Surface Morphology Analysis Using Fractal Theory in Micro Electrical Discharge Machining

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Surface morphology obtained using micro electrical discharge machining (micro EDM) not only provides important information, such as wear and lubrication properties, but also shows many micro peaks and craters, which are very different from that machined using traditional methods. Hence, surface morphology evaluations in micro EDM processes using traditional evaluation parameters are not appropriate. In this study, a novel evaluation method, which includes a wavelet filter and the fractal theory, was proposed and analyzed in detail to evaluate the surface morphology in order to achieve the required precision and concision. First, a novel evaluation method was proposed, and the effectiveness of the method was investigated. Second, a series of relevant experiments based on titanium alloys and high-speed steels were conducted to validate the effectiveness of the new method, the analysis results were presented. Third, the fractal trend in the surface morphology was summarized based on the titanium alloy and high-speed steel materials; the rules governing the relationship between the fractal dimension and the surface morphology were proposed. Finally, the phenomena observed in the analysis process were investigated and discussed in detail.

1. Introduction

Micro EDM is an advanced machining method because of a number of advantages. It can be used to machine materials that possess high hardness and strength. Moreover, the contact between the electrodes and the machined workpieces is direct; hence, micro EDM is a popular machining technique to fabricate small, complicated, and high-precision components. Various parameters influence the machining performance of micro EDM processes, such as material removal and tool wear rate, but the surface morphology is the most important among them. The conventional method of evaluating the surface morphology in micro EDM processes using the arithmetic mean deviation (Ra) or root mean square (RMS) of the profiles is no longer applicable. Moreover, Zhang, Cui, Ding, and Guo studies and Ramasawmy and Blunt consider that the conventional method is unsuitable because the machining accuracy resulting from micro EDM processes is considerably higher than that of the conventional method. In addition, the Ra of the profiles for the conventional method, which mainly assesses surface roughness for a wide range, is not accurate enough. Therefore, a variety of methods is introduced to solve this problem. Polynomial fitting is a convenient and efficient way that has the ability to handle less complicated data, and its main advantage is the reasonable flexibility, which is free from the influence of measuring areas. The method of filter is another prevailing approach for roughness evaluations; it mainly consists of Gaussian and wavelet filters. The former is an ideal filter under the condition that the surfaces of the workpieces are regarded as random processes, and the latter can separate the signals from the feature of the surface structures and has good time-frequency localization feature. In addition, the method of motif is a technique that can evaluate the roughness and waviness completely using only seven parameters.

There are various types of approaches to characterize the surface morphology of a workpiece, but the diversity and complexity make it difficult to represent the surface morphology perfectly without using various parameters. Furthermore, this results in complex parameters especially when they are linked with the machining parameters. Therefore, a concise method should be developed to evaluate the surface morphology easily and clearly, and the fractal theory has become considerably popular in recent years, which has mainly one evaluation parameter—the fractal dimension. In the field of machining, a few researchers have made effective use of fractal analysis. Li, Xie, and Deng tried to find the relationship between the wetting properties and the characterization of irregular micro structured surfaces, and the fractal dimension is an important parameter, which measured the contact angle in their research. Zhang, Lu, and Friedrich show that once the weights of the materials in the wear mechanism changed, the fractal dimension of the wear debris may vary with the weights in their study of the wear behavior. Yin and Komvopoulos conducted a number of experiments based on the slip-line model; they integrated this model with another described by fractal geometry in order to establish the connections between the fractal parameters and other properties of the surface, and then they obtained numerous important results. Therefore, surface morphology, which has its own characteristics that contain several micro peaks and craters, includes useful information that reflects the properties of machining, and is an essential factor that it is significant for evaluating these properties using the fractal theory. The objective of this study is to combine the machining parameters with the fractal theory and investigate the relationship between the surface morphology characterized by the fractal analysis and the processing rules, which has not been established.

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2. Materials and Methods

2.1 The characteristics of the surface morphology in micro EDM

Figure 1 shows that different surface morphologies are obtained by different machining methods, and the surface morphology obtained by traditional grinding has a clear texture, but that machined by micro EDM does not possess this property due to the randomness of the discharge. Figure 2 shows the Ra and RMS of the high-speed steel machined using micro EDM, and it fails to establish the relationship between the discharging energy and the surface morphology. Hence, it is hard to evaluate the surface morphology resulting from micro EDM processes using traditional evaluation methods such as the Ra and RMS. As the surface morphology machined by micro EDM has fractal characters, the newly proposed method in this study can serve as a new tool showing good accuracy and concision in evaluating these types of surfaces. Therefore, the traditional evaluation method seems unsuitable when the surface morphology includes craters, peaks, blowholes, and cracks, and they are the result of the high energy discharge, solidification of workpiece materials, vaporization of the dielectric, and thermal stress.

