Discontinuous Lüders Band Propagations of α -Iron in the Blue-brittleness Temperature Range*

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The present work was concerned with an experimental study of a Lüders band propagation in the blue-brittleness range and its analysis of some parameters, that is, dislocation structure, stress dependence of the band propagation velocity, and others.

The specimen used was an α -iron containing 0.006wt% carbon. A correlation between the stress drop and the decay time during the band propagation was obtained by using an oscillographic recorder. Also, dislocation structures were observed by means of transmission electron microscopy.

Dislocation structures were tangled and the average dislocation density was about 8×10^9 /cm² in the temperature range from 160 to 250°C. The magnitude of the stress drop did not directly depend on the amount of dislocations generated and its multiplication rate, but on the width of the Lüders band propagated. The propagation velocity of the Lüders band could be expressed by the same relation as the stress dependence of dislocation velocity. The dislocation velocity exponent m derived from the relation between the stress drop and the decay time was approximately 18 in the temperature range investigated. With increasing temperature, the initial velocity of the Lüders propagation increased: about 55.6 cm/sec at 160°C, 312.4 cm/sec at 200°C, and 595.5 cm/sec. This temperature dependence of the velocity could mainly be associated with that of the stress concentration factor caused by the dynamical strain aging near the Lüders band front.

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I. Introduction

Most of *bcc* metals such as Fe, Nb, etc.^{(1)~(3)} which contain very small amounts of interstitial elements are accompanied with a Lüders band formation and its propagation at a yield point during the tension test.

As indicated by Johnston and Gilman⁽⁴⁾, dislocations are abruptly multiplied when stress falls from an upper yield point to a lower stress. This yielded region rapidly encompasses the cross section of a specimen, that is, a Lüders band propagates toward the tensile direction along the gauge length. The characteristic of Lüders band propagation depends on the grain size⁽⁵⁾⁽⁶⁾, temperature⁽⁷⁾, applied stress⁽⁸⁾⁽⁹⁾ and strain rate⁽¹⁰⁾⁽¹¹⁾. Especially, plastic strain distribution around the propagating front which is considered very complicated⁽¹²⁾ has not been made clear in many respects, although dislocation structures have been observed by a transmission electron microscopy⁽¹³⁾. It has already been shown, however, in many investigations^{(14)~(16)} that the behavior of the Lüders band propagation should intimately be concerned with the dislocation motion and the stress dependence of propagation velocity would be given by the same relation as that of dislocation velocity⁽¹⁷⁾⁽¹⁸⁾.

At room temperature a Lüders band propagates at a limited stress. On the other hand, when the deformation is performed at a higher temperature where the interaction between solute atoms and mobile dislocations takes place, the Lüders band propagation is accompanied by a serrated flow⁽¹⁹⁾, that is, with rapid

stress drops in a stress-strain curve, and then after the band propagation many strain bands are observed on specimen surfaces⁽²⁰⁾⁽²¹⁾. Namely, the following relation has been recognized that these rapid stress drops are associated with the repetition of discontinuous band propagations. Although these discontinuous propagations have an important role in the relation between the serrated flow and the strengthening mechanism in the work-hardening region where serrations occur frequently, the mechanism of the Lüders band propagation in the blue-brittleness range has not been studied in details.

In the present paper, dislocation structures and the discontinuous band propagation mechanism were investigated, when a polycrystalline iron was tensile tested in the blue-brittleness temperature range where solute carbon atoms strongly interact with mobile dislocations.

II. Experimental Procedures

The specimen used in this experiment was prepared by vacuum melting the re-electrolytic iron made by Showa Denko K.K. Chemical analysis of the material is given in Table 1. Test pieces were sheets of 0.5 mm in thickness which were cold rolled from a rod of 10 mm dia and cut by an electrical discharge machine to have a gauge length of 20 mm and a width of 4 mm. They were sealed in evacuated quartz capsules under a vacuum of about 5×10^{-5} mmHg, which were homogenized for 1 hr at 730°C and then quenched

Table 1 Chemical composition of specimen (wt%).

