Grain Size Distribution in Recrystallized Alpha-Titanium

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A quantitative optical microscopy study was made on the grain size distribution in one-, two- and three-dimensions in recrystallized titanium (0.2% O_{eq}) by the linear intercept, planimetric and serial sectioning methods. Analysis of the results revealed that all the size distributions were logarithmic-normal and that the mean values of the chord length $\bar{l}$, grain area $\bar{A}$, and grain volume $\bar{V}$ were in good agreement with the values associated with the maximum of the distribution curves (which were approximately $\bar{l}$, $\bar{A}$ and $\bar{V}$ at 50% of the cumulative frequency). There existed an intrinsic spatial distribution of grain sizes, which could not be deduced from either one- or two-dimensional measurements quantitatively without knowledge of the shape and size of the constituent grains. The interrelations of $\bar{l}$, $\bar{A}$ and $\bar{V}$ were expressed by the equations

$$\bar{l} = \alpha (\bar{A})^{1/2}, \bar{l} = \beta (\bar{V})^{1/3} \text{ and } \bar{A} = \gamma (\bar{V})^{2/3}$$

where α , β and γ are approximately unity. It was therefore concluded that the grain size measured either in one- or two-dimensions could be utilized to establish the three-dimensional grain size. The construction of three-dimensional grains by stacking serial sections revealed that the number of faces, vertices and edges of a polyhedral grain increased with increase in grain size (namely the cumulative frequency) and the larger the grain size the more isotropic the grain became.

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I. Introduction

The choice of the parameter used to represent the grain size of metals is very important in understanding the effects of grain size on certain physical and chemical properties. Theoretically, the grain size can be expressed in terms of line, plane or space parameters, or functions thereof. However, an unambiguous definition of the grain size is required for the rigorous interpretation of the effect of grain size on a given property.

The grain size and shape in three dimensions are closest to the ideal for describing the geometrical feature of polycrystals and hence for correlating the effect of grain size on properties. Direct measurement of the grain size in three dimensions is, however, not always feasible in practice. Most definitions used in practice are based on the grain size in two dimensions⁽¹⁾ in accordance with ASTM specification E 112-H 63⁽²⁾. Many authors prefer the average grain area $\bar{A}$ on the plane of polish. Nevertheless, the one-dimensional grain size defined by the mean linear intercept chord length $\bar{l}$ is also recommended as a fundamental measurement⁽³⁾⁽⁴⁾.

Even though the grain size in one dimension may not be the geometric parameter which governs directly the influence of grain size on a particular chemical or physical property, it is the easiest, quickest and most accurate measurement to make. Moreover, as Smith and Guttman⁽⁵⁾ have shown the mean linear intercept value yields on exact measure of the volume-to-surface ratio and a good index of grain volume, as long as the grain shape is not highly abnormal. In view of the ease in which it can be measured, it is therefore highly desirable

to establish the relationship between the one-dimensional measurement of the grain size in terms of the mean intercept length and those based on two- and three-dimensional measurements.

In a previous paper⁽⁶⁾ the present authors characterized the grain size in swaged and recrystallized unalloyed titanium wire by measurements on a planar section of : (a) the intercept chord length distribution $f(l)$, (b) the mean linear intercept $\bar{l}$ and (c) the mean grain area $\bar{A}$. It was concluded from these measurements that the chord length distribution was log-normal, there existed a distribution of grain sizes in space, $\bar{l}$ was approximately equal to $\sqrt{\bar{A}}$ and there occurred a change in shape towards a more regular shape as the average grain size increased. The present investigation is a continuation of a study into the character of the grain size and shape in unalloyed titanium, with emphasis being placed on relating the grain size distributions measured in one- and two-dimensions to those existing in three-dimensions. Special attention was placed on checking the previous conclusions⁽⁶⁾ based on the one- and two-dimensional measurements.

II. Experimental Procedure

1. Material and specimen preparation

The special lot of titanium prepared by the Battelle Memorial Institute and listed in the previous paper⁽⁶⁾ was used for the present investigation. The chemical analysis is given in Table 1. As-received rods of 1/4 in. diameter were swaged at room temperature to 1/16 in. diameter wires without any intermediate annealing, representing a 93% reduction in area. Specimens 1/3

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(1) E. L. Kenyon : *Symposium on Industrial Metallurgy*, ASM, Cleveland, Ohio, (1949), p. 153.