2.2 Fractal theory and main fractal methods

Fractal geometry is a new branch of science, which considers irregular-geometry morphology, and the objects that have the character of fractal geometry are almost universal such as mountain ranges, coastlines, rivers, and clouds. The examples mentioned above all have the character of self-similarity, which means that a partially enlarged view of a fractal object resembles its complete view. If an object has a fractal feature, then the research issue may be simplified using only a few parameters. The traditional methods may be very complex because the conventional models are often related to several parameters, which increase the difficulty of analysis.

Evaluating the surface morphology of micro EDM processes using the profile curves of the surface is an important approach. Furthermore, the profile curves of the surface morphology of micro EDM processes may be analyzed using the fractal theory because of properties such as irregularity and self-similarity. There are different definitions and computing methods for fractal dimension, and no common approach that evaluates all the fractal objects, because their properties vary and the effects of different approaches are distinct. In this study, a power spectrum method, which represents the traditional way, a box-counting method, which represents the common way, and a structure function method, which represents the new way, were checked respectively to investigate a suitable method to evaluate the surface morphology of micro EDM processes. The expression for the power spectrum method\(^\text{[11]}\) is as follows:

\[
P(\omega) = \frac{1}{|\omega|^{5-2D}}
\]  

where \(P(\omega)\) is the power spectrum; \(\omega\) is the frequency, and \(D\) is the fractal dimension. First, a set of frequencies is processed by Fourier transform in practice, and then a double logarithmic curve is plotted; if \(k\) is regarded as the slope of the curve, the fractal dimension can be calculated using the following equation:

\[
D = 2.5 - k
\]  

The expression for the box-counting method\(^\text{[12]}\) is as follows:

\[
D = -\lim_{r \to 0} \frac{\log N(r)}{\log r}
\]  

where \(r\) is the size of the measuring box, and \(N(r)\) is the number of boxes required.

If the profile curves are regarded as spatial sequences, the expression for the structure function method\(^\text{[13]}\) can be written as follows:

\[
S(\tau) = \left\langle [Z(x + \tau) - Z(\tau)]^2 \right\rangle = C\tau^{4-2D}
\]  

As the data of the profile curves extracted in practice are discrete, the heights of the profile curves are set to \(x[i]\) (\(i = 0, 1, \ldots, N-1\)), where \(N\) is the sample number, and then the expression of its discrete form can be expressed as follows:
\[
S(n) = \sum_{i=0}^{N-n} (x(i + n) - x(i))^2 \quad (2-5)
\]
Equation (2-5) can be simplified to the following expression according to the theory of fractional Brownian motion:
\[
S(n) = G \cdot n^{4-2D} \quad (2-6)
\]
where \(G\) is constant, and then a double logarithmic curve is plotted; if \(k\) is regarded as the slope of the curve, the fractal dimension can be calculated using the following equation
\[
D = 2 - k/2 \quad (2-7)
\]

2.3 Experiments based on two different workpieces

In the field of aerospace, titanium alloys and high-speed steels are widely used, and hence were chosen as the workpieces in this study. Thereafter, several set of relevant experiments were conducted; resistance–capacitance pulse generator was used, and a series of micro-holes were drilled. Ultimately, the workpieces were observed under atomic force microscope (AFM), as seen in Fig. 3, where \(C\) is the capacitor; \(U\) is the voltage, and \(E\) is the discharging energy, the discharging time of No.1–No.4 and No.9–No.12 is 160 ns, the discharging time of No.5–No.8 and No.13–No.16 is 380 ns.

2.4 Evaluation rule

First, the calculation methods for the fractal dimension should be selected; the power spectrum method, the box-counting method, and the structure function method are the three typical approaches in the field of the fractal theory, and hence, they were analyzed to check whether the surface morphology obtained using micro EDM had the characters of fractal dimension. In this process, as Fig. 4 shows, a random outline of these workpieces was extracted, and then the three methods were used to make comparisons (eight profiles would be extracted from one workpiece in the whole evaluation process).

