Effects of Material Anisotropy upon the Cutting Mechanism*

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This paper describes the effect of anisotropy of the material upon the cutting mechanism. The experiment was performed by orthogonal cutting of aluminum plate which is cut off at various inclined angles to the rolling direction, and the cutting phenomenon corresponding to the change of the inclined angle to the rolling direction was examined. Moreover, the obtained experimental values are analyzed on the basis of Hill's theory.

The results showed that the effects of anisotropy of the material appeared clearly at the cutting phenomenon, that is, the specific cutting force $K_s$ showed such a two cycle fluctuation that $K_s$ gains a maximum value at 30° and 120° respectively and a minimum value at 75° and about 150° when the inclined angle $\theta$ changes between 0° and 180°.

The relation between the shear angle $\phi$ and the inclined angle $\theta$ proved to be in fully reverse phase to the $K_s$-$\theta$ curve.

The theoretical values based on Hill's theory showed qualitative coincidence with the experimental value of shear angle $\phi$, but it was difficult to obtain theoretical conformity with the experimental value of shear stress $\tau_s$.

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I. Introduction

Recently the theory of plasticity including strain hysteresis relating to the problem of the plastic working—the so-called theory of anisotropic plasticity—has become a subject of active experimental and theoretical studies(1). On the other hand, in a cutting process, very few investigations have treated the cutting phenomenon considering strain hysteresis(2)~(5). It is, therefore, the present condition that the major part of the dynamic analysis of the cutting phenomenon is performed under the assumption of isotropy of a material. The assumption of isotropy, however, is unsuitable to the analysis of a cutting mechanism of the material which has undergone the progress of high strain, that is, such a material had strong anisotropy at the previous stage of cutting and therefore the analysis considering the effect upon anisotropy is deemed to be necessary for the above analysis.

So, in this paper, the cold-rolled material of aluminum was used as a material to be cut, an orthogonal cutting was performed by changing the angle $\theta$ between the rolling direction and the cutting direction, and a cutting phenomenon changing with the $\theta$ was examined. Some information obtained by the result concerning the effect of anisotropy upon a cutting mechanism is presented.

II. Experimental Procedure

1. Specimen preparation

Aluminum plate (Al–2.4% Mg alloy 5052) with a finished thickness of 15 mm by hot rolling and with a chemical composition shown in Table 1, was used as a cutting material. After removing the prehysteresis by heat treatment at 540°C for 30 h in a salt bath, cold rolling was performed by changing the rolling reduction 60% to 90% by a two-high rolling mill. Thus a rolled plate with different anisotropy strength was produced. The speci-

Table 1 Chemical composition of Specimen (wt%).

<table>
<thead>
<tr>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Cr</th>
<th>Zn</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si+Fe=0.45</td>
<td>0.10</td>
<td>0.10</td>
<td>2.4</td>
<td>0.2</td>
<td>0.2</td>
<td>R.</td>
<td></td>
</tr>
</tbody>
</table>

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men is cut off from the sample at an interval of 15° within a range from 0° to 90°. The specimen is shaped like a narrow paper tablet with a size of 80 × 30 × 1.5 (mm).

2. Experimental for cutting

Cutting was based on an orthogonal cutting method using the longitudinal feed of a vertical milling machine. The specimen, which has been fixed to a tool dynamometer set on the table of the milling machine, was cut and the measurement of the cutting force and cutting ratio was performed. Experimental conditions are shown in Table 2. A large rake angle such as 30° and 40° was used to prevent the generation of built-up edge. Even at these large rake angles a slight built-up edge was still observed, though its effect appears to be a negligible small. Consequently, the effect upon the built-up edge was neglected in this paper for the analysis of experimental data. Figure 1 is an orthogonal cutting model.

<table>
<thead>
<tr>
<th>Tool dimensions</th>
<th>Rake angle α</th>
<th>30, 40 (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relief angle γ</td>
<td>9 (deg)</td>
<td></td>
</tr>
<tr>
<td>Nose radius r</td>
<td>15 (µm)</td>
<td></td>
</tr>
<tr>
<td>Tool material</td>
<td>SKH 9</td>
<td></td>
</tr>
</tbody>
</table>

Cutting conditions:
- Cutting speed V = 300 (mm/min)
- Depth of cut t = 0.1 (mm)
- Cutting fluid: none

Table 2 Experimental conditions.

III. Experimental Results and Discussion

1. Change of specific cutting force $K_s$

Figure 2 shows the relation between the inclined angle $\theta$ (the angle between the rolling direction and the cutting direction) and the specific cutting force $K_s$ (hereafter referred to as $K_s$). This experimental result makes clear some respects concerning the effect of the anisotropy of the material.

