Dislocations in Ferromagnetic Materials

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Magnetoelastic interaction between stress around dislocation lines and magnetostriction was investigated. When a magnetic wall passes across dislocations, energy due to this interaction varies with its position. Variation of this energy changes by $10^{-5}$ erg/cm/dislocation in order of magnitude. Supposing that this energy change is the main origin of variation of internal energy with the wall position a formula of coercive force and initial susceptibility are deduced. The calculated values of coercive force and initial susceptibility of iron are 0.1 Oe and 49, respectively. These values seem to agree well with the observed values for annealed iron. Temperature dependence of coercive force and its variation by plastic deformation are interpreted by the present theory.

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I. Introduction

Hitherto, magnetic characteristics of ferromagnetic materials have been investigated disregarding the origin and the exact distribution of internal stress. Recently, correlation between structure-sensitive character of metal and the defects such as vacancies, dislocations and voids have been investigated in the field of plasticity of metal. In this paper, the effect of stress around dislocation lines on the mechanism of displacement of the magnetic wall will be studied and the behaviour of coercive force in reference to the plastic deformation of magnetic materials will be discussed.

II. Displacement of a Magnetic Wall

In an ideal strain-free single crystal of a ferromagnetic substance a magnetic wall should, under the action of a very small external field, move immediately into its new equilibrium position. But, experimental fact shows that there is a certain resistance to the wall displacement which has to be overcome by adding a certain finite field. This fact means that there are local variations of the internal energy of the specimen with the wall position. They can arise through any deviation from the ideal lattice due to lattice imperfections, strain, etc. In this paper, stress around dislocation is assumed to be a dominant lattice perturbation.

The distribution of dislocations in the crystal are not clear, but it has been noted that the number of dislocation lines in an annealed metal may be in the order of $10^6$ to $10^8$/cm$^2$ and that around every dislocation line, there is a stress field which is inversely proportional to the distance from its center. As the result of interaction between this stress and magnetostriction, the density of magnetoelastic energy varies around a dislocation line.
Furthermore, magnetoelastic energy is so much larger than magneto-crystalline anisotropy energy in the neighbourhood of a dislocation core, that the direction of magnetization must deviate from that of the non-stressed portion by the action of the stress field, and accordingly internal magnetostatic energy therefore should appear. These magnetoelastic and magnetostatic energies should contribute to the internal energy.

Now in a magnetic domain boundary or in a magnetic wall direction of magnetization changes in a gradual way over many atomic planes. Hence, if a magnetic wall moves across a dislocation line, magnetoelastic energy varies with the position of the wall due to interaction between stress around the dislocation and magnetostriction. In the vicinity of a dislocation, generally distribution of internal poles varies when the magnetic wall passes through the dislocation. Then, strictly speaking, change of wall energy should be the sum of contributions of magnetoelastic and magnetostatic energies, but the latter is negligibly smaller than the former and accordingly, it is assumed that the energy of the wall varies only with the change of magnetoelastic energy.

III. Change of Wall Energy due to Stress Around a Dislocation

If the direction of magnetization and the stress at a point within the material are denoted by $\alpha_i$ and $\tau_{ij}$, respectively, magnetoelastic energy can be given by,\(^{(1)}\)

$$W'(\theta) = -\frac{3}{2}(\lambda_{100} \sum \alpha_i^2 \tau_{ii} + 2 \lambda_{111} \sum \alpha_i \alpha_j \tau_{ij}). \quad (1)$$

($ijk$) : (xyz)

In the following the magnetoelastic energy due to the interaction between the stress around the dislocation and the spin orientation in a domain is calculated.

In Fig. 1, let the directions of magnetization of the domains in the both side of a domain wall lie along ± Y-direction respectively. If $\theta(X)$ denotes the angle between spins and Y-direction at a point $X$ in the domain wall, then $\theta(X)$ is expressed by,$^{(1)}$

$$\sinh \left\{ \frac{K(1+P)}{A} \right\}^{1/2} X = -\left( \frac{1+P}{P} \right) \cot \theta. \quad (2)$$

$$P = \frac{9}{4} \left( C_{11} - C_{12} \right) \frac{2^2}{K}$$

where, $K$ is magneto-crystalline anisotropy energy, $A$ is exchange energy, $C_{11}$, $C_{12}$ are moduli of elasticity and $\lambda$ is the magnetostriction constant. Direction of magnetization, mentioned above, will be altered by the stress field around a dislocation line in the vicinity of it, but at some distance from the dislocation, magnetoelastic energy due to the stress of the dislocation becomes far smaller than magneto-crystalline anisotropy energy, so that it is assumed here that the effect of stress field around the dislocation on the spin orientation is neglected for the following calculations. First the case of screw dislocation is to be considered.