(2) ASTM designation E 112-60 T.

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(5) C. S. Smith and L. G. Guttman : *Trans. Met. Soc. AIME*, 218 (1953), 81.

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Table 1 Chemical analysis of Battelle titanium*.

O	C	N	H	Fe	O _{eq} ** (at%)
420	53	53	36	90	0.18

* Analysis is in ppm by weight; see Ref. (6)

** Interstitial content expressed as oxygen equivalent; defined by H. Conrad: Acta Met., 14(1966), 1631.

in. long were cut from the swaged wire for the annealing treatments and the grain size evaluation.

After swaging, the specimens were chemically polished with a HNO₃-HF solution, washed under running water and rinsed in ethyl alcohol to be dried. Heat treatments for recrystallization and grain growth were carried out under a vacuum of less than 10⁻⁵ Torr at several elevated temperatures. The specimens were placed next to a monitoring thermocouple in the vacuum system; it usually took 6 minutes after placing the pre-heated furnace over the vacuum system to bring the temperature to that desired. The annealing time was strictly observed from the instant at which the temperature of the specimens became that desired, and the temperature and vacuum of the system were monitored constantly during the annealing treatment. After annealing, the furnace was slid away from the vacuum system and the specimens were cooled in vacuum to room temperature.

In the present work, five specimens, designated as A, B, C, D and E, were chosen for the grain size distribution study. Table 2 shows the annealing temperatures and times for these specimens. Previous work⁽⁶⁾ established that the results for a given grain size were not dependent on the combination of time and temperature used to obtain the size and hence the results for a given annealing treatment can be considered to be characteristic of the grain size, independent of the treatment. After annealing, the specimens were mounted in a cold self-setting resin and conventional metallographic techniques were employed to prepare the flat plane of polish ready for etching. Any scratches inherited from the buff-polishing were removed by chemically polishing in a solution consisting of 30% by volume HF and 30% by volume HNO₃. Grain boundaries on the plane of polish were well revealed by subsequent etching in an aqueous solution of 2 vol% HF plus 2 vol% H₂O₂ for about ten seconds.

Table 2 Heat treatments and values of mean linear intercept, $\bar{l}$ mean grain area $\bar{A}$ and mean grain volume $\bar{V}$.

Specimen	Annealing temp. (°C)	Time (hr)	$\bar{l}$ (cm)	$\bar{A}$ (cm ²)	$\bar{V}$ (cm ³)
A	600	1/6	7 × 10 ⁻⁴	5 × 10 ⁻⁷	7 × 10 ⁻¹⁰
B	600	1	10 × 10 ⁻⁴	8 × 10 ⁻⁷	12 × 10 ⁻¹⁰
C	500	16	17 × 10 ⁻⁴	27 × 10 ⁻⁷	50 × 10 ⁻¹⁰
D	700	1/2	22 × 10 ⁻⁴	44 × 10 ⁻⁷	90 × 10 ⁻¹⁰
E	800	1/2	44 × 10 ⁻⁴	160 × 10 ⁻⁷	800 × 10 ⁻¹⁰

2. Determination of grain size

(1) One-dimensional grain size

About 12 to 21 overlapping photomicrographs with

a suitable magnification (× 600~× 150 originally and enlarged × 3.5) were taken for each specimen, and they were constructed into a huge map in which at least 400 grains were included. Several random lines were drawn on this map and the mean linear intercept chord length, $\bar{l}$ was calculated by the equation

$$\bar{l} = L/(N-1) \quad (1)$$

where L was the total length of drawn lines and N the number of intercepts of lines with grain boundaries. To minimize the experimental error, at least 400 intercepts were counted. This mean linear intercept chord, $\bar{l}$ on the micrographs was in good agreement with that counted on the microscope directly by bringing the surface in and out of focus to establish the intersection of the cross-hair in the eye piece of the microscope with the grain boundaries.

To determine the one-dimensional grain size distribution, at least 400 chords l were measured individually with a Vernier caliper on the enlarged micrographs, and the frequency $f(l)$ was obtained for each class of l . Hence this method of measuring the one-dimensional grain size distribution is designated as the linear intercept method in this paper.