(i) The value of $K_s$ showed such a two cycle fluctuation that $K_s$ gains a maximum value at 30° and 120° respectively and a minimum value at 75° and about 150° when the inclined angle $\theta$ changes between 0° and 180°.

(ii) The effect of the rolling reduction upon $K_s$ changes at almost the same cycle on the 70, 80 and 90% rolled specimen, while the periodicity for the fluctuation of $K_s$ has been lost in the 60% rolled specimen and also the regulation become smaller. Moreover, the cutting of the annealed specimen (the rolling reduction is equal to 0%) was performed to compare with the rolled specimen, but nothing in the fluctuation of $K_s$ could be seen and $K_s$ was obtained with a constant straight line.

The cause of such loss of periodicity in $K_s$ means the decrease in the anisotropic strength of the specimen.

The anisotropic strength was presumed from the X-ray intensity of the preferred orientation in the rolling texture corresponding to the rolling reduction.$^6$

Fig. 1 Schematic diagram of orthogonal cutting model.

Fig. 2 Variation in specific cutting force $K_s$ with inclined angle to rolling direction $\theta$. 
According to the result, the rolling reduction becomes higher, the anisotropic strength becomes stronger too. The orientation (112) [111] appeared to the final and stable orientation in the rolling texture at the rolling reduction of 70% and the X-ray intensity of this orientation also increases as the rolling reduction increases. On the one hand, the orientation (113) [332] besides the orientation (112) [111] appears at the 60% rolled specimen and moreover, the strong X-ray intensity was shown in the former. It is considered that the fact that such a distribution of a crystal grain cannot be shown by only the one preferred orientation leads to the appearance of the turbulence of periodicity in $K_s$ during cutting.

Figure 3 shows the effect of the rake angle on $K_s$ in the 90% rolled specimen.

For the experiment performed at two rake angles of 30° and 40°, the values of $K_s$ fluctuate at the same period in both cases. The value of $K_s$ decreases as the rake angle becomes bigger and, moreover, the rate of fluctuation is low. The result means that the effect of the anisotropy of the material is reduced in the case of the cutting by a sharp cutting edge.

2. Change of shear stress

It is expected that shear plane become a flow zone having some thickness at low cutting speed as in this experimental condition. But in this paper, in order to simplify the analysis, $\tau_s$ was calculated by the following formula, assuming a single shear plane:

$$\tau_s = (F_H \cos \phi - F_v \sin \phi) \sin \phi/t.w,$$

(1)

where $F_H$ and $F_v$ are the principal and the thrust component of force of cutting resistance respectively, $\phi$ is the shear angle (the value obtained from a cutting ratio), $t$ is the depth of cut, and $w$ is the cutting width.

The relation between the value of $\tau_s$ calculated and $\theta$ is given in Fig. 4. According to this figure, there is more or less a phase lag in the state of fluctuation of $\tau_s$ in the 70%, 80% and 90% rolled specimen, but $\tau_s$ fluctuates at almost the same period. In the 60% rolled specimen, however, dispersion appears at the experimental point and the specimen has no periodicity.

Figure 5 shows the result obtained by varying the rake angle of the 90% rolled specimen. A remarkable difference appears at the absolute value of $\tau_s$ as the rake angle changes and the value of $\tau_s$ at $\alpha$ of 40° was about 20% less than the value of $\tau_s$ at $\alpha$ = 30°.

Fig. 4 Variation in shear stress $\tau_s$ with inclined angle to rolling direction $\theta$.

Fig. 5 Variation in shear stress with inclined angle to rolling direction $\theta$. Rake angles: 30° and 40°.
3. Change of shear angle $\phi$

The relation between the shear angle and the inclined angle $\theta$ with respect to the rolling reduction is shown in Fig. 6 and the effect of anisotropy of the material strongly appears at a shear angle too.

The value of the shear angle showed such a periodical fluctuation that the value of $\theta$ a maximum at $75^\circ$ and $150^\circ$ and a minimum at $30^\circ$ and $120^\circ$.

The value of a shear angle proved to be in fully reverse phase to the $K_s-\theta$ curve (Fig. 2) as shown previously.

This reverse phase relation can be explained by the fact that $K_s$ is in an inversely proportional relation to $\phi$. $A_s$ is evident from some models of the cutting of a shear plane, the following relation between shear angle $\phi$ and a shear plane area $A_s$ exists:

$$A_s = t \cdot w / \sin \phi,$$

(2)

where $t \cdot w = \text{const.}$ so that the shear plane area becomes larger as $\phi$ becomes smaller. Therefore, even if $\tau_s$ is assumed to be constant irrespective of $\theta$, the value of $K_s$, the force required to produce a chip will become larger at a large shear plane area. In fact, however, $\tau_s$ changes with $\theta$ as shown in Fig. 4 and, moreover, $\tau_s$ becomes larger at the place where $\phi$ is small, that is, at the place where the shear plane area is large. It can be said, therefore, that $K_s$ becomes a fully reverse phase to $\phi$.

