Compressive Behavior of Open-Cell Titanium Foams with Different Unit Cell Geometries

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Mechanical properties of open-cell titanium foams with different cell geometries (truncated octahedron and rhombic dodecahedron cells) were examined through compressive tests. These foams were manufactured through the electron beam melting (EBM) process. The compressive behavior depends on the porosity, cell geometry and the cell orientation. Titanium foams with truncated octahedron cells showed high strength compared to those of rhombic dodecahedron cells. This is due to the short cell edges in truncated octahedron cells. In addition, the parallel and oblique cell edges against the compression direction are effective to increase the compressive strength. Macroscopic shear bands caused by ordered cell geometry were observed in some titanium foams.

Keywords: titanium foam, electron beam melting, truncated octahedron, rhombic dodecahedron

1. Introduction

Metal foams are a class of lightweight materials with novel physical, mechanical, thermal, electrical and acoustic properties. They offer potential for lightweight structures, energy absorption, and thermal management. Titanium foams are used for structural applications or for bone-replacement implants because of their low density, high strength and excellent corrosion resistance.

Powder metallurgical (PM) process with spacer materials has been used for the manufacturing of magnesium and titanium foams. Due to the non-spherical shape of the spacer materials, the cell structure of metal foams becomes homogeneous, and the pore shape becomes heterogeneous. Recently, additive manufacturing (AM) technology has been focused on many industrial applications. In the case of metallic parts, selective electron beam melting (EBM), selective laser sintering (SLS), and selective laser melting (SLM) have enabled to produce highly porous interconnected structures. The arbitrary pore shapes and pore sizes are in the range from 100 to 200 μm, which can create complex and ordered structure titanium foam that is distinguished from previous approaches.

AM process enables to manufacture open-cell geometry consisting of periodic space-filling polyhedrons. Though there are several space-filling polyhedrons, the present study focusses on two polyhedrons, truncated octahedron and rhombic dodecahedron. A truncated octahedron, which is one of Archimedean solids, has 14 faces (8 hexagons and 6 squares), 36 edges and 24 vertices. A rhombic dodecahedron, which is one of Catalan solids, has 12 rhombic faces, 24 edges and 14 vertices. Of course, titanium foams with these polyhedron cell geometries have been reported in the previous study. However, the relationship between the porosity, cell edge length, cell diameter and cell orientation has not been discussed systematically.

In the present study, mechanical properties of titanium foams with truncated octahedron and rhombic dodecahedron cells are investigated systematically. Optimal cell geometry for mechanical properties such as plateau stress and energy absorption is clarified through the compressive tests.

2. Experimental Procedure

Periodic truncated octahedron and rhombic dodecahedron open-cell geometries are constructed by using a commercial 3D-CAD software. 3D images of the unit-cells are shown in Fig. 1. Octa_A and Octa_B are the truncated octahedron cells with the edge length of a. In the x-y-z orthogonal coordinate system, Octa_B has cell geometry after 54.7 deg rotation around y-axis and 45 deg rotation around z-axis of Octa_A. Dodeca_A, Dodeca_B and Dodeca_C are the rhombic dodecahedron cells with the edge length of a. In the x-y-z orthogonal coordinate system, Dodeca_B has cell geometry after 90 deg rotation around x-axis of Dodeca_A. Dodeca_C has cell geometry after 54.7 deg rotation around y-axis of Dodeca_A. Here, the compression direction is parallel to z-axis.

In the case of truncated octahedron unit cell with the edge length, a, the volume is expressed as

\[ V_{Octa} = 8 \sqrt{2} a^3. \]  

(1)

By assuming the cylindrical shape of the cell edges, the volume of cell edges in the unit cell is calculated as

\[ V_{a} = \frac{\pi}{4} t^2 \times \frac{1}{3} \times a \times 36 = 3 \pi a t^2 \]  

(2)

where \( t \) is the diameter of cylindrical edges. Therefore, the nominal porosity, \( p_{N} \), becomes

\[ p_{Octa} = \frac{V_{Octa} - V_{a}}{V_{Octa}} = 1 - \frac{3 \sqrt{2} \pi}{16} \left( \frac{a}{t} \right)^2. \]  

(3)

In the case of rhombic dodecahedron unit cell with the edge length, a, the volume is expressed as

\[ V_{Dodeca} = \frac{16}{9} \sqrt{3} a^3. \]  