(1) In Fig. 2 position of the screw dislocation is $(x_0, 0, 0)$. The axis of the dislocation is lying along the $z$ axis and it makes an angle $\alpha$ with the $Z$ axis in the YZ plane. Stress components around a screw dislocation are given by the following expression.$^{(2)}$

\[
\begin{align*}
\tau_{xx} &= -\tau_{yy} \frac{y}{x^2 + y^2}, \\
\tau_{yz} &= -\tau_{zx} \frac{x}{x^2 + y^2}, \\
\tau_{zz} &= \frac{G b}{2\pi},
\end{align*}
\]

where $G$ is shear modulus and $b$ is the amount of Burger's vector. Substituting Eq. (3) into Eq. (1) and assuming $\lambda_{100} = \lambda_{111} = \lambda$, magnetoelastic energy in the wall is given by,

$$I^{t}_x = \int_2 \int \left( -\frac{3}{2} \lambda \tau_{ij} \right) \sin 2(\theta - \alpha - \frac{x}{x^2 + y^2}) dx dy $$

where the width of the wall is supposed to be $2\delta$. Now, since the relation between $\theta$ and $X$ is given by Eq. (2), we have

$$I^{t}_x = 3\pi \lambda \tau_{ij} \left\{ \cos 2\alpha \int_2 \left( \frac{b_0 \sinh aX}{b_0^2 + \sinh^2 aX} \right) dx \\
+ \sin 2\alpha \int_2 \left( \frac{b_0^2}{b_0^2 + \sinh^2 aX} \right) dx \\
- \frac{1}{2} \sin 2\alpha \int_2 \frac{dx}{aX} \right\}. \quad (5)$$

where

$$a = \left\{ \frac{K(1+P)}{A} \right\}^{1/2}, \quad b_0 = \left( \frac{1+P}{P} \right)^{1/2}.$$ 

Putting $X = x + x_0$, Eq. (6) is integrated over $2\delta$, then total magnetoelastic energy is given by,


(2) T. Read: Dislocations in Crystals. (1953), Mc.Graw-Hill.
\[ \Gamma^g = \frac{3 \lambda}{2} Gb \left( \frac{A}{K} \right)^{1/2} \left[ \sin 2\alpha \ln \left( \frac{1 + P^{1/2} + \tanh ax_0}{1 + P^{1/2} - \tanh ax_0} \right) + 2 \cos 2\alpha \left( \frac{\tan^{-1} (P^{1/2} \cosh ax_0)}{1 + P^{1/2} \cosh ax_0} - \frac{-\tan^{-1} (P^{1/2} \cosh ax_0)}{1 + P^{1/2} \cosh ax_0} \right) \right] \]  

(6)

For the case of edge dislocation the result obtained above is a little bit changed. In Fig. 1, xz plane is the slip plane on which the dislocation lies. Angle between x and X is denoted by \( \alpha \). The stress components around a edge dislocation are given by \( \tau \).

\[ \tau_{xx} = \frac{\kappa}{b_0} \frac{3(x^2 + y^2)}{x^2 + y^2} \]  

\[ \tau_{yy} = \frac{\kappa}{b_0} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}, \quad \tau_{xy} = \frac{\kappa}{b_0} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2} \]  

(7)

where \( \kappa \) is Poisson’s ratio. Then, \( \Gamma^g \) should be added to the wall energy and then \( \gamma \) varies with the change of \( \Gamma^g \) as the wall passes a dislocation line.

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\[ H_0 = \frac{1}{2} J_5 \left( \frac{d\gamma}{d\alpha} \right)_{\alpha_{\text{max}}} \]  

(9)

The relation between \( \Gamma^g \) and \( x_0 \) calculated by Eqs. (6) and (8) are shown in Figs. 3 and 4. As shown in the figures, total magnetoelastic energy varies when the magnetic wall passes across a dislocation line.

