n_0}$ developed at the non-cracked part raises $V(E)$ for a given current $I$, acting to reduce $I_c$ and to raise $n$-value. In this way, it affects on $V$–$I$ ($E$–$I$) curve and hence on critical current and $n$-value. Another affecting factor is shunting current at the cracked part as shown below.

Figure 7 shows the change in electric field ($E$) with current ($I$) and the change in electric field ($E$) with REBCO-transported current ($I_{RE}$) at the cracked part in logarithmic scale for $(1 - f)(L_0/s)^{1/n_0} = 0.2, 0.5$ and 0.9. (a), (b) and (c) refer to the calculation results for representative specimen lengths of $L = (a)$ 1.5 cm, (b) 6.0 cm and (c) 18.0 cm. The $E$–$I$ and $E$–$I_{RE}$ curves are shown with solid and dotted curves, respectively. In this calculation, the REBCO-transported current $I_{RE}$ was calculated by $I_{c0}(1 - f)(L_0/s)^{1/n_0}[V_c/(E_c L)]^{1/n_0}$. The shunting current $I_s = I - I_{RE}$ is equal to the difference in current between $E$–$I$ and $E$–$I_{RE}$ curves as indicated by $\leftrightarrow$ in Fig. 7. The $I_s$ increases with increasing $E$. The increment of $I_s$ with $E$ is enhanced with decreasing ligament parameter (with increasing crack size) and with increasing specimen length. Namely, the shape of the $E$–$I$ curve varies with increasing specimen length due to enhanced shunting current. The critical current $I_c$ is estimated by the criterion of $E_c = 1 \mu V/cm$. Comparing the $I_c$ values ($I$ at $E = 1 \mu V/cm$ in $E$–$I$ curve) for $L = 1.5$ (a), 6.0 (b) and 18 cm (c) under a given $(1 - f)(L_0/s)^{1/n_0}$, for instance 0.2, the $I_c$ value for $L = 18$ cm is highest and that for $L = 1.5$ cm is lowest. In this way, the shunting current, which increases with specimen length, acts to raise $I_c$ for longer specimen. On the other hand, the slope of the $E$–$I$ curves in the logarithmic scale in the range of $E = 0.1$ to $10 \mu V/cm$, corresponding to $n$-value, decreases with increasing $I_c$. Accordingly, the shunting current acts to raise $I_c$ and to reduce $n$-value, while the voltage developed at the non-cracked part acts to reduce $I_c$ and to raise $n$-value. The critical current and $n$-value are determined by the competition of these contradictory factors.

The critical current $I_c$ under $E_c = 1 \mu V/cm$ criterion increases with increasing specimen length, as shown in Fig. 6(a) and has been observed actually for DyBCO and SmBCO conductor tapes in our preceding works. This is attributed to the dominancy of the shunting current, which acts to raise $I_c$, over the voltage developed at the non-cracked part, which acts to reduce $I_c$. Actually, voltage at the non-cracked part is not developed for small ligament parameter and hence gives no influence on $I_c$. Though it is developed for intermediate and large ligament parameter, its influence on critical current is minor since $I_c$ is estimated at low $E$ ($E_c = 1 \mu V/cm$) where the voltage developed at the non-cracked part is rather low. In this way, the specimen length-dependence of critical current under an existent crack is accounted for from the viewpoint of the enhanced current shunting at the cracked part.

On the other hand, the specimen length-dependence of the $n$-value is complex, as mentioned in (3) in the Subsection 3.3.1. The $n$-value in this work is the average slope of the relation of $\ln(E)$ to $\ln(I)$ in the range of $E = 0.1$ to $10 \mu V/cm$. The tendency that $n$-value decreases with increasing specimen length $L$ for $(1 - f)(L_0/s)^{1/n_0} \leq 0.8$ (Fig. 6(b)), which has been observed actually for DyBCO and SmBCO conductor tapes in our preceding works, is accounted for by the increase in shunting current, which reduces the slope of the $E$–$I$ relation in logarithmic scale. However, for large $(1 - f)(L_0/s)^{1/n_0}$ such as 0.90 and 0.95,
3.4 Influence of specimen length on $n - I_c$ relation

It has been shown experimentally that the $n$-value decreases with decreasing critical current\(^{6,7,9,11}\). However, the specimen length-dependence of the relation of $n$-value to critical current has not been investigated. As shown in Fig. 6, the cracking of the REBCO layer reduces critical current and $n$-value and the reduction is dependent on the specimen length and crack size (ligament parameter). In this section, the influence of specimen length on the relation between $n$-value and critical current is discussed.