(4)

By assuming the cylindrical shape of the cell edges, the volume of cell edges in the unit cell is calculated as
V_{Dodeca} = \frac{\pi}{4} t^2 \times \frac{1}{3} \times a \times 24 = 2\pi at^2 \quad (5)

where \( t \) is the diameter of cylindrical edges. Therefore, the nominal porosity becomes

\[ p_{Dodeca}^N = \frac{V_0^{Dodeca} - V_{Dodeca}}{V_0^{Dodeca}} = 1 - \frac{3\sqrt{3}\pi}{8} \left( \frac{a}{t} \right)^2. \quad (6) \]

The nominal porosities of both truncated octahedron cell and rhombic dodecahedron cell are plotted as a function of the normalized edge length in Fig. 2(a). The normalized porosity increases with increasing the normalized edge length. The edge length of the rhombic dodecahedron unit cell is longer than that of the truncated octahedron unit cell.

When the volume of the unit cell is equivalent to the volume of sphere with the diameter of \( d \), each nominal porosity is expressed as

\[ p_{Octa}^N = 1 - 3^\frac{1}{2} \times 2^{-\frac{3}{2}} \times \pi^{\frac{1}{2}} \times \left( \frac{d}{t} \right)^{-2}, \quad (7) \]

\[ p_{Dodeca}^N = 1 - 3^\frac{1}{2} \times (2\pi)^{\frac{1}{2}} \times \left( \frac{d}{t} \right)^{-2}. \quad (8) \]

The nominal porosities are plotted as a function of the normalized cell diameter in Fig. 2(b). Relationship between the normalized porosity and normalized cell diameter of two unit cells becomes almost the same. The nominal porosities of the present titanium foam specimens are 80% and 90%. Since the thickness of all cell edges are fixed at 1 mm, the cell diameters of specimens with 80% and 90% porosities are about 6 mm and 8 mm, respectively.

Open-cell titanium foams with different porosities and cell geometries were manufactured in vacuum through 3D EBM process. Arcam A2X machine designed for titanium alloys was used in this study. Commercially pure titanium, Grade 2, powder was used as a starting material. The chemical composition is shown in Table 1. The shape of the compressive specimens have cylindrical shape with 30 mm in diameter and 30 mm in height. Building direction was parallel to the cylindrical axis. No heat treatment was carried out after EBM process. Mechanical properties of the titanium foam specimens were examined by compressive tests at room temperature using a Shimadzu Autograph AG-50kNISD. Crosshead speed was fixed at 10 mm/min.

### 3. Results

#### 3.1 Effect of porosity

Photographs of cylindrical titanium foam specimens are shown in Fig. 3. Vertical direction is parallel to the compression direction. Thickness of the cell edge is 1 mm in all specimens. It is noted that the cell diameters of truncated oc-
Octahedron unit cells are the same as those of rhombic dodecahedron cells at the same porosity.

Compressive stress-strain curves of ten kinds of titanium foam specimens are shown in Fig. 4. The plateau stresses of Octa_A (80% and 90%) are 28.9 MPa and 7.4 MPa, respectively. The plateau stresses of Octa_B (80% and 90%) are 18.4 MPa and 5.3 MPa, respectively. The plateau stresses of Dodeca_A (80% and 90%) are 17.9 MPa and 3.7 MPa, respectively. The plateau stresses of Dodeca_B (80% and 90%) are 17.3 MPa and 3.7 MPa, respectively. The plateau stresses of Dodeca_C (80% and 90%) are 17.3 MPa and 3.7 MPa, respectively. The plateau stresses of Dodeca_C (80% and 90%) are 17.3 MPa and 3.7 MPa, respectively. Yield stress of dense Grade 2 titanium manufactured through 3D EBM process has been reported as 540 MPa, which was much higher than those of the present titanium foams. The absorbed energy up to 50% strain of Octa_A (80% and 90%) are 6.5 MJ/m$^3$ and 2.2 MJ/m$^3$, respectively. The absorbed energy of Octa_B (80% and 90%) are 4.1 MJ/m$^3$ and 1.6 MJ/m$^3$, respectively. The absorbed energy of Dodeca_A (80% and 90%) are 4.1 MJ/m$^3$ and 1.3 MJ/m$^3$, respectively. The absorbed energy of Dodeca_B (80% and 90%) are 4.1 MJ/m$^3$ and 1.2 MJ/m$^3$, respectively. The absorbed energy of Dodeca_C (80% and 90%) are 5.9 MJ/m$^3$ and 1.8 MJ/m$^3$, respectively. Both the plateau stress and absorbed energy of titanium foams with 80% porosity were higher than those of 90% porosity.