The effect of the rolling reduction upon the shear angle is larger as the rolling reduction is higher.

To compare with the annealed specimen the value was written together. According to the result, no change in the shear angle due to $\theta$ could occurs.

4. Direction of cutting force $R$

The direction of the cutting force $R$ is shown by the angle $(\beta - \alpha)$ between the cutting direction and the cutting force $R$. Where the angle $(\beta - \alpha)$ can be determined from the following formula:

$$\beta - \alpha = \tan^{-1} F_v/F_H.$$

(3)

As mentioned later, the values are significant for the theoretical shear angle analysis.

Figure 7 shows the relation between the angle $(\beta - \alpha)$ and the inclined angle $\theta$. It is shown that the values of the angle $(\beta - \alpha)$ to the inclined angle $\theta$ fluctuates within the range of about $8^\circ$ for every rolling reduction.

From this fact, there is little effect of the rolling reduction upon the angle $(\beta - \alpha)$. In other words, it means that the ratio of $F_H$ and $F_v$ is nearly independent of the rolling reduction and the inclined angle $\theta$.

IV. Consideration Based upon Plane Strain Theory

It is already known that the strain in the deformation area of a cutting process except in the neighbourhood of a cutting edge is in the plane strain state.

In this paper, the consideration of the shear stress $\tau_s$ and the shear angle $\phi$ using Hill's solution was performed and some examinations concerning the conformity were per-
formed.

1. Shear stress

When the rolling direction of a rolled plate with orthotropy is fixed as axis x, the direction perpendicular to the axis x in the plate surface is fixed as axis y, and the direction of plate thickness is fixed as axis x, the yield condition at the state of plane strain of a plate is expressed by the following formula(7):

\[
\frac{(\sigma_x - \sigma_y)^2}{4(1-C)} + \tau_{xy}^2 = T^2, \tag{4}
\]

where C is an anisotropic constant and T is shearing yield stress concerning axis x and y.

On the one hand, a slip line generating at a single axis tensile test of a test piece cut off at an inclined angle \(\theta\) to the rolling direction can be regarded as an approximate coincidence with a shear plane. So a slip line shall be taken as a polar coordinate, stress components concerning this shall be taken as \(\sigma_\alpha\), \(\sigma_\beta\), and \(\tau_{\alpha\beta}\), and \(\Phi\) shall be taken as a counterclockwise orientation of a curve to the axis (the rolling direction), the coordinate conversion of the stress components are given by the following expression:

\[
\begin{align*}
\sigma_x &= \sigma_a \cos^2 \Phi + \sigma_\beta \sin^2 \Phi + 2\tau_{\alpha\beta} \sin \Phi \cos \Phi, \\
\sigma_y &= \sigma_a \sin^2 \Phi + \sigma_\beta \cos^2 \Phi - 2\tau_{\alpha\beta} \sin \Phi \cos \Phi, \\
\tau_{xy} &= -(\sigma_a - \sigma_\beta) \sin \Phi \cos \Phi \\
&\quad + \tau_{\alpha\beta} (\cos^2 \Phi - \sin^2 \Phi). \tag{5}
\end{align*}
\]

Thus, the yield condition showed by eq. (4) can be replace by

\[
f(\sigma_\alpha, \sigma_\beta, \tau_{\alpha\beta})
\]

\[
= \frac{1}{1-C}[(\sigma_a + \sigma_\beta) \cos 2\Phi - 2\tau_{\alpha\beta} \sin 2\Phi]^2
\]

\[
+ [(\sigma_a - \sigma_\beta) \sin 2\Phi + 2\tau_{\alpha\beta} \cos 2\Phi]^2 = 4T^2, \tag{6}
\]

since a slip line (shear plane) coincide with a direction which a elongation is nothing.

\[
\frac{\partial f}{\partial \sigma_\alpha} = - \frac{\partial f}{\partial \sigma_\beta} = 0.
\]

That is, eq. (6) can be rewritten as

\[
\begin{align*}
\cos 2\Phi ([(\sigma_a + \sigma_\beta) \cos 2\Phi - 2\tau_{\alpha\beta} \sin 2\Phi] \\
&\quad + \sin 2\Phi [(\sigma_a - \sigma_\beta) \sin 2\Phi + 2\tau_{\alpha\beta} \cos 2\Phi] = 0.
\end{align*} \tag{7}
\]

solving for \(\tau_{\alpha\beta}\) from the eqs. (6) and (7), \(\tau_{\alpha\beta}\) is given by the following equation:

\[
\tau_{\alpha\beta} / T = (1 - C \sin^2 2\Phi)^{1/2}. \tag{8}
\]

A model of cutting described on the basis of an anisotropic theory is shown in Fig. 8, and \(\Phi\) in formula (8) can be written as \(\Phi = \theta - \phi\) from its definition. The shear stress \(\tau_{\alpha\beta}\) on the slip line corresponds to the shear stress \(\tau_s\) in a cutting.