(2) Two-dimensional grain size

The grain map described in the previous section was also used to measure the intercept grain area, $\bar{A}$ on the plane of polish with a planimeter. The areas of at least 300 individual grains were measured to obtain the grain area distribution $f(A)$. The mean grain area, $\bar{A}$ was calculated by the equation

$$\bar{A} = \sum [f(A) \cdot A] / N \quad (2)$$

where $f(A)$ is the frequency of the grain area class A and N the total number of grains measured. This method of measuring the grain intercept area is designated the planimetric method. Since any plane of polish would represent a random section of the specimen, only one plane was used for this distribution study.

(3) Three-dimensional grain size

Serial sectioning was employed to establish the three-dimensional grain size. After etching the grain boundaries on the plane of polish, a Shimadzu microhardness tester type M was used to introduce two diamond indentations at both ends of the polished surface to locate clearly where micrographs should be successively taken on the newly appearing planes during the serial sectioning. Another two indentations of smaller size were made without any load to determine the depth of the serial section. Then, the specimen was carefully polished on a buff-polisher until the depth-indentations had disappeared; in this way the depth of sectioning could be estimated (the depth of the indentations without load could be estimated from the edge length of the indentation, since the geometry of the diamond indenter is known). When the depth-indentations were buffed-off (representing about 1-2 microns) the specimens were again etched with the dilute HF-H₂O₂ etching solution to reveal grain boundaries, and 12 to 21 overlapping

photomicrographs were taken. This procedure was continued until the total thickness polished-off was about $2.5 \bar{l}$.

For each grain size, the grain boundaries were traced from the photomicrographs onto tracing paper. From a series of such traces, the appearance or disappearance of a particular grain was determined. If a grain started to grow and then shrink away within a series of photomicrographs, it was selected to measure the three-dimensional volume, V . Usually a grain was contained in about 10 or more serial sections. The grain area A surrounded by grain boundaries for a selected grain on each serial section was then measured with a planimeter. The area times the estimated thickness of the section was summed up to yield the grain volume:

$$V = \Sigma (A \cdot d) \quad (3)$$

where d is the thickness of a section. This method is designated the serial sectioning method.

Measurements on about 100 three-dimensional grains were made for determining the volume distribution $f(V)$. The mean grain volume, $\bar{V}$ was calculated from

$$\bar{V} = \Sigma [f(V) \cdot V] / N \quad (4)$$

where $f(V)$ is the frequency of grain volume of class V and N is the total number of grains measured.

To visualize the actual grains in three-dimensions, a series of grain models representing a range in spatial sizes were made for specimen C. It was felt that the results obtained for this grain size might be representative for the range of annealing treatments (grain sizes) considered here.

III. Experimental Results

1. One-dimensional grain size

The mean linear intercept grain size $\bar{l}$, defined by eq. (1), was measured both directly under a microscope and indirectly on the enlarged microphotographs. Both measurements on at least 400 grains gave the same value for $\bar{l}$. It was 7, 10, 17, 22 and 40 microns for specimens A, B, C, D and E, respectively; these values are recorded in Table 2.

The frequency $f(l)$ in percent measured on 400 grains versus the logarithm of the chord length, l is depicted in Fig. 1. It is seen that the distribution curves are nearly symmetrical and the peak height becomes lower with increasing grain size. The values of l associated with the maximum frequency ("peak" values, l_p) are 6.8, 9.5, 17.5, 21.5 and 40 microns for specimens A, B, C, D and E, respectively, which are in good agreement with $\bar{l}$ for these specimens. In the present work, the maximum chord length $l_{\max}$ is 2.1~3.0 times the value of $\bar{l}$; previous work⁽⁶⁾ yielded a factor of 2.0~2.5.

The size distribution for the linear intercept chord length is further depicted in Fig. 2, which is a plot of the cumulative frequency in percentage versus the logarithm of l . To be noted from this figure is that the data points for the most part follow a straight line for each grain size, with deviations at both the small and large chord lengths. The chord length at 50% of

Fig. 1 Distribution of the grain size in one-dimension in recrystallized titanium.

Fig. 2 Cumulative frequency of the chord length, l measured on 400 grains of recrystallized titanium (one-dimension).

the cumulative frequency is in good agreement with the mean linear intercept chord length $\bar{l}$. It is therefore concluded that the distribution of the linear intercept chord length is of the logarithmic-normal type, as was found in the previous work⁽⁶⁾.