C	N	Mn	Si	S	O	P	Al
0.006	0.0007	0.0005	0.0015	0.006	0.0013	0.0007	0.001

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into iced water by crushing the capsules. Average grain size obtained was about $30\mu\text{m}$. The tensile test was carried out using an Instron type tensile machine at a strain rate of about $4.2 \times 10^{-4}/\text{sec}$ (0.5 mm/min). The testing temperature used were room temperature, 50, 100, 160, 200 and 250°C , and the deformation was performed in a silicone oil bath up to 250°C above 50°C .

At these temperature where Lüders band propagated with a rapid repetition of stress drops, an oscillographic recorder was used to measure the successive variation of stress with propagating time of the band. Moreover, the dislocation structures were observed by a transmission electron microscope operating at 100 kV .

III. Experimental Results

1. Tension test

When a polycrystalline iron which was rapidly cooled into iced water from solid solution temperature was deformed in tension at room temperature, a Lüders band propagated through the specimen at a constant stress, i.e. at a lower yield stress. On the other hand, at the testing temperature of the blue-brittleness range, the Lüders band propagated discontinuously accompanying repeated stress drops.

Typical examples of the stress-strain curves and optical strain marking observed on the specimen surface corresponding to the repeated stress drops are shown in Fig. 1. Lüders bands propagation with frequently repeated stress drops were observed in temperature range of $160\text{--}250^\circ\text{C}$ and the largest drop

Fig. 1 Optical micrographs and stress-strain curves obtained during Lüders band propagation at different temperatures.

of stress occurred at 200°C . The number of the deformation bands formed on the specimen surface after the band propagation at regular intervals coincided with that of stress fluctuation on the stress-strain curve. It may, therefore, be seen from these observations that the discontinuous repeated propagation of Lüders band in this temperature range proceeds with a rapid displacement of the band front during the stress drop and ceases after the displacement of a certain distance along the specimen before this process is repeated. Once the band propagation ceased, the flow stress increases up to a given stress level where Lüders band can propagate again and then the band suddenly begins to move toward the undeformed region. These repeated Lüders band propagations continue until the specimen is strained throughout the gauge length. Strain marking appeared between deformed and undeformed regions became also more distinct at 200 and 250°C than in the case of 160°C . On the deformation above 250°C Lüders band propagated irregularly and no Lüders band was observed at 280°C .

2. Propagation of Lüders band and dislocation structures

Photograph 1 shows the dislocation structures in the Lüders band when tensiled in the temperature range of room temperature to 250°C . Lüders strain at each deformation temperature was about 3%. Dislocation distribution after the band propagation showed the tangled and cell structures were partly formed. These cell structures became more pronounced with a rise in temperature. Figure 2 shows the relation between deformation temperature and the dislocation density which was measured⁽²²⁾ from dislocation structures. The average dislocation density was about $7 \times 10^9/\text{cm}^2$ at room temperature where the Lüders band smoothly propagated and was about $8\text{--}9 \times 10^9/\text{cm}^2$ at $160\text{--}250^\circ\text{C}$ where a pronounced repetition of stress drop appeared. Thus the variation of dislocation densities at different temperatures is not so distinct that the rate of generation and multiplication of dislocations within the band front may be suggested to be constant in this temperature range.

Dislocation structures in a region where the Lüders band propagated with rapid stress drops were also observed. Photograph 2 shows some examples of the dislocation structures produced at 200°C after the propagation accompanying two stress drops. The structures correspond to the deformed regions A and C, and the undeformed ones B and D. The dislocations which might be introduced during quenching of the specimen were observed only in the undeformed region, but in the deformed one the remarked tangle structures similar to those in Photo. 1 were observed.

Figure 3 shows the relation between the width of the deformed region (L) caused by the displacement of a Lüders front by one stress drop and the dislocation density obtained in the deformed region corresponding to the stress drop. At each deformation temperature, the average dislocation density was

Photo. 1 Dislocation structures after Lüders band propagation at different temperatures; (a) room temperature, (b) 160°C, (c) 200°C, (d) 250°C.

Fig. 2 Relation between dislocation density at Lüders strain and deformation temperature.

Fig. 3 Relation between dislocation density and width of the Lüders band propagated during stress dropping at different temperatures.

Photo. 2 Dislocation structures observed in the regions where the Lüders band propagated ((A) and (C)) and did not propagate ((B) and (D)) during the occurrence of two stress drops at 200°C.