IV. Coercive Force

The energy of the wall should vary in a more or less irregular way depending upon the position of the wall and there may be a position of minimum energy, and the wall will naturally take up these positions. The wall may be displaced from these positions by the application of an external field, which in effect exerts a pressure on the wall tending to displace it so as to increase the magnetization in the direction of the field. Then, the coercive force is a measure of the magnitude of the maximum restoring force on the wall and this threshold field, \( H_0 \), necessary for transitional motion of the wall separating domains magnetized oppositely is expressed by, usually,

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(9)

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For a screw dislocation, by Eq. (6),

\[ \frac{d\Gamma^g}{dx_0} = 3 \lambda Gb \left( \sin 2\alpha \frac{b_0}{b_0 + \sinh ax_0} + \cos 2\alpha \frac{b_0 \sinh ax_0}{b_0 + \sinh ax_0} \right) \]  

(10)

Maximum or minimum points of \( \frac{d\Gamma^g}{dx_0} \) come from \( \frac{d^2\Gamma^g}{dx_0^2} = 0 \; \text{satisfy} \; \sinh ax_0 = \frac{b_0}{b_0} \frac{(1 - \sin 2\alpha)}{\cos 2\alpha} \) or \( -\frac{b_0}{b_0} \frac{(1 + \sin 2\alpha)}{\cos 2\alpha} \).

Inserting Eq. (11) into Eq. (10),

\[ \frac{d\Gamma^g}{dx_0} = \frac{3}{2} \lambda Gb (1 + \sin 2\alpha) \quad \text{or} \quad -\frac{3}{2} \lambda Gb (1 - \sin 2\alpha). \]

Then,

\[ H_0^g = \frac{3 \lambda Gb}{4 J_5} (1 + \sin 2\alpha) \]  

(12)

Similarly, for an edge dislocation, the following expression for \( H_0^g \) is obtained.

\[ H_0^g = \frac{3 \lambda Gb}{4 J_5} (1 - \cos 2\alpha) \]  

(13)

The variation of the wall energy is the sum of the contribution of magnetoelastic energy of each dislocation, but this contribution varies with the following factors; 1. Kind of dislocation. 2. Direction of the dislocation axis in the wall. 3. Distance between the dislocation line and the midplane of the wall. If distribution of the dislocation axis is isotropic, then the contribution of the total sum of magnetoelastic energy should be zero on an average and consequently, energy of the wall does not change at any point in the material. But, strictly
speaking, there is a fluctuation of local distribution of the dislocation. The contribution due to this fluctuation may be the cause of coercive force. Suppose the volume of a magnetic domain is $L_3$ and the total number of dislocations in the wall is $n$, then dislocation density effective as obstacles may be given by the following expression,

$$ m = m_0 \left( n \ln \frac{L}{2\delta} \right)^{1/2}. $$

Then, coercive force $H_C$ is given by,

$$ H_C = \frac{n^{1/2} L^{1/2}}{2 J_s L} \ln \frac{L}{2\delta} d \eta \left|_{\max} \right. $$

The density of dislocation per unit area is denoted by $N$ and supposing that the axis of these dislocations distribute isotropically and the dislocation contributes to the increase of surface energy only when the axis of it lies within a range which makes an angle $2\theta$ with the plane of the wall. The density of dislocation nearly parallel to the plane is given by,

$$ \rho_{11} = \frac{N L^3}{2\pi} \left( \int_{\pi/2/\pi}^{\pi/2} \sin \beta d\beta \int_{0}^{2\pi} d\phi = 2N\delta L. \right) $$

And then,

$$ m = 2\delta \left( \ln \frac{L}{2\delta} \right)^{1/2} N^{1/2}. $$

Consequently,

$$ H_C = \frac{3\lambda G b}{2J_s L} \left( \ln \frac{L}{2\delta} \right)^{1/2} N^{1/2}. \quad (14) $$

