X-ray Study of Phason Strains in an AlCuFeMn Decagonal Phase

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The peak profiles, shifts, and splittings of Bragg reflections, which are inherent in the phason strain of a quasicrystal, are carefully examined using a precise powder X-ray diffraction method for a wide range of different AlCuFeMn alloys and annealing temperatures. Based on the peak-shift dependences of the Bragg reflections on the phason momentum $Q_{\perp}$, two kinds of new approximant phases are identified. These phases have linear phason strains $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, and $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$, corresponding to lattice spacings $a = 3.814$ nm and $b = 2.005$ nm, and $a = 9.985$ nm and $b = 2.005$ nm in the quasiperiodic plane of the orthorhombic system, respectively. [doi:10.2320/matertrans.M2016446]

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1. Introduction

The broadening and shifting of Bragg reflection peaks by characteristic strains are of great relevance to quasicrystal stability and to the transformation mechanism from a quasicrystal to an approximant crystal phase. A suitable linear phason strain can transform a decagonal quasicrystal to a one-dimensional quasicrystal or to an approximant crystal, and cause peak shifts that increase monotonically with the phason momentum $Q_{\perp}$ ¹⁾. We have studied the broadening and shifting of Bragg reflection peaks for decagonal phase of Al-Ni-Co^{2,3)} and Al-Pd-Mn⁴⁾ by the high-resolution single-crystal X-ray diffraction.

In an AlCuFeMn alloy system, Tasi *et al.*⁵⁾ discovered a one-dimensional quasicrystal composed of Al₆₅Cu₂₀Fe₁₀Mn₅ with a characteristic periodicity of 1.84 nm. Furthermore, Yang *et al.*⁶⁾ discovered two new one-dimensional quasicrystals, also composed of Al₆₅Cu₂₀Fe₁₀Mn₅, with characteristic periodicities 2.99 and 4.14 nm. And they discussed the relation between linear phason strains and the peak shifts observed in these phases, and showed that these three phases, with lattice constants 1.84, 2.99, and 4.14 nm, have linear phason strains of τ^{-6} , $-\tau^{-8}$, and $-\tau^{-6}$, respectively.

Until now, neither the ideal decagonal phase nor any related approximant phase has been found in an AlCuFeMn alloy system. The aim of the present work is therefore to search for this ideal decagonal phase or for some new decagonal phase showing an appropriate linear phason strain and to discuss the phason strains and phase transformations, in a wide range of AlCuFeMn alloys with different compositions to Al₆₅Cu₂₀Fe₁₀Mn₅.

2. Experiment

AlCuFeMn alloy ingots, formed in the composition area with manganese much more preferentially than the one-dimensional quasicrystal Al₆₅Cu₂₀Fe₁₀Mn₅ were prepared by melting highly pure Al (99.99%), Cu (99.99%), Fe (99.99%), and Mn (99.99%) in an Ar atmosphere using an arc furnace.

These ingots were sealed in a quartz tube. After annealing

at 800°C for 72 h and quenching in water, all the ingots produced were divided into two specimens. One specimen was used without further processing, while the other was re-annealed at 880°C for 72 h and again quenched in water. The specimens were crushed and ground into fine powder using an agate mortar. X-ray diffraction experiments were performed using CuK α radiation with a powder diffractometer PANalytical X'pert Pro. The CuK β radiation was excluded by a Ni-filter. Rapid measurements were achieved with a silicon semiconductor array detector. Since Fe and Mn atoms produce a strong X-ray fluorescence background in diffraction patterns, weak Bragg reflections were absorbed into the background when applying the usual scan-speed. In order to measure the peak shifts precisely, the scan-speed was therefore strongly reduced over a limited angular range, to display the distinct weak peaks above the background. The position resolution of this detector corresponds to the width of the individual semiconductor devices. The reciprocal-space momentum resolution is 0.0065 nm⁻¹, estimated from the peak profile of Si powder diffraction.