$7-8 \times 10^9/\text{cm}^2$ and did not depend on the width of the deformed region.

3. A relation between stress drop and width of the deformed region

Figure 4 shows the relation between the values of stress drops appeared during Lüders band propagation and the width of deformed region at different temperatures. It is clear that the value of stress drop is proportional to its width measured along the direction of the tension axis. A plot of these values gives a good

Fig. 4 Relation between the stress drop and the width of Lüders band after the band propagation at different temperatures.

Fig. 5 Schematic representation of a Lüders band and the stress-time curve during stress dropping from the B to C stress level at which the propagation is stopped.

agreement with a solid line evaluated from the relation between a stress-strain curve and a cross head velocity. The linear relation is illustrated in Fig. 5. A part of elastic elongation which corresponds to AB transforms into the plastic one with a stress drop $\Delta\sigma$ from B to C , and then a Lüders band propagates producing the strain ϵ_L along the original length L_0 . Consequently a stress relaxation takes place due to the constriction of the load cell spring. Furthermore, the stress increases again until the value of stress D reaches the same value of stress B , holding the relation of $AB//CD$. Now, the time expended during the stress increasing from C to D which is equal to the stress level of B is denoted as Δt and the cross head velocity is V_c mm/sec. The length of specimen elongated by a stress drop $\Delta\sigma$ is given by the following equation;

$$\Delta L (\propto \Delta\sigma) = V_c \times \Delta t. \quad (1)$$

If the value of plastic strain after the band propagation is constant, the following relation can be derived:

$$L = L_0(1 + \epsilon_L) = \Delta L(1 + \epsilon_L)/\epsilon_L \quad (2)$$

Combining eqs. (1) and (2), and substituting the values of $V_c = 8.33 \times 10^{-3}$ /sec and $\epsilon_L \sim 0.03$ and also using Δt obtained from the load-time relationship in this work, eq. (3) is obtained:

$$\Delta\sigma = 2.77 \times L. \quad (3)$$

Thus, it would be clear that the stress drop is a linear function of the width of the deformed region, i.e. a

Fig. 6 Oscillographic traces showing the load-time history of abrupt stress drop during the Lüders band propagation at 160–250°C.

Lüders elongation.

4. Stress-time curves

As shown in Fig. 1, the stress falls so rapidly that a usual X-Y recorder could not follow up a time trace during the stress drop. Therefore, an oscillographic recorder was used. Some examples of the recording trace are shown in Fig. 6. With increasing deformation temperature, the expended time in a stress drop became shorter and the times at 160 and 250°C were about 0.5 and 1/50 sec for one drop, respectively.

As described above, the stress drop was proportional to the width of the deformation region. Therefore, the velocity of a Lüders band propagation can be derived from a time trace during the band front displacement.

As has already been reported by many investigators, it has been indicated that the Lüders band propagation would closely be associated with the behavior of dislocation motion^{(13)~(15)} and the stress dependence of the band velocity could be written in a similar relation to that⁽¹⁶⁾⁽¹⁷⁾ of dislocation motion. That is, the average velocity of dislocation is generally expressed in terms of the applied stress by the next equation⁽²³⁾:

$$v = B\sigma^m \quad (4)$$

where σ is an applied stress, m the stress-velocity exponent, and B a constant. The following relation between strain rate and average dislocation velocity is also given by⁽²⁴⁾

$$\dot{\epsilon} = \phi b \rho v, \quad (5)$$

where $\dot{\epsilon}$ is the strain rate, ϕ a geometrical constant, b the Burgers vector and ρ the mobile dislocation density.

The real strain rate inside the Lüders band front has been related to the strain gradient $(d\epsilon/dl')$, as treated by Hart⁽¹²⁾.

$$\dot{\epsilon} = V_B(d\epsilon/dl') \quad (6)$$

where V_B is the propagation velocity of the Lüders band front, l' the front width, and ϵ the strain inside the band front. From eqs. (5) and (6), we obtain

$$V_B = \phi b \rho v / (d\epsilon/dl'). \quad (7)$$

Although the strain distribution of a Lüders band front $\epsilon(l')$, the mobile dislocation density ρ and the dislocation velocity v might be dependent on the applied stress, especially the velocity would be most

sensitive to the applied stress than the other factors. Also, $d\varepsilon/dl'$ can be assumed to be approximately constant. Hence, eqs. (4) and (7) lead to the following relation:

$$V_B = \rho K_1 \sigma^m \quad (8)$$

where $K_1 = \phi b B / (d\varepsilon/dl')$ is constant.