The values of \(C\) and \(T\) were determine by the following process. When the direction of the tension of the test piece is inclined at \(\theta\) to the rolling direction, the tensile yield stress \(\sigma\) of the direction under the plane strain is expressed as follows(7):

\[
\sigma = 2T \left( \frac{1 - C}{1 - C \sin^2 2\Phi} \right)^{1/2}. \tag{9}
\]

When formula (9) is solved by substituting the tensile yield stress \(\sigma_{0.2}\) at 0° of and 45° into the formula, \(C\) and \(T\) are decided. The value obtained is as follows:

\[
C = -0.3, \quad T = 52 \text{ [MPa]}. \]

The value of \(\tau_{\alpha\beta}\) calculated from formula (8) for the 90% rolled material taken as an example and the experimental value of \(\tau_s\) obtained previously are shown Fig. 9. The two curves show to be in reverse phase relation and it can be said that the analysis of the shear stress based on the plane strain theory is difficult for the material used in the experiment.
2. Shear angle

When the inclination of the direction of the maximum shear stress \( \tau_{\text{max}} \) to the rolling direction is taken as \( \psi \), and the inclination of the direction of the maximum shear strain velocity \( \dot{\gamma}_{\text{max}} \) is taken as \( \psi' \), the following relation is obtained(7):

\[
\tan 2\psi' = (1 - C) \tan 2\psi. \tag{10}
\]

Formula (10) shows that the direction of \( \tau_{\text{max}} \) is not coincident with the direction of \( \dot{\gamma}_{\text{max}} \) in the presence of anisotropic constant \( C \).

So, assuming that the angle made by the cutting direction with the rolling direction (axis x) is \( \theta \), the shear plane coincides with the direction of the maximum shear strain velocity, and the angle made by the resultant force of a cutting force with direction of the maximum shear stress is 45\(^\circ\), from the model of cutting shown in Fig. 8, \( \psi \) and \( \psi' \) can be written as follows:

\[
\psi = \theta + (\beta - \alpha) - \frac{\pi}{4}, \quad \psi' = \theta - \phi. \tag{11}
\]

Accordingly, the change of the shear angle \( \phi \) for \( \theta \) will be obtained theoretically from the formulae (10) and (11), if \( (\beta - \alpha) \) is decided by the cutting experiment.

Figure 10 shows a comparison between the experimental value and the theoretical value for the case at 90\% rolled material in the same manner as in the previous subsection. In the figure the coincidence between their absolute values cannot be seen, but it is found that they fluctuate almost at the same period. Accordingly, the phenomena of the change of a shear angle \( \phi \) by \( \theta \) can be explained theoretically in a qualitative way.

As mentioned above, Hill's theory is not substantially applicable to analysing the shear stress and shear angle. As one of the causes, the calculation of the experimental values was carried out on the assumption of a single shear plane model.

V. Conclusion

The cold rolled material of aluminum was used as cutting material. Orthogonal cutting was performed by varying the inclined angle \( \theta \), and the cutting phenomenon corresponding to the change in \( \theta \) was examined.

1. The change in specific cutting force with inclined angle \( \theta \) showed such a periodic fluctuation that the value is maximum at 30\(^\circ\) and 120\(^\circ\) and minimum at 75\(^\circ\) and 150\(^\circ\).

2. The effect of the rolling reduction upon the specific cutting force, changes at almost the same period for the 70\%, 80\% and 90\% rolled material. On the one hand, the turbulence of the force periodicity occurs for the 60\% rolled material and, moreover, the regulation is small.

3. The curve showing the relation between a shear angle \( \phi \) and \( \theta \) was in fully reverse phase to the curve \( K_s - \theta \).

4. The value of the angle \( (\beta - \alpha) \) of the cutting force was shown a periodic fluctuation within a range of about 8\(^\circ\) in the case of the rake angle 30\(^\circ\).
(5) The theoretical value by the Hill's theory showed qualitative coincidence with an experimental value of shear angle \( \phi \), but the value came to be in reverse phase to shear stress \( \tau \), so that it was difficult to obtain theoretical conformity with the experimental value.

REFERENCES

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