Within a quasiperiodic plane, a reciprocal-lattice vector Q is specified using a five-dimensional (5D) indexing scheme¹⁾:

$$\begin{aligned} e_i^{\parallel} &= \sqrt{\frac{2}{5}} \left(\cos \frac{2\pi}{5} i, \sin \frac{2\pi}{5} i \right) & Q_{\parallel} &= \frac{2\pi}{A_{5D}} \sum_i n_i e_i^{\parallel} \\ e_i^{\perp} &= \sqrt{\frac{2}{5}} \left(\cos \frac{4\pi}{5} i, \sin \frac{4\pi}{5} i \right) & Q_{\perp} &= \frac{2\pi}{A_{5D}} \sum_i n_i e_i^{\perp} \end{aligned} \quad (1)$$

where $i = 0, 1, 2, 3$, or 4 , $Q_{\parallel}$ denotes the measured momentum transfer vector, and $Q_{\perp}$ the phason momentum vector. We will henceforth use a five-dimensional (5D) indexing scheme to specify the peak indices along two orthogonal two-fold symmetry axes in the quasiperiodic plane as $m0\bar{n}\bar{n}0$ and $0mn\bar{n}\bar{m}$. By introducing an appropriate linear phason strain, the Bragg peaks move from positions corresponding to a decagonal symmetry to those of a periodic symmetry. The peak shift produced by the linear phason strain is given by

$$\Delta Q_{\parallel} = M \cdot Q_{\perp}, \quad (2)$$

where M is the phason matrix

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$$M = \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{pmatrix}. \quad (3)$$

Thus, in powder X-ray diffraction, the ten equivalent reflections represented by $m0\bar{n}\bar{n}0$ in a decagonal symmetry separate into one kind of two equivalent reflections represented by $m0\bar{n}\bar{n}0$, and two kinds of four equivalent reflections represented by $n0\bar{m}0n$ and $0m0\bar{n}n$ with rectangular symmetry. Moreover, the ten equivalent reflections represented by $0mn\bar{n}\bar{m}$ under a decagonal symmetry separate into one kind of two equivalent reflections represented by $0mn\bar{n}\bar{m}$ and two kinds of four equivalent reflections represented by $nm0\bar{m}\bar{n}$ and $mn\bar{n}0\bar{m}$ with rectangular symmetry.

For example, if a linear-phason strain θ_1 is introduced, the $10\bar{1}\bar{1}0$ reflection has periodic indices $h = 3, 5, 8$, and 13 for $\theta_1 = -\tau^{-4}, \tau^{-6}, -\tau^{-8}$, and τ^{-10} , respectively. On the other hand, if a linear-phason strain θ_2 is introduced, the $011\bar{1}\bar{1}$ reflection has periodic indices $k = 3, 5$, and 8 for $\theta_2 = \tau^{-5}, -\tau^{-7}$, and τ^{-9} , respectively. One-dimensional periodic distances are thus calculated from the shifts in the peak positions.

Moreover, by adding the index l along the ten-fold symmetry periodic-axis, the above indices become $m0\bar{n}\bar{n}0l$ and $0mn\bar{n}\bar{m}l$, respectively. In this paper, we calculate the reciprocal-space momenta $Q_{\parallel}$ and $Q_{\perp}$ corresponding to $Q_{\parallel} = 1/d = 2\sin\theta/\lambda$ (without the factor 2π).

3. Results

Precise measurements of the Bragg peak profiles and positions were done using powder X-ray diffraction. Figure 1 shows the resulting patterns for $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$ annealed at 800°C . Each Bragg reflection is indexed in the manner described above. All the observed reflections are consistent with the strong electron-diffraction spots in the one-dimensional quasicrystal discovered by Tasi *et al.*⁵⁾

The peak profiles of individual decagonal reflections are asymmetrical or are split asymmetrical. The direction of the asymmetry reflects the sign of $Q_{\perp}$, while the size of the shifts is proportional to $Q_{\perp}$. This effect is indicative of linear phason strain. To analyze this requires an accurate measurement of the reflections with a large $Q_{\perp}$. Long-duration X-ray measurements were therefore done carefully between 2.5 and 3.2 nm^{-1} to cover a range of $Q_{\parallel}$. Four peaks, indexed as $10\bar{1}\bar{1}00$, $10\bar{1}\bar{1}01$, $00\bar{1}\bar{1}03$, $10\bar{1}\bar{1}02$, were observed in separate

Fig. 1 X-ray diffraction patterns for $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$ annealed at 800°C . (a) Clear peaks are designated $m0\bar{n}\bar{n}0l$ or $0mn\bar{n}\bar{m}l$. (b) The most intense peak overlaps several peaks.

samples of different compositions (respectively, $\text{Al}_{65}\text{Cu}_{24}\text{Fe}_3\text{Mn}_8$, $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_7\text{Mn}_{10}$, $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_9\text{Mn}_8$, and $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$) annealed at 800°C and 880°C . Shifted peak positions were calculated and displayed together with the measured X-ray diffraction patterns (Figs. 2–4), and listed in Table 1.