3.2 Effect of shape of unit cell
The plateau stresses of truncated octahedron cells were higher than those of rhombic dodecahedron cells. Energy absorption of truncated octahedron cells were also higher than those of rhombic dodecahedron cells. As shown in Fig. 2(b), the cell diameter of these cells are the same. Therefore, the difference is not due to the size effect.

3.3 Effect of compression direction
The plateau stress and the absorbed energy of Octa_A specimen were higher than those of Octa_B specimen. The plateau stress and the absorbed energy of Dodeca_C specimen were higher than those of Dodeca_A and Dodeca_B specimens. The plateau stress and the absorbed energy of Dodeca_A and Dodeca_B are almost the same. These differences were caused by the anisotropic deformation of open-cell titanium foams.

3.4 Macroscopic shear bands
Photographs of the titanium foam specimens at the compressive strain of 20% are shown in Fig. 5. Octa_A and Dodeca_A specimens showed relatively uniform deformation. On the other hand, the macroscopic shear bands are observed in Octa_B, Dodeca_B and Dodeca_C specimens. Oscillations of the stress-strain curves observed in these specimens [Fig. 4 (b), (d) and (e)] are due to the shear band formation.

4. Discussion
Mechanical properties of the present titanium foam specimens are listed in Table 2. In many metal foams, the relationship between the plateau stress and the relative density has been expressed by

$$\sigma_P = C\sigma_S \left(\frac{\rho^*}{\rho_S}\right)^n.$$  

(9)

where $C$ is the constant, $\sigma_S$ is the yield stress of the cell wall material, $\rho^*$ is the density of the foam, $\rho_S$ is the density of the cell wall material and $n$ is the density exponent. In the present open-cell titanium foams, the value of $n$ is almost equal to two, which is independent of the cell geometry. If the metal foam has completely plateau region, the relationship between the absorbed energy and the relative density becomes identical to that of plateau stress. However, the value of $n$ in energy absorption was slightly smaller than that of plateau stress. This is due to the oscillation of compressive stress-strain curves caused by the formation of macroscopic shear bands.

The strength of the present titanium foams depended on the cell geometry. Titanium foams with truncated octahedron cells showed higher strength than those with rhombic dodecahedron cells. This is due to the cell edge length. In the case of open-cell metal foams, the main mechanism of com-

![Fig. 3 Photographs of open-cell titanium foam specimens manufactured through EBM process. Octa_A (a,b), Octa_B (b-g), Dodeca_A (c-h), Dodeca_B (d-j), Dodeca_C (e,j). Each nominal porosity is 80% (a-e) and 90% (f-j).]
pressive deformation is the bending or buckling of the cell edges. According to the simple beam theory, the short beams show high resistance to bending or buckling. As shown in Fig. 2(a), the cell edge length of the truncated octahedron cells is shorter than that of rhombic dodecahedron cells. Therefore, we can conclude that the truncated octahedron cell geometry is effective to increase the strength compared to the rhombic dodecahedron cell geometry.

The strength of the present titanium foams also depended on the compression direction. This is due to the orientation of the cell edges. Orientation of the cell edges are classified into three types: parallel, perpendicular and oblique cell edges against the compression direction (z-axis). These cell edges are illustrated as light gray, black and dark gray colors in Fig. 6. The difference between Octa_A and Octa_B specimens is due to the number of oblique edges, 24 oblique edges in Octa_A and 18 oblique edges in Octa_B [Table 2]. Oblique cell edges have high resistance to the compressive deformation compared to perpendicular cell edges. Therefore, Octa_A specimen consisting of many oblique cell edges showed high compressive strength compared to Octa_B specimen. On the other hand, Dodeca_C specimen has 6 parallel edges, which have highest resistance to compressive deformation. Therefore, Dodeca_C specimen showed the highest compressive strength compared to Dodeca_A and Dodeca_B specimens, which have no parallel edges. The difference between Dodeca_A and Dodeca_B specimens is due to the same reason as octahedron specimens. Dodeca_A specimen consisting of many oblique edges, 24 oblique edges, showed high compressive strength compared to Dodeca_B specimen consisting of 12 oblique edges.