2. Two-dimensional grain size

The grain areas of 300 grains on a plane of polish were measured individually with a planimeter, and the mean grain area, $\bar{A}$ defined by eq. (2) was calculated, yielding 5.0, 8.2, 27, 44 and $160 \times 10^{-7} \text{ cm}^2$ for specimens A, B, C, D and E, respectively (Table 2). The frequency $f(A)$ is plotted against the logarithm of the grain area, A in Fig. 3. The trends in the distribution for the two-dimensional grain size are similar to those for the one-dimensional grain size: the frequency curves are symmetrical, the peak height decreases with increasing grain size, and the width of the curve becomes broader as the grain size increases. The peak values, A_p of the distribution curves are in accord with the mean grain areas, $\bar{A}$ within the experimental errors.

Fig. 3 Distribution of the grain size in two-dimensions in recrystallized titanium.

Fig. 4 Cumulative frequency of the grain area, A measured on 300 grains of recrystallized titanium (two-dimensions).

The maximum grain area, $A_{\max}$ is between 4.5 and 5.0 times $\bar{A}$.

Depicted in Fig. 4 is the cumulative frequency, $\Sigma f(A)$ versus the logarithm of the grain area A . Within the experimental errors a straight line can be drawn through a given set of data points in all cases, the deviation from the straight line only becoming appreciable beyond 95% of the cumulative frequency. Also to be noted, is that the mean grain area, $\bar{A}$ calculated by eq. (2) is in accord with the grain area at 50% of the cumulative frequency. On the basis of Figs. 3 and 4 it is concluded that the grain size distribution in two-dimensions is of the logarithmic-normal type.

3. Three-dimensional grain size

The volume of grain was calculated by eq. (3), measuring the grain area with a planimeter and multiplying it by the thickness determined from the disappearance of the depth-indentations. For each grain size the volumes of at least 100 grains (mostly more than 100) were measured. The frequency, $f(V)$ having the volume,

Fig. 5 Distribution of the grain size in three-dimensions in recrystallized titanium.

Fig. 6 Cumulative frequency of the grain volume, V measured on 100 grains of recrystallized titanium (three-dimensions).

V is plotted against the logarithm of V in Fig. 5. It is seen from the figure that the shape of the distribution curve is symmetrical for all cases. The peak values are in accord with the mean volumes calculated from eq. (4), which are 7, 12, 50, 90 and $800 \times 10^{-10} \text{ cm}^3$ for specimens A, B, C, D and E, respectively (Table 2), and the maximum volume, $V_{\max}$ is about 5 times the "peak" volume, V_p .

In Fig. 6 the cumulative frequency, $\Sigma f(V)$ in percentage is depicted against the logarithm of the grain volume, V . The data points follow mostly a linear relationship in all cases. A slight deviation from the straight line is observed in the large volume range; however, the deviation is less than that for the linear intercept or the grain area shown in Figs. 2 and 4. Again, the grain volume at 50% of the cumulative frequency is in a good accord with the mean volume and the "peak" volume. This all indicates that the grain volume distribution is of the logarithmic-normal type.

4. Three-dimensional model of recrystallized grains

To visualize the grain in three dimensions a number of grains were modeled by cutting the grain out of a series of photomicrographs and stacking them at a

suitable thickness. Such three-dimensional grain models are illustrated in Photo. 1 as a function of the cumulative frequency $\Sigma f(V)$ for specimen C with $\bar{l}$ of 17 microns. The arrow on the grain surface indicates the direction of swaging (wire axis). No simple relationship between the swaging direction and the grain shape was evident upon close examination of these models; however, the shape became more isotropic as the grain size increased. This was also indicated by the fact that the Euler's indices used to characterize a polyhedron increased with increasing grain size; see Table 3. Here N_f , N_v , and N_e are the number of faces, vertices and edges respectively determined for each grain.

Photo. 1 Three-dimensional grain models of recrystallized titanium of $\bar{l}=17$ microns as a function of the cumulative frequency. Arrows on the surface indicate the direction of swaging.

Table 3 The Euler's indices for polyhedral grains of specimen C with mean linear intercept grain size of 17 microns.