Figure 2(a) shows the powder X-ray diffraction patterns for $\text{Al}_{65}\text{Cu}_{24}\text{Fe}_3\text{Mn}_8$ annealed at 800°C . In these patterns, the peaks correspond to a decagonal type and an FCC type with $a = 0.590 \text{ nm}$ indicated by the arrows. For peaks of the decagonal type, the peak positions calculated using $\theta_1 = \tau^{-6}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6361 \text{ nm}$, and $c = 1.234 \text{ nm}$ most accurately reproduce the peak shifts and splittings. Here, $\theta_1 = \tau^{-6}$ and $A_{5D} = 0.6361 \text{ nm}$ result in moving the $10\bar{1}\bar{1}00$ peak to the periodic position with index 5 and reciprocal-space momentum 2.6242 nm^{-1} . Similarly, $\theta_2 = \tau^{-5}$ and $A_{5D} = 0.6361 \text{ nm}$ move the $011\bar{1}\bar{1}0$ peak to the periodic position with index 3 and reciprocal-space momentum 2.9949 nm^{-1} . These linear phason strains θ_1 and θ_2 therefore correspond to the periodicities 1.905 and 1.002 nm in the one-dimensional problem, respectively. The peaks indicated by arrows represent an NaCl-type structure because the clearly observed peaks obey the rule $h + k + l = 2n$. On the other hand, one half of an $\text{Al}_{65}\text{Cu}_{24}\text{Fe}_3\text{Mn}_8$ ingot was melted during re-annealing at 880°C . Figures 2(b) and 2(c) show the X-ray diffraction patterns for $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_7\text{Mn}_{10}$ annealed at 800°C and re-annealed at 880°C , respectively. The former result shows the same linear phason strain as was mentioned above. On the other hand, the latter result shows a new approximant phase of decagonal quasicrystal and a second crystalline phase. In Fig. 2(c), the splitting of the asymmetrical double peaks is clearly narrower than in Fig. 2(b). The shifts and splittings of the decagonal-type peaks in Fig. 2(c) are best reproduced with $\theta_1 = \tau^{-10}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6368 \text{ nm}$, and $c = 1.234 \text{ nm}$. Here, $\theta_1 = \tau^{-10}$ makes the $10\bar{1}\bar{1}00$ reflection move to the periodic position with index 13 and reciprocal-space momentum 2.6033 nm^{-1} , while $\theta_2 = \tau^{-5}$ makes the $011\bar{1}\bar{1}0$ reflection move to the periodic position with index 3 and reciprocal-space momentum 2.9916 nm^{-1} . The linear phason strains θ_1 and θ_2 correspond to the one-dimensional periodicities 4.994 nm and 1.003 nm , respectively.

Figures 3(a) and 3(b) show the powder X-ray diffraction patterns for $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_9\text{Mn}_8$ annealed at 800°C and re-annealed at 880°C . In Fig. 3(a), $\theta_1 = \tau^{-6}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6361 \text{ nm}$, and $c = 1.230 \text{ nm}$ yield the most accurate the peak shift. On the other hand, Fig. 3(b) shows a one-dimensional quasi-crystalline phase and another crystalline phase. Each decagonal-type peak is very narrow compared to those in Fig. 3(a). The shifts and splittings of the decagonal-type peaks in Fig. 3(b) are best reproduced with $\theta_1 = -\tau^{-8}$, $\theta_2 = 0$, $A_{5D} = 0.6340 \text{ nm}$, and $c = 1.2285 \text{ nm}$. Here, $\theta_1 = -\tau^{-8}$ results in the $10\bar{1}\bar{1}00$ reflection moving to the periodic position with index 8 and reciprocal-space momentum 2.6035 nm^{-1} . This linear phason strain θ_1 corresponds to the one-dimensional periodicity 3.0728 nm . The phase transformation from the approximant to the one-dimensional quasicrystal is therefore produced by re-annealing at 880°C .

Figures 4(a) and 4(b) show the powder X-ray diffraction patterns for $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$ annealed at 800°C and re-annealed at 880°C . In Fig. 4(a), setting $\theta_1 = -\tau^{-6}$, $\theta_2 = 0$, $A_{5D} =$

Table 1 Calculation of the ideal peak position and the split-peak positions for decagonal quasicrystals.