Second, after a suitable fractal method was selected, the raw data was required to be de-noised. Filter is an inevitable procedure to analyze the data. For the surface morphology of the workpiece undergoing micro EDM, the high precision requires the filter to have the ability to retain maximum low frequency signals and remove the high frequency noise as far as possible. Although there are several filters to choose, the wavelet method is better because it is able to reconstruct the function, which has a perfect feature of linear or higher order polynomial; a low entropy is observed, which results from the sparsely distributed wavelet coefficients, and good de-correlation, which is the main advantage because the signals are...
de-noised in the wavelet domain instead of the frequency domain. That is, the wavelet method is an advanced filter tool that can de-noise the signal far better than traditional methods, which is based on the theory of Fourier transform or other algorithms. Therefore, it was chosen as the filter tool to investigate the relevant data. Moreover, the profiles of DB8 wavelet basis were similar to that of the surface morphology obtained using micro EDM processes. The surface morphology was analyzed using wavelet filter whose wavelet basis is DB8. In this process, the fractal dimensions of eight profiles that belonged to each workpiece were calculated respectively, according to filter layers. The fractal dimension varies according to the filter layers, and hence, if the filter layers are four, in one workpiece, 32 groups of fractal dimensions (every layer has eight groups of fractal dimensions) will be obtained.

Finally, the fractal dimensions of eight profiles were summarized to one group of data that represented the surface morphology of a workpiece according to the filter layers, for example, if the filter layers are four, four groups of fractal dimensions (every layer has one group of fractal dimension) will be obtained, and then the data of each workpiece collected according to the filter layers were plotted in some particular order such as discharging energy; the relationship between the machining parameters and the fractal dimensions was established, and a suitable wavelet filter layer was found at the same time.

In summary, the novel evaluation rule can be divided into six steps, as seen in Fig. 5.

3. Results and Discussion

3.1 Results

3.1.1 Titanium alloy

A random outline of the workpieces, as seen in Fig. 4, was extracted and analyzed by using the power spectrum method, the box-counting method, and the structure function method, and the results were presented in Fig. 6. It could be seen that the linearity of the power spectrum method was so poor that it seemed unsuitable for the evaluation of the surface morphology obtained from micro EDM processes, and the other two methods had better linearity. Hence, in the following section, both the methods are used to analyze the related data, and then a suitable method is selected.

As Fig. 4 shows, for every workpiece, eight profiles, which
include three horizontal, three vertical, and two diagonal profiles, were extracted to calculate the fractal dimension. Therefore, the fractal dimensions calculated using the two fractal methods with the condition of one, two, three, and four layers of wavelet filter were shown in Fig. 7 respectively. In the Figures, “the sample numbers” in the horizontal ordinate represented the profiles extracted from the workpiece (for example, Sample Number 1 implies Profile 1); it could be found from the four Figures that the variation in the fractal dimensions was in the range of approximately 0.15, which demonstrated isotropic property. It could be concluded from Fig. 7 that the values of the fractal dimensions decreased with the increment of the layers of the wavelet filter, which meant that the values of the fractal dimensions were considerably different from the real situation of the profiles.

The fractal dimensions of the eight profiles of each workpiece were summarized to one datum, and then the data of all the workpieces were collected and plotted according to the filter layers, as seen in Fig. 8. It could be found from Fig. 8 that in the box-counting method, the overall trend in the fractal dimension was from high level to low level under the condition of a single layer and double layer of the wavelet filter. However, when they were filtered using three and four layers, the above trend was not observed; hence, it indicated that the fractal dimension analyzed using the box-counting method may be inaccurate when the fractal dimension of the surface morphology was sufficiently small. This phenomenon made the curves of the box-counting method to fluctuate in a disorderly manner, and hence, it was unsuitable for the analysis of the surface morphology obtained from micro EDM processes. In the structure function method, it could be seen that the overall trend in the fractal dimension was from high level to low level under the condition of all the filter layers of the wavelet. Considering the difference between the results of the filter layers of the wavelet in the box-counting method and the structure function method, the main conclusion was the uniform trend in the structure function, in which the fractal dimension decreased with the increment of the discharging energy under the condition of constant discharging time (decided by the value of the capacitor). This implies that the filter layer is irrelevant to the consistency of the trend in the structure function method, which shows the relationship between the fractal dimension and the discharging energy. Therefore, it provides a new perspective of evaluating the surface morphology obtained from micro EDM processes.

Above all, the structure function method is more compatible than the box-counting method for the analysis of the surface morphology obtained from micro EDM processes. Under the condition of constant discharging time (decided by the value of the capacitor) – larger the discharging energy, lower the fractal dimension—is the trend that is observed. However, for the fractal theory, fractal dimension is a relative concept, which cannot simply define that a high fractal dimension represents a rough surface, while a low fractal dimension represents a smooth one. Based on this, it is influential to combine the fractal dimension with the real circumstance of the surface. Combining Fig. 3 with the rules, and it indicates that higher the fractal dimension, finer the surface morphology, while lower the fractal dimension, rougher the surface morphology. Furthermore, the one-filter layer is suitable for the surface morphology obtained from micro EDM processes.