Cumulative freq. $\Sigma f(V)$ (%)	Volume (cm^3)	Face N_f	Vertex N_v	Edge N_e
5	10×10^{-10}	14	17	29
10	14×10^{-10}	16	23	37
25	24×10^{-10}	18	22	38
50	50×10^{-10}	20	36	54
75	85×10^{-10}	22	38	58
90	130×10^{-10}	26	45	69
95	170×10^{-10}	28	47	73

IV. Discussion

1. Grain size distribution in space

One objective of the present work was to check the previous conclusion⁽⁶⁾ that for swaged and recrystallized titanium there exists a range of grain sizes in space. This conclusion was based mainly on the low value (~ 0.5) obtained for the ratio of the peak linear intercept grain size l_p to the maximum size $l_{\max}$. Several authors⁽⁷⁾⁻⁽⁹⁾ have pointed out that this ratio is a measure of the uniformity of grain size in space, stating that a uniform size should yield values for this ratio of 0.7 to 0.95 depending on the shape of the grain. More vigorous analyses of the grain size distribution are based on specific grain shapes: the sphere⁽¹⁰⁾, the ellipsoid of revolution⁽¹¹⁾, the cylinder⁽¹¹⁾⁽¹²⁾ and particles with a constant number of flat circular ends⁽¹²⁾

and so on. However, these cannot be applied directly to experimental data on actual metals, since these shapes do not satisfy the important requirement of space-filling. Rutherford et al.⁽¹³⁾ and Smith⁽¹⁴⁾ have discussed the topology of grains, and Smith concludes, based on a collection of experimental results from different branches of science, that the grain shape is determined by the requirements both of space-filling and of minimum surface energy. The shape that best fits these requirements is the regular tetrakaidecahedron.

The present serial sectioning results, Figs. 5 and 6, clearly establish that a size distribution in space does exist for the present material and that this is approximately log-normal, supporting the earlier conclusions based on the linear intercept analysis⁽⁶⁾. The present results therefore provide some support for the use of the ratio of $l_p/l_{\max}$ (or $A_p/A_{\max}$) for indicating the existence of a size distribution in space.

Of additional interest are the relationships which exist between the size distribution expressed by the one-, two- and three-dimensional measurements. Again, a completely rigorous expression for those relationships requires the assumption of a specific grain shape, for example a sphere⁽¹⁵⁾. Even though such an assumption may not lead to completely erroneous conclusions, nevertheless results based on the assumption of spherical grains must be suspect for metal grains. Therefore, the approach taken here is to compare directly the experimental data from measurements in the different dimensions.

A log-log plot of $\bar{l}$ versus $\bar{A}$ is depicted in Fig. 7. This indicates that $\bar{l}$ is proportional to $(\bar{A})^{1/2}$ giving

$$\bar{l} = \alpha (\bar{A})^{1/2} \quad (5)$$

where α is nearly unity. A similar relationship was found in the earlier work⁽⁶⁾. A log-log plot of $\bar{l}$ versus $\bar{V}$ is given in Fig. 8 yielding

$$\bar{l} = \beta (\bar{V})^{1/3} \quad (6)$$

Fig. 7 The relationship between the mean linear intercept chord, $\bar{l}$ and mean grain area, $\bar{A}$ in recrystallized titanium.

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(12) R. L. Fullman : *Trans. AIME*, **197** (1953), 447.

(13) J. J. B. Rutherford, R. H. Aborn and E. C. Bain; *Metals and Alloys*, **8** (1937), 345.

(14) C. S. Smith : *Metal Interfaces*, ASM, Cleveland, Ohio, (1952), p. 65.

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Fig. 8 The relationship between the mean linear intercept chord, $\bar{l}$ and mean grain volume, $\bar{V}$ in recrystallized titanium.

Fig. 9 The relationship between the mean grain area, $\bar{A}$ and mean grain volume, $\bar{V}$ in recrystallized titanium.

where β is also nearly unity. From eqs. (5) and (6) it is expected that the relationship between $\bar{A}$ and $\bar{V}$ is given by

$$\bar{A} = \gamma (\bar{V})^{2/3} \quad (7)$$

with γ being approximately unity. Figure 9 shows that eq. (7) is in fact followed. The deviation of the smallest grain size from the straight line in this figure is attributed to: (a) the determination of the thickness involves certain experimental errors, which become proportionally more important the smaller the grain size, and (b) the number of serial sections per grain are fewer the smaller the grain size. Of significance regarding the values of α , β and γ is that analyses based on space-filling polyhedra yield values of approximately unity for these coefficients⁽¹⁶⁾.