Present experimental results revealed that the cell geometry of Octa_A lattice was effective to increase the compressive strength and the absorbed energy. This result can be not only valid for titanium foams but also for other metal foams. The formation of shear bands can be discussed. At initial period of linear elasticity, no shear band was formed. After the yielding, the shear band generated in the direction of the maximum shear stress of the cylindrical specimen. In the case of conventional metal foams such as ALPORAS aluminum foams, local deformation region like shear band generates in the perpendicular plane against the compression direction. The difference in the compressive deformation is probably due to two reasons. One is the ordered and homogeneous cell geometry in the present 3D EBM titanium foams. In the case of PM titanium foams with disordered cell geometry, no shear band formation has been observed\textsuperscript{15}. Present cell geometry is similar to the bulk single crystal. Therefore, the shear band during the compression was formed in the direction of maximum shear stress. Another is the high buckling strength of the present titanium foams. Modulus and strength of titanium are higher than aluminum. Therefore, the local buckling was limited and the shear band was formed. It is recommended that the soft cell wall materials are effective to reduce the shear band formation. On the other hand, the shear band formation was limited in the titanium foams with 90% porosity against 80% porosity. In the case of high porosity, the local buckling occurred preferentially because of long cell edges.

5. Conclusions

Open-cell titanium foams with different porosities and unit cell geometries were manufactured through 3D EBM.
Compressive tests at room temperature achieved the following experimental results.

1. Plateau stress and absorbed energy of open-cell titanium foams increased with decreasing the porosity.
2. Plateau stress and absorbed energy of the titanium foams with the truncated octahedron unit cells were higher than those of the titanium foams with rhombic dodecahedron unit cells. This is due to the short cell edges in truncated octahedron unit cells compared to in rhombic dodecahedron cells.
3. Plateau stresses of Octa_A and Dodeca_C specimens were higher than those of other specimens. The reason is due to the orientation of the cell edges. Parallel edges against the compression direction are effective to in-

### Table 2

<table>
<thead>
<tr>
<th>Unit cell</th>
<th>Nominal Porosity, pN (%)</th>
<th>Cell diameter, d/t</th>
<th>Plateau stress, σp/MPa</th>
<th>Energy absorption, W/MJm$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octa_A</td>
<td>80</td>
<td>5.7</td>
<td>28.9</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>8.0</td>
<td>7.4</td>
<td>2.2</td>
</tr>
<tr>
<td>Octa_B</td>
<td>80</td>
<td>5.7</td>
<td>18.4</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>8.0</td>
<td>5.3</td>
<td>1.6</td>
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<tr>
<td>Dodeca_A</td>
<td>80</td>
<td>5.8</td>
<td>17.9</td>
<td>4.1</td>
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<td></td>
<td>90</td>
<td>8.2</td>
<td>4.7</td>
<td>1.3</td>
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<tr>
<td>Dodeca_B</td>
<td>80</td>
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<td>17.3</td>
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<tr>
<td></td>
<td>90</td>
<td>8.2</td>
<td>3.7</td>
<td>1.2</td>
</tr>
<tr>
<td>Dodeca_C</td>
<td>80</td>
<td>5.8</td>
<td>23.9</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>8.2</td>
<td>5.4</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Fig. 5 Photographs of the compressive test specimens at the compressive strain of 20%. Macroscopic shear bands are observed in Octa_B, Dodeca_B and Dodeca_C specimens.

Fig. 6 Parallel, perpendicular and oblique cell edges against the compression direction are colored by light gray, black and dark gray, respectively. (a) Octa_A, (b) Octa_B and (d) Dodeca_B consist of perpendicular and oblique cell edges. (c) Dodeca_A consists of only oblique cell edges. (e) Dodeca_C consists of parallel and oblique cell edges.
crease the initial flow stress.

(4) Metal foams consisting of Octa-A lattice have a potential for energy absorbing applications.

In addition, the macroscopic shear bands were formed in some specimens. It caused the decrease in the absorbed energy. Mechanism of the shear band formation can be explained by both the ordered cell geometry and the buckling strength of the base material. Disordered cell geometry and soft material are probably effective to reduce the shear band formation. These results are not only valid for the present titanium foams but also for other metal foams such as aluminum foams.

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REFERENCES