**3.1.2 High-speed steel**

High-speed steel was chosen as another workpiece, and the machining conditions were the same as that of titanium alloys, as seen in Fig. 4. The mode of sampling data was identical to that of the titanium alloy, and the mode of the data processing was based on the rules mentioned previously, in which the surface morphology was de-noised by one layer of wavelet filter for which the wavelet basis was DB8. Moreover, the fractal dimension of the surface morphology using the structure function method was calculated and the overall
trend in the eight surface morphologies was showed to investigate whether isotropic property was confirmed, and combined the real situation of the surface morphology gauged by the AFM with the overall trend of the fractal dimension.

Figure 9 shows the fractal dimension of 64 profiles of the high-speed steels, and thereafter the fractal dimensions of all the eight profiles were averaged to one and were plotted in Fig. 10. It shows that, similar to the trend in the titanium alloy, the values of the fractal dimensions varied from a high level to a low level. Correlating this trend with Fig. 3, a rule could be found that larger the discharging energy, lower the fractal dimension, and smaller the discharging energy, higher the fractal dimension under the condition of the same discharging time (decided by the value of the capacitor). Furthermore, under the condition of the same discharging time (decided by the value of the capacitor), the fine surface morphology results in a higher fractal dimension, and the rough surface morphology leads to a lower fractal dimension, which is similar to the relation in the titanium alloy summarized previously.

Above all, the rules found in the titanium alloy are also suitable for the high-speed steel, while it is hard to determine the traditional parameters, such as Ra and RMS, for the assessment criterion of surface morphologies obtained from micro EDM processes.

3.2 Discussion

Considering the algorithms of the fractal methods, for the box-counting method, after the level of scale is determined, the number of boxes required is sealed, and in the process where the level of scale approaches zero, the number of boxes required will have a corresponding value. Moreover, its fractal dimension is defined by the gradient of the least square fitting, which is obtained from calculating the logarithm of the number of boxes and the level of scale. The mechanism of this method is more suitable for continuous function, while the sample numbers of the surface morphology are clearly discrete. When the data used are discrete, the results may be unstable and error may occur. Compared to the box-counting method, the expression for the structure function method can be discretization, and the core of this method is the differences between every point and the remaining points; the differences are squared to be the results, which will be averaged to characterize the level of measure. This mechanism, on one hand, is compatible with the discrete data, and on the other hand, considers all the points of the experimental data; hence, it improves the computational accuracy and reduces the error. According to the definition of the fractal theory, which is
entirely different from the definition of the Euclidean geometry, the fractal dimension of the ideal straight line is 1. Note that in Fig. 7 (c) and (d), the fractal dimensions are approximately 1.1, but the profiles of the surface morphology deviate from the straight line, and the fractal dimension of the sample numbers may not describe the real situation of the eight pro-

Fig. 8 The overall trend of the fractal dimension. (a) The fractal trend of No.1–4 (left) and No.5–8 (right) titanium alloys by one layer of the wavelet filter, (b) The fractal trend of No.1–4 (left) and No.5–8 (right) titanium alloys by two layers of the wavelet filter, (c) The fractal trend of No.1–4 (left) and No.5–8 (right) titanium alloys by three layers of the wavelet filter, (d) The fractal trend of No.1–4 (left) and No.5–8 (right) titanium alloys by four layers of the wavelet filter.
files accurately and integrally. Moreover, the fractal dimension is lower because more layers of the wavelet filter misses the useful low frequency signals, and at the same time eliminates the high frequency noise. Therefore, the previous experience that the suitable filter layers of wavelet would be four layers seems useless for the analysis of the surface morphology obtained from micro EDM processes using the integrated method, which includes the fractal theory and the wavelet method.

From the viewpoint of the wavelet filter, the surface morphology of one layer filter, already demonstrates the principal characters of the former original rendering. Moreover, the interference due to the high frequency noise is removed effectively. Hence, it can be inferred from the above discussion that a single layer of the wavelet filter is more suitable for the analysis of the surface morphology obtained from micro EDM processes synthesizing each type of the above situation.