Index $m0\bar{n}\bar{n}0l$	Ideal peak position/nm ⁻¹	Split-peak positions/nm ⁻¹		
		$m0\bar{n}\bar{n}0l$	$n0\bar{m}0nl$	$0m0\bar{n}nl$
(a) Al ₆₅ Cu ₂₄ Fe ₃ Mn ₈ at 800°C; $\theta_1 = \tau^{-6}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6361$ nm and $c = 1.234$ nm				
10 $\bar{1}\bar{1}$ 00	2.6030	2.6242	2.5788	2.6174
10 $\bar{1}\bar{1}$ 01	2.7263	2.7465	2.7032	2.7399
00 $\bar{1}\bar{1}$ 03	2.9152	2.8965	2.9375	2.9033
10 $\bar{1}\bar{1}$ 02	3.0664	3.0844	3.0459	3.0785
(b) Al ₆₅ Cu ₁₈ Fe ₇ Mn ₁₀ at 800°C; $\theta_1 = \tau^{-6}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6364$ nm and $c = 1.230$ nm				
10 $\bar{1}\bar{1}$ 00	2.6018	2.6230	2.5775	2.6157
10 $\bar{1}\bar{1}$ 01	2.7259	2.7461	2.7027	2.7392
00 $\bar{1}\bar{1}$ 03	2.9214	2.9027	2.9436	2.9093
10 $\bar{1}\bar{1}$ 02	3.0681	3.0861	3.0475	3.0799
(c) Al ₆₅ Cu ₁₈ Fe ₇ Mn ₁₀ at 880°C; $\theta_1 = \tau^{-10}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6368$ nm and $c = 1.234$ nm				
10 $\bar{1}\bar{1}$ 00	2.6002	2.6033	2.5804	2.6185
10 $\bar{1}\bar{1}$ 01	2.7234	2.7264	2.7046	2.7410
00 $\bar{1}\bar{1}$ 03	2.9134	2.9107	2.9316	2.8972
10 $\bar{1}\bar{1}$ 02	3.0636	3.0662	3.0468	3.0792
(d) Al ₆₅ Cu ₁₈ Fe ₉ Mn ₈ at 800°C; $\theta_1 = \tau^{-6}$, $\theta_2 = \tau^{-5}$, $A_{5D} = 0.6361$ nm and $c = 1.230$ nm				
10 $\bar{1}\bar{1}$ 00	2.6030	2.6242	2.5787	2.6170
10 $\bar{1}\bar{1}$ 01	2.7270	2.7473	2.7038	2.7404
00 $\bar{1}\bar{1}$ 03	2.9218	2.9031	2.9440	2.9098
10 $\bar{1}\bar{1}$ 02	3.0692	3.0871	3.0485	3.0810
(e) Al ₆₅ Cu ₁₈ Fe ₉ Mn ₈ at 880°C; $\theta_1 = -\tau^{-8}$, $\theta_2 = 0$, $A_{5D} = 0.6340$ nm and $c = 1.2285$ nm				
10 $\bar{1}\bar{1}$ 00	2.6117	2.6035	2.6137	2.6137
10 $\bar{1}\bar{1}$ 01	2.7356	2.7278	2.7375	2.7375
00 $\bar{1}\bar{1}$ 03	2.9272	2.9345	2.9254	2.9254
10 $\bar{1}\bar{1}$ 02	3.0775	3.0706	3.0792	3.0792
(f) Al ₆₅ Cu ₂₀ Fe ₇ Mn ₈ at 800°C; $\theta_1 = -\tau^{-6}$, $\theta_2 = 0$, $A_{5D} = 0.6360$ nm and $c = 1.229$ nm				
10 $\bar{1}\bar{1}$ 00	0.26034	0.25823	0.26087	2.6087
10 $\bar{1}\bar{1}$ 01	0.27276	0.27074	0.27327	2.7327
00 $\bar{1}\bar{1}$ 03	0.29236	0.29426	0.29189	2.9189
10 $\bar{1}\bar{1}$ 02	0.30702	0.30523	0.30747	3.0747
(g) Al ₆₅ Cu ₂₀ Fe ₇ Mn ₈ at 880°C; $\theta_1 = -\tau^{-6}$, $\theta_2 = 0$, $A_{5D} = 0.6360$ nm and $c = 1.232$ nm				
10 $\bar{1}\bar{1}$ 00	2.6034	2.5823	2.6087	2.6087
10 $\bar{1}\bar{1}$ 01	2.7270	2.7068	2.7321	2.7321
00 $\bar{1}\bar{1}$ 03	2.9186	2.9377	2.9140	2.9140
10 $\bar{1}\bar{1}$ 02	3.0681	3.0502	3.0726	3.0726

0.6360 nm, and $c = 1.229$ nm reproduces the peak shift most accurately. Here, $\theta_1 = -\tau^{-6}$ makes the 10 $\bar{1}\bar{1}$ 00 reflection move to the periodic position with index 11 and reciprocal momentum 0.25823 nm⁻¹. This linear phason strain θ_1 corresponds to the one-dimensional periodicity 4.2598 nm. This phase is one of the one-dimensional quasicrystals discovered by Yang *et al.*⁽⁶⁾ and is stable at 800°C and at 880°C.