It would be reasonably considered⁽²⁵⁾ that a certain stress concentration exists near the band front during the band propagation. Therefore, taking K_0 as the stress concentration coefficient, eq. (8) can be expressed as follows:

$$V_B = K_1 \rho (K_0 \sigma)^m. \quad (9)$$

When a stress relaxation (a stress drop) occurs, the following relation will be derived:

$$\Delta l = \varepsilon_L dl_0 = V_c \cdot dt - l_0 \cdot d\sigma / E, \quad (10)$$

where V_c is the cross head velocity, E is the Young modulus of the combined testing system including the specimen, and Δl is the elongation produced during a band propagation with a strain along the length l_0 at a time interval Δt .

From the fact that a very rapid stress drop appears, the band velocity can be very fast and independent of V_c ; therefore $V_c \ll V_B$. Also, since the Lüders band velocity is defined as $dl_0/dt = V_B$, eq. (11) is obtained from eq. (10):

$$\varepsilon_L V_B = -l_0/E \cdot d\sigma/dt. \quad (11)$$

Then, the following equation is given from eqs. (9) and (11):

$$\varepsilon_L \cdot K_1 \cdot \rho (K_0 \sigma)^m = -l_0/E \cdot d\sigma/dt. \quad (12)$$

Integrating eq. (12) and taking natural logarithms,

$$\ln(K_0 \sigma) = 1/1 - m \cdot \ln\{K_0(m-1)\} + 1/1 - m \cdot \ln(A/Ct + 1) \quad (13)$$

where $A = \varepsilon_L E K_1 \rho / l_0$, and C is the integration constant. When $t=0$, $\sigma = \sigma_0$ (σ_0 is the stress at the onset of the stress drop), we obtain

$$\ln(K_0 \sigma) = 1/1 - m \cdot \ln(A/Ct + 1) + \ln(K_0 \sigma_0). \quad (14)$$

Therefore,

$$\ln(\sigma/\sigma_0) = 1/1 - m \cdot \ln(A/Ct + 1) \quad (15)$$

where

$$A/C = \varepsilon_L \rho E K_1 (m-1) (K_0 \sigma_0)^m / l_0 \sigma_0 = \alpha. \quad (16)$$

Assuming $A/C \cdot t \ll 1$,

$$\ln(\sigma/\sigma_0) \simeq 1/1 - m \cdot \ln(\alpha t). \quad (17)$$

Using eq. (17) the relation between σ/σ_0 and time t at 160, 200 and 250°C are shown in Fig. 7. The result also shows that this relation can be given by this equation except the latter course of the band propagation. The value of α was decided by extrapolation of eq. (17) at each temperature. Substituting these α values in eq. (15), the stress-time relations are plotted in Fig. 8(a)–(c). The stress-velocity exponents m which were determined from the slopes of the straight lines

Fig. 7 Log. plot of stress-time curves during stress dropping at different temperatures.

Fig. 8 Stress-time relation in the stress drop during Lüders band propagation at (a) 160°C, (b) 200°C and (c) 250°C.

at different temperatures are given in Table 2. The values of m were 17–18 and were independent of deformation temperature.

Table 2 Values of parameters derived from the present work.

Temp. (°C)	<i>m</i>	$\alpha(\text{sec}^{-1})$	σ_0 (kg/mm ²)	V_0 (cm/sec)	$K_0 \left(\frac{m=18}{\sigma_0=20 \text{ kg/mm}^2} \right)$
160	18.4	1.292×10^3	21.2	55.6	1.75
200	17.4	7.322×10^3	20.0	312.6	1.93
250	18.4	1.448×10^4	19.7	595.9	2.00 ²⁶

5. Velocity of Lüders band propagation

As mentioned above, the stress drop was proportional to the width of a propagated region during the band propagation. The variation of their width with time is expressed as follows:

$$dl/dt = dl_0/dt + \varepsilon_L dl_0/dt + l_0 \cdot d\varepsilon_L/dt. \quad (18)$$