Comparing the analysis result of the titanium alloys with that of the high-speed steels, a conclusion can be drawn that the machining rules are mainly determined by the processing method and irrespective of the type of workpiece, because micro EDM is a method that involves random discharge, irrespective of the type of workpiece. On the surface, this character of micro EDM leads to the uniform distribution of the craters and peaks; the surface machined using micro EDM is isotropy.

For workpieces machined by micro EDM, the fractal dimension of surface morphology is hardly affected by the directions of surface morphology profiles due to the process of random discharge. When the workpiece is machined by micro wire electrical discharge machining (micro WEDM), the fractal dimension of surface morphology will be influenced by the directions of surface morphology profiles, this is because on the one hand, the existence of wire-cut machining direction causes the generation of the surface morphology follows a sequential order; on the other hand, the shaking of the wire electrode in the machining process leads to different degrees of fluctuation on the surface morphology. These phenomena affect the fractal dimensions if the line profiles are extracted from different direction. According to the presented evaluation method, the extracted line profiles will have a wide range of the fractal dimensions, so workpieces machined by micro WEDM are not suitable for the evaluation method.

As to the relationship between $Ra (RMS)$ and fractal dimension, From Fig. 2 and Fig. 10, it could be found that sometimes larger $Ra (RMS)$ induces the higher fractal dimension, sometimes not. The link between fractal dimension and $Ra (RMS)$ is not obvious due to their different definition, according to the definition of fractal dimension, it reflects the fine degree of the surface morphology; while the value of $Ra (RMS)$ reflects the fluctuation of the surface morphology. Whereas, $Ra (RMS)$ and fractal dimension are both useful parameters for evaluation of surface morphology in micro EDM, In the case of same discharging energy, such as No.3 and No.5 ($E = 8.91 \mu J$), it is hard to distinguish between them using only by fractal dimension. Note that the different discharging time between No.3 and No.5 (No.4 and No.6), the former is 160 ns, while the latter is 380 ns. As seen in Fig. 2, it could be found that when the discharging energy is $8.91 \mu J$ (15.84 $\mu J$), then the $Ra (RMS)$ of which discharging time is 160 ns is always lower than the $Ra (RMS)$ of which discharging time is 380 ns. In the view of mechanism, when the discharging time is equal, the surface morphology is mainly dominated by discharging time, once the discharging time is
longer, then the peaks on the surface will be higher and the
craters on the surface will be deeper, resulting in higher value
of surface roughness. So, longer discharging time leads to
higher value of Ra (RMS) under the condition of the same
discharging energy.

4. Conclusion

In this study, a novel evaluation method that combined the
wavelet filter with the fractal theory was proposed, and it is
suitable for the evaluation of surface morphologies of titan-
um alloys and high-speed steels obtained from micro EDM
processes. From the above analysis and discussions, the fol-
lowing conclusions can be drawn.

1) It is hard to evaluate the surface morphology obtained
from micro EDM processes using traditional evaluation
methods such as Ra and RMS. As the surface morphology
obtained from micro EDM processes has fractal characters,
the newly proposed method in this study can serve as a new	tool showing good accuracy and concision in evaluating these
types of surfaces.

2) A single layer of the wavelet filter is more suitable for
the analysis based on the fractal theory, which demonstrates
the principal characters of the former original rendering, and
at the same time, the interference due to the high frequency
noise is removed effectively.

3) In the analysis of the surface morphology obtained from
micro EDM processes, the structure function method is a
more suitable way because its expression can be transformed
to a discrete version, and the calculation principle considers
the relationship between every point and the remaining
points. Furthermore, it improves the accuracy and reduces the
error.

4) The values of the fractal dimensions increase with the
decrease in the discharging energy, which means that higher
the fractal dimension, finer the surface morphology, and lower
the fractal dimension, rougher the surface morphology, under
the condition of constant discharging time (decided by the
value of the capacitor). If discharging energy is the same, Ra
(RMS) can be useful parameters to determine which work-
piece has the longer discharging time, the higher the value of
Ra (RMS) is, the longer the discharging time is. In addition,
this machining rule is irrespective of the type of workpiece.

Above all, for the titanium alloy and high-speed steel ma-
terials, which are widely used in the field of aerospace, these
rules can serve as guidance and references for relevant re-
searchers and workers to conduct related experiments and
process the associated workpieces required. In addition, most
of the experiments are already completed by the authors of
this study to investigate more details of the connection be-
tween the fractal dimension of the surface morphology ob-
tained from micro EDM processes and a variety of machining
parameters. Moreover, it can be expected that more useful
information and important guidelines will be found to pro-
mote relevant research in the field of evaluation of the surface
morphology obtained from micro EDM processes.

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