Figure 9 shows the plot of (a) critical current against ligament parameter \((1 - f)(L_0/s)^{1/n_0}\), (b) $n$-value against \((1 - f)(L_0/s)^{1/n_0}\) and (c) $n$-value against critical current. As stated in Section 2, the ligament parameter \((1 - f)(L_0/s)^{1/n_0}\) has a physical meaning of the critical current ($I_c$) normalized with respect to the critical current $I_{c0}$ in the non-cracked state under the condition of large crack and high $R_t$. In practice, $R_t$ is low and shunting current flows and therefore $I_c$, more or less, shifts upwards from the ligament parameter \((1 - f)(L_0/s)^{1/n_0}\). The results in Fig. 9(a) show that, as a first approximation, $I_c$ increases almost linearly with \((1 - f)(L_0/s)^{1/n_0}\) when the specimen is short. Such a tendency has been observed in practice\(^{6,7,11}\). It is noted that $I_c$ increases with increasing specimen length $L$ and shifts upwards slightly from the value of $I_{c0}(1 - f)(L_0/s)^{1/n_0}$ due to the current shunting that is enhanced more in longer specimen.

The $n$-value decreases sharply with decreasing ligament parameter at high \((1 - f)(L_0/s)^{1/n_0}\) region (Fig. 9(b)), indicating that once cracking occurs, $n$-value is significantly reduced even the crack is small. In contrast to critical current, $n$-value is strongly affected by the specimen length. It is noted that, when the specimen is short ($L = 1.5$ and $3.0$ cm), the $n$-value decreases with decreasing ligament parameter (with increasing crack size), sharply in the region of large ligament parameter, gradually in the region of intermediate ligament parameter and then sharply in the region of small ligament parameter. On the other hand, when the specimen is long ($L > 10$ cm), the $n$-value decreases more sharply in the region of large ligament parameter and then more gradually in the regions of intermediate and small ligament parameter in comparison with the result of short specimens. In this way,
the relation of $n$ to $(1 - f)(L_0/s)^{1/n_0}$ shifts downwards sharply with increasing specimen length.

Due to the features of critical current and $n$-value mentioned above, critical current $I_c$ increases with increasing $L$ for any $(1 - f)(L_0/s)^{1/n_0}$, though the increment is small. On the other hand, $n$-value is sensitive to the specimen length $L$. As a result, $n$-value decreases sharply with decreasing critical current when specimen length increases (Fig. 9(c)) and the $n - I_c$ relation shifts downwards with increasing specimen length.

The results in Fig. 9(c) were obtained for a given value of $R_t = 2 \mu \Omega$. In practice, $R_t$ value is dependent on the species of the stabilizer, resistance at REBCO-stabilizer interface, fracture morphology of the coated layer and so on. Then the influence of $R_t$ on $n - I_c$ relation was examined by varying $R_t$ value. The results for the specimen length $L = (a) 1.5$ cm, (b) 6.0 cm and (c) 18 cm are shown in Fig. 10. It is noted that (a) for any specimen length, $n - I_c$ relation shifts upward for higher $R_t$ and (b) the decrease in $n$-value with decreasing critical current becomes sharper for longer specimen and lower $R_t$.

It is noted that, while $s$ was taken to be the length of the cracked part in the model of Fang et al., it could be taken to be the current transfer length. The constitutial equations for $V$–$I$ curve are expressed by eqs. (2) and (3) in both cases under a condition of $s \ll L_0$. In the present work, the ligament parameter $(1 - f)(L_0/s)^{1/n_0}$ was used to synthesize the $V$–$I$ curve, in which discrete value of $s$ was not needed.

4. Conclusions

The specimen length-dependence of critical current and $n$-value in coated conductor tape with a local crack was studied by modeling analysis for the specimen length range of 1.5–18 cm, where the influence of cracking on voltage–current curve is reflected sharply in voltage–current curve. Main results, which can be utilized for analysis and interpretation of the experimental results under applied tensile stress in laboratory scale of specimen length, are summarized as follows.

(1) The voltage developed at the cracked part is equal to the overall voltage and the voltage–current curve of the cracked section is the same as that of the overall specimen for any specimen length (<18 cm) when the ligament parameter is small (crack is large). On the other hand, when the ligament parameter is large (crack is small), the voltage developed in the non-cracked part became higher and contributed more largely to the overall voltage with increasing specimen length.

(2) Under an existent large crack (small ligament parameter), critical current increases but $n$-value decreases with increasing specimen length due to the enhanced shunting current. When the crack size is small (ligament parameter is large such as 0.9), critical current increases with specimen length due to the enhanced shunting current. On the other hand, $n$-value decreases with increasing specimen length due to the enhanced shunting current but then it increases with increasing specimen length due to the enhanced voltage-development in the non-cracked part.

(3) Critical current increases with ligament parameter. This tendency is enhanced slightly with increasing specimen length. The relation of $n$-value to ligament parameter is more sensitive to the specimen length in comparison with that of critical current to ligament parameter. When the specimen is short ($L = 1.5$ and 3.0 cm), the $n$-value decreases with decreasing ligament parameter (with increasing crack size), sharply in the region of large ligament parameter, gradually in the region of intermediate ligament parameter and then sharply in the region of small ligament parameter. When the specimen is long, the $n$-value decreases more sharply in the region of large ligament parameter and then more gradually in the regions of intermediate and small ligament parameter in comparison with the relation of $n$-value to ligament parameter of short specimens.

(4) The $n$-value decreases sharply with decreasing critical current when specimen is long and when electric
resistance in the shunting circuit is low. The relation of $n$-value to critical current shifts downwards with increasing specimen length and decreasing electric resistance in the shunting circuit.

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