In Table 1, from the comparison between compositions and lattice constants A_{5D} and c , excluding (e) one-dimensional quasicrystal of $\theta_1 = -\tau^{-8}$, $\theta_2 = 0$, as Fe increases and Cu decreases along (a)–(d)–(f), and Mn decreases and Cu increases along (b)–(f) and (c)–(g), the lattice constant tends to decrease somewhat.

4. Discussion

We found two types of approximant for the AlCuFeMn decagonal quasicrystal, with linear phason strains $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, and $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$, respectively. The first type of approximant is formed from the one-dimensional quasicrystal with $\theta_1 = \tau^{-6}$ and $\theta_2 = 0$, found by Tsai *et al.*⁽⁵⁾, by intro-

ducing the further linear phason strain $\theta_2 = \tau^{-5}$. The second type of approximant is formed from the first type of approximant by a τ^4 -fold decrease in θ_1 , corresponds to a τ^2 -fold increase in the periodicity.

We simulated the peak shifts due to linear phason strains on the quasi-periodic plane, and calculated the lattice constants of the approximants. Figure 5 (a) shows the peak positions for an ideal decagonal quasicrystal. The peak positions of the 10 $\bar{1}\bar{1}$ 0 and 011 $\bar{1}\bar{1}$ reflections are indicated by arrows on the $Q_{||x}$ ($m0\bar{n}\bar{n}0$) and $Q_{||y}$ ($0mn\bar{n}\bar{m}$) axes in the quasi-periodic plane, respectively.

Figure 5 (b) shows the shifts in the peak positions induced by the linear phason strains $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, as determined using eqs. (2) and (3). As a result of these shifts, the peak positions refer to an approximant with an orthorhombic reciprocal lattice. The shifted 10 $\bar{1}\bar{1}$ 0 reflection is located in the tenth cell from the center and corresponds to a reciprocal-space momentum of 2.6242 nm⁻¹, as calculated using $A_{5D} = 0.6361$ nm and $\theta_1 = \tau^{-6}$. The estimated lattice constant $a = 3.811$ nm is hence double that of the one-dimensional periodicity 1.905 nm. The odd-index peaks on the $Q_{||x}$ -axis fall

Fig. 2 Peak profiles and simulated peak positions for (a) $\text{Al}_{65}\text{Cu}_{24}\text{Fe}_3\text{Mn}_8$ annealed at 800°C , and $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_7\text{Mn}_{10}$ annealed at (b) 800°C and (c) 880°C . The dotted lines represent identical decagonal peak positions, and the solid lines the split-peak positions shifted relative to the identical positions as a result of the linear phason strains (a) $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, and (b) $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$. The peaks indicated by arrows in (a) describe an NaCl-type structure. Moreover, the peaks indicated by arrows in (c) indicate crystalline phases.

outside of this axis upon application of the one-dimensional periodicity. Similarly, the shifted $011\bar{1}\bar{1}$ reflection is located in the sixth cell from the center and has a reciprocal-space momentum of 1.9966 nm^{-1} , as calculated using $A_{5D} = 0.6361 \text{ nm}$ and $\theta_2 = \tau^{-5}$. The estimated lattice constant $b = 2.003 \text{ nm}$ is therefore double the one-dimensional periodicity 1.002 nm . The odd-index peaks on the $Q_{||}$ -axis enter this window from outside of the window range upon application of the one-dimensional periodicity. Figure 5 (c) shows the orthorhombic peak positions shifted by the linear phason strains $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$. The shifted $10\bar{1}\bar{1}0$ and $011\bar{1}\bar{1}$ reflections are located in the twenty-sixth and the sixth cells from the center and have reciprocal-space momentums of 2.6033 nm^{-1} and 2.9916 nm^{-1} , calculated using $A_{5D} = 0.6368 \text{ nm}$, and $\theta_1 =$

Fig. 3 Peak profiles and simulated peak positions for $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_9\text{Mn}_8$ (a) annealed at 800°C and (b) re-annealed at 880°C , respectively. The dotted lines represent identical decagonal peak positions and solid lines split-peak positions shifted relative to the identical positions as a result of the linear phason strains (a) $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, and (b) $\theta_1 = -\tau^{-8}$ and $\theta_2 = 0$. The peaks indicated by arrows in (b) show some crystalline phases.