Since the Lüders strain will hardly change during the band propagation, $d\varepsilon_L/dt \approx 0$ and eq. (18) leads to

$$dl/dt = dl_0/dt \cdot (1 + \varepsilon_L). \quad (19)$$

Also, differentiating eq. (5) with t ,

$$d\sigma/dt = \sigma_0 \cdot \alpha / (1 - m \cdot 1/(\alpha t + 1)^{m-1}). \quad (20)$$

The velocity of the Lüders band propagation V_B is written by

$$V_B = dl_0/dt = 1/(1 + \varepsilon_L) \cdot dl/dt \\ = 1/(1 + \varepsilon_L) (d\sigma/dt) (dl/d\sigma). \quad (21)$$

Substituting eqs. (3) and (20) in (21), and taking $\varepsilon_L = 0.03$;

$$V_B = 1/2.85 \times \alpha \sigma_0 / (m-1) (\alpha t + 1)^{m/m-1}. \quad (22)$$

This equation gives the variation of V_B as a function of time t where $V_0 = 1/2.85 \times \alpha \sigma_0 / (m-1)$ corresponds to the initial velocity at the beginning of the stress drop. The values of α , σ_0 and m derived from the present work and V_B calculated by using these factors are given in Table 2.

The variations of the propagation velocity at each temperature were obtained by substituting the values $m=18$ and $\sigma_0=20 \text{ kg/mm}^2$ in eq. (22) and its plot is shown in Fig. 9.

Fig. 9 Variation of Lüders band velocity with time at different temperatures.

These results show that the decay of the velocity appears very remarkable at 250°C at which the initial velocity is the greatest, and became slower with decreasing temperature.

IV. Discussion

A sharp yield phenomenon appears on the stress-strain curve, when a polycrystalline iron containing even a small amount of carbon and/or nitrogen atoms is deformed in tension at room temperature. At the upper yield point, the Lüders band being accompanied by the rapid dislocation generation and its multiplication nucleates in some localized region and the yield stress reduces with a displacement of the front across the transverse direction of the specimen. After the stress falls to a lower yield point, the Lüders band propagates toward the direction of the tensile axis under the lower yield stress. However, when the deformation is performed in the temperature range of 160–250°C where the interaction between dislocation and solute atoms becomes pronounced during deformation, the band propagates with the repeated stress drops, i.e. serrations as shown in Fig. 1.

Although this phenomenon seems to be a simple repetition of an upper and a lower stress, the yield process shows a different behavior from that of deformation at room temperature.

The strain produced by the band propagation is about 3% at each deformation temperature and also the dislocation densities obtained do not show the temperature dependence. Therefore, the Lüders strain which corresponds to the dislocation density produced during the band propagation may be assumed to be almost constant with respect to the deformation temperature. It is also expected that the value of stress drop ($\Delta\sigma$) may be associated with only the magnitude of the front displacement. Thus the repeated discontinuous propagation of the Lüders band observed at a temperature where a dynamical strain aging becomes remarkable is obviously related to the distance of the front displacement during the stress drop.

If during the displacement of the front toward the undeformed matrix, no fresh mobile dislocations are generated in the region where the band front already propagated, the behavior of the Lüders band propagation will be governed only by that of the dislocations inside the front.

The stress dependence of a propagation velocity can be expressed by eq. (8), but in addition to the applied stress σ , a certain stress concentration between the deformed and undeformed regions should be considered⁽²⁶⁾ in eq. (9). If the value of K_0 is taken as a stress concentration coefficient, $K_0\sigma$ must be taken into account as a stress contributing to the actual band propagation.

The initial velocity of the propagation varies with deformation temperature as given in Table 2. At room temperature, the propagation velocity obtained was 0.02 cm/sec, but at elevated temperatures of 160, 200 and 250°C the velocities are 55.6, 312.4 and 595.9

cm/sec respectively. Thus it is obvious that the band begins to propagate at a higher velocity with rising temperature.

We will discuss the factors which cause an appreciable variation in propagating velocity. As expressed in eq. (9), the velocity of the band propagation V_B could be related to the applied stress σ_0 , the stress-velocity exponent m , the mobile dislocation density ρ , and the stress concentration coefficient K_0 . From the experimental results, the value of m is approximately 18 and σ_0 is about 20 kg/mm² which corresponds to the stress at the onset of the propagation at each deformation temperature.