The present experimental results show that the relationship between $\bar{l}$ and $\bar{A}$ may be expressed (if taken as a function of the cumulative frequency) by the equation

$$\bar{l} = \alpha' (\bar{A})^{1/2} \quad (8)$$

where α' is a constant which depends on the cumulative frequency. Also, the relation between $\bar{l}$ and $\bar{V}$ may be expressed by

$$\bar{l} = \beta' (\bar{V})^{1/3} \quad (9)$$

where β' is a constant which also depends on the

cumulative frequency. Taking the derivatives of eqs. (8) and (9) it is deduced that

$$d\bar{l} = \alpha' d\bar{A} / 2 (\bar{A})^{1/2} \quad (10)$$

$$\text{and} \quad d\bar{l} = \beta' d\bar{V} / 3 (\bar{V})^{2/3} \quad (10a)$$

Since all of the grain size distributions in one-, two- and three-dimensions are log-normal, the interrelationship among these three distributions can be written as

$$f(\bar{l}) d\bar{l} = f(\bar{A}) d\bar{A} \quad (11)$$

$$\text{and} \quad f(\bar{l}) d\bar{l} = f(\bar{V}) d\bar{V} \quad (11a)$$

Substituting and rearranging we obtain

$$f(\bar{A}) = (\alpha')^2 f(\bar{l}) / 2 \bar{l} \quad (12)$$

$$\text{and} \quad f(\bar{V}) = (\beta')^3 f(\bar{l}) / 3 \bar{l}^2 \quad (12a)$$

Accordingly, if one knows the variation of α' and β' with the cumulative frequency it is possible to deduce the size distribution in two- or three-dimensions from that in one-dimension. At present, however, there is no way of obtaining this except by direct measurement. The results shown in Figs. 2 and 6 indicate that $(\beta')^3$ varies approximately in a linear manner from 0.5 to 1.35 as the cumulative frequency increases from zero to 100%.

2. Grain shape

Apart from the existence of a grain size distribution in space, an examination of the grain models in Photo. 1 revealed that the Euler's indices used to characterize a polyhedron increase with increasing size of the grain; see Table 3. In all cases, the Euler's theory for a polyhedron given by the equation

$$N_f + N_v - N_e = 2 \quad (13)$$

is satisfied, where N_f , N_v and N_e are the number of faces, vertices and edges respectively. The fact that the Euler's indices vary with the grain size implies that the shape of the grain changes; the present results indicate that the grain becomes more regular or isotropic as its size becomes larger. Of interest in this regard is that in the previous study⁽⁶⁾ it was concluded on the basis of the shape factor $\delta = \pi \bar{A} / 2 (\bar{l})^2$ that the grain shape changes to a more regular form with increase in grain size. The shape factor δ is the ratio of the lengths of the perimeter and intercept in the average two-dimensional grain section⁽¹⁵⁾.

A further important point regarding the results presented in Table 3 is that as the grain size increases the number of faces increases beyond that of the tetrakaidcahedron, which is considered to best approximate the combined conditions of minimum surface energy and space-filling. This increase in number of faces, and in turn number of edges and vertices, results from the fact that a spatial distribution of sizes is present, placing the larger grain in contact with smaller grains and thereby creating a larger number of faces than if the grains were all of one size.

V. Conclusions

- (1) Quantitative optical microscopy studies on swaged

(16) E. E. Underwood: *Quantitative Microscopy*, McGraw-Hill, (1968), p. 91.

and recrystallized α -titanium (0.2% O_{eq}) established that the grain size distributions in one-, two- and three-dimensions are of the logarithmic-normal type for all grain sizes, the arithmetic mean values being in good accord with the most frequent values and with those at 50% cumulative frequency.

(2) The interrelationships between the mean linear intercept chord length $\bar{l}$, the mean grain area $\bar{A}$ and the mean grain volume $\bar{V}$ are expressed by the simple equations: $\bar{l} = \alpha(\bar{A})^{1/2}$, $\bar{l} = \beta(\bar{V})^{1/3}$ and $\bar{A} = \gamma(\bar{V})^{2/3}$ where α , β and γ are approximately unity. It was therefore concluded that any grain size conveniently measured on the plane of polish in either one- or two-dimensions could be used to define the grain size.

(3) The three-dimensional measurements (serial sectioning) established that there exists an intrinsic distri-

bution of sizes in space and that the shape of a given grain in such a distribution varies with its size—the number of faces, vertices and edges increasing with size or volume. It is concluded that the increase in these geometric parameters results from the existence of the spatial size distribution, for thereby a given grain will generally be in contact with some grains of a smaller size.

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