V. Coercive Force of Deformed Materials

When magnetoelastic energy density due to internal stress has the same order of magnitude as magneto-crystalline anisotropy energy, contribution of this magnetoelastic term to the wall energy must be taken into account. When the amplitude of the variation of internal stress from point to point in the material is $2\delta$ and the distribution of it is isotropic, the width of the magnetic wall has been given by,

$$ 2\delta = 6 \left( \frac{A}{K(1+P)} \right)^{1/2}, $$

$$ P = \frac{3\lambda}{2K}. $$

While, based on the dislocation theory on work-hardening of a pure metal, internal stress is supposed to be proportional to the square root of the dislocation density. So that, from Eq. (14) we get

$$ H_C = C_0 F(z) = C_0 \left( \frac{A}{J_s (K+3/2\lambda z)} \right)^{1/2} \cdot z \left( \ln \left( \frac{L}{2} \frac{K+3/2\lambda z}{A} \right)^{1/2} \right)^{1/2}, \quad (15) $$

where $C_0$ is a proportional constant which does not depend on temperature. Now, putting the actual value $\lambda = 2.5 \cdot 10^{-6}$ cm/cm, $K = 3.4$ ergs/cc, $J_s = 485$ gauss and $A$ for nickel, which are temperature dependent, into

the above expression, then dependence of coercive force of nickel on the magnitude of internal stress can be obtained. The result of calculation for the case, $\delta = 1, 2, 4, 6, 8, 12$ Kg/mm$^2$ are shown in Fig. 5. ($F(z)$ curve) As can be seen in Fig. 5, the coercive force for the case $z=1$ kg/mm$^2$ which is thought to correspond to annealed nickel takes a maximum value at about 100$^\circ$C. For the larger value of $z$ the maximum point of the coercive force curve appears at a lower temperature. Some experimental evidence for the present results are summarized in the following.

Dietrich and Kneller(4) measured temperature dependence of the coercive force of a nickel single crystal at temperatures from $-200^\circ$C to $400^\circ$C and they found that; 1. the value of coercive force becomes maximum at about $100^\circ$C. 2. when the nickel single crystal is strained by tension, maximum coercive force appears at a lower temperature with increase of working. Further, a linear relation of coercive force with $z$ is shown in Fig. 6. A relation similar to this theoretical one has been obtained by Kneller et al.(4)

VI. Initial Susceptibility

Initial susceptibility is a measure of the intrinsic

Fig. 6 Relation between coercive force and stress. (Theoretical curve)

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restoring force on the magnetic wall for small displacement. If the magnetoelastic energy $\Gamma'$ changes with $x_0$, the initial susceptibility, $\chi_0$, is given by the following expression,

$$\chi_0 = \frac{dJ}{dH|_{H=0}} = \frac{4 J_s^2}{R_0} B \left( \frac{d\Gamma'}{dx_0^2}|_{H=0} \right)^{-1},$$

where $B$ is a factor which depends on the number of the effective wall for the increase of magnetization and $R_0$ is a factor which depends on the number of dislocation effective to the increase of energy of the wall.

The minimum position of the wall energy curve can be given by, $d\Gamma'/dx_0 = 0$. Then, by Eq. (10), we have

$$\sinh \alpha x_0 = -b_0 \tan 2\alpha.$$

And then, for a screw dislocation,

$$\frac{d \Gamma_s}{dx_0^2}|_{H=0} = 3 \lambda G b \left( \frac{A}{K} \right)^{1/2} \cos^2 2\alpha \left( \sin^2 2\alpha + P \right)^{1/2}.$$

Similarly, for an edge dislocation,

$$\frac{d \Gamma_e}{dx_0^2}|_{H=0} = 3 \lambda G b \left( \frac{A}{K} \right)^{1/2} \sin w \sin^2 2\alpha \cos^2 2\alpha + P^{1/2}.$$

Accordingly, initial susceptibility can be given by,

$$\chi_0 = \frac{16 J_s^3}{3 \lambda G b} \left( \frac{A}{K} \right)^{1/2} \frac{B}{R_0}. \quad (16)$$

The initial susceptibility and coercive force for both nickel and iron at room temperature are calculated by putting the observed value of $J_s$, $\lambda$, etc. into Eqs. (14) and (16). The results are shown in Table 1. For the sake of comparison, the experimental values of $H_c$ and $\chi_0$ are also shown in the same table, and the coincidence of theoretical and experimental results are satisfactory.

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