Although the fraction of mobile dislocation to the total dislocation density is obscure, the dislocation densities obtained can be regarded as a constant value at different temperatures as described above. Furthermore, since the Lüders deformation is a process in which the band front propagates toward the undeformed matrix, and also further deformation seems not to occur in the propagated area, it will be supposed that most of the dislocations may be mobile only in the limited front region. Therefore, the mobile dislocation density would not also vary with deformation temperature.

The constant B in eq. (4) gives a frictional stress for dislocation motion and the motion might be affected by a resistance due to the P-L effect⁽²⁷⁾ which would result from the interaction between the mobile dislocation and solute atoms at elevated temperatures. For this reason, in the latter course of the propagation in which the velocity becomes very slow, the effect for the constant B due to the formation of a solute atmosphere cannot be ignored. A plot of eq. (15) shows the deviation from the straight line drawn in the final course of the propagation, and this deviation may be attributed to the influence of the atmosphere formation of solute around the mobile dislocations. However, in the case of a very high velocity of the propagation as in this experiment, the temperature dependence of this value might be insignificant.

By considering the above-mentioned fact and the temperature dependence of various factors, it can be thought that the variation of the initial velocity with the deformation temperature might mainly depend on the magnitude of the stress concentration coefficient K_0 . It has been reported that the coefficient at the head of a Lüders front where dislocations generate from the grain boundary might be 2⁽²⁶⁾ as a maximum value. In this study, supposing that the most remarkable dislocation locking takes place at 250°C where $K_0 = 2$, the values at 160 and 200°C provide 1.75 and 1.93 respectively. Although it is very difficult to determine the precise value of K_0 , it can qualitatively be understood that the effect of the stress concentration at the head of the band becomes greater with increasing temperature, since the rate of formation of a solute atmosphere depends on the diffusion rate of solute. Consequently, a part of the dislocations in the front region may be locked by solute atoms and act as a disturbance for the mobility of the dislocations so

that the active front region where the generation and/or multiplication of dislocations are performed may be limited and hence the width of the front will become much narrower.

Also, it is considered that a strain gradient in the front where the Lüders strain is produced becomes more steep with rising temperature, and then the higher stress concentration easily occurs. The front width was not measured in this work, but the extent of the sharpness of strain marks observed in the band front may suggest the magnitude of the front width.

The Lüders band begins to propagate again by this stress concentration produced in such a manner as mentioned above and at the same time the stress falls. Consequently, the propagating velocity continuously reduces with decreasing stress. The variation of this velocity is shown in Fig. 9 as a function of time and the velocity decays sharply within a short time.

Since the onset of the propagation is carried out at a very high speed, the effect of an atmosphere formation may be insignificant. However, when all the mobile dislocations can strongly be locked by the Cottrell atmosphere as the velocity becomes slower during the band propagation, the propagation ceases. These deformation processes in the Lüders propagation are repeated and the band discontinuously propagates along the specimen until all the gauge length is strained in the blue-brittleness temperature range.

V. Summary

The results discussed above for the behavior of the discontinuous propagation of a Lüders band in the blue-brittleness temperature range include the following:

- (1) The dislocation densities and its distribution observed after the Lüders band propagation do not depend on the deformation temperature and are approximately constant.
- (2) In the temperature range of 160–250°C, which corresponds to the blue-brittleness temperature, the Lüders band propagates with the repetition of an abrupt stress drop. The magnitude of this drop is proportional to the displacement distance of the band front.
- (3) The stress-velocity exponent, m , which was obtained by analysis of a stress-time relation does not vary with the deformation temperature under the experimental conditions and gives a constant value of about 18.
- (4) By considering of stress concentration at the head of the Lüders band front, the velocity of the Lüders band which propagates during one stress drop can be expressed by the same relation as that of the stress dependence of dislocation velocity.
- (5) The magnitude of a stress concentration becomes greater at elevated temperatures, at which the effect of dynamical strain aging is more remarkable. Also, the Lüders band begins to propagate at a higher velocity as the stress concentration becomes greater.

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