Fig. 4 Peak profiles and simulated peak positions for $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$ (a) annealed at 800°C and (b) re-annealed at 880°C , respectively. The dotted lines represent identical decagonal peak positions and the solid lines the split-peak positions shifted relative to the identical positions as a result of the linear phason strains $\theta_1 = -\tau^{-6}$, $\theta_2 = 0$.

Fig. 5 Simulated peak positions in a quasi-periodic plane for: (a) an ideal decagonal quasicrystal; (b) an approximant crystal with linear phason strains of $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, with lattice constants $a = 3.811$ nm and $b = 2.003$ nm; (c) an approximant crystal with linear phason strains $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$, with lattice constants $a = 9.999$ nm and $b = 2.003$ nm. Peak positions at the $10\bar{1}\bar{1}0$ and $011\bar{1}\bar{1}$ reflections are denoted by arrows on the $Q_{||x}$ ($m0\bar{n}\bar{n}0$) and $Q_{||y}$ ($0mn\bar{n}\bar{m}$) axes, respectively.

τ^{-10} and $\theta_1 = \tau^{-5}$, respectively. Therefore, the estimated lattice constant $a = 9.998$ nm is double the one-dimensional periodicity 4.994 nm. Similarly, the lattice constant $b = 2.006$ nm is calculated to be double the one-dimensional periodicity 1.003 nm.

The first type of approximant appears at 800°C with compositions $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_9\text{Mn}_8$, $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_7\text{Mn}_{10}$, and $\text{Al}_{65}\text{Cu}_{24}\text{Fe}_3\text{Mn}_8$. The second type of approximant appears at 880°C , with a composition $\text{Al}_{65}\text{Cu}_{18}\text{Fe}_7\text{Mn}_{10}$, and transforms into the first type. Moreover, we confirmed the existence of two-types of one-dimensional quasicrystal, which have linear phason strains $\theta_1 = \tau^{-6}$ and $\theta_2 = 0$, and $\theta_1 = -\tau^{-8}$ and $\theta_2 = 0$, found by Yang *et al.*⁶⁾ For the composition of $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$, the first type of approximant, with $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$ at 800°C , changes into a one-dimensional quasicrystal with $\theta_1 = -\tau^{-8}$ and $\theta_2 = 0$ at 880°C . Furthermore, for the composition of $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_{10}\text{Mn}_5$, located near the composition of $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$, studied by Tasi *et al.*⁵⁾ and Yang *et al.*⁶⁾, there are three-types of one-dimensional quasicrystals with $\theta_1 = \tau^{-6}$ and $\theta_2 = 0$, $\theta_1 = -\tau^{-8}$ and $\theta_2 = 0$, and $\theta_1 = -\tau^{-6}$ and $\theta_2 = 0$ at 880°C . Therefore, an ideal decagonal quasicrystal may be stable above 880°C around these compositions.

On the other hand, a one-dimensional quasicrystal with $\theta_1 = -\tau^{-6}$ and $\theta_2 = 0$ is formed at 800°C and 880°C with composition $\text{Al}_{65}\text{Cu}_{20}\text{Fe}_7\text{Mn}_8$. This linear phason strain $\theta_1 = -\tau^{-6}$ is another type compared to the series of the above-mentioned linear phason strains $\theta_1 = \tau^{-6}$, $-\tau^{-8}$, and τ^{-10} .

The AlCuFeMn alloy system can consist of a one-dimensional quasicrystal and many approximants, each with a different linear phason strain. In the future, an ideal decagonal crystal may be found at a higher temperature. Further experiments on AlCuFeMn alloy systems will yield more detailed information on the transformation from a decagonal quasicrystal to approximants via one-dimensional quasicrystals.

5. Conclusion

We discovered two types of approximants in an AlCuFeMn alloy system, arising from the linear phason strains $\theta_1 = \tau^{-6}$ and $\theta_2 = \tau^{-5}$, and $\theta_1 = \tau^{-10}$ and $\theta_2 = \tau^{-5}$. These correspond, respectively, to $a = 3.811$ nm and $b = 2.003$ nm, and $a = 9.998$ nm and $b = 2.006$ nm for an orthorhombic system.

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