Fracture of Particles in a Particle/Metal Matrix Composite under Plastic Straining and Its Effect on the Young's Modulus of the Composite

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Fracture of Al_2O_3 particles in an Al_2O_3 particle/ aluminum alloy metal matrix composite (MMC) under plastic straining was observed by *in-situ* tensile testing in a scanning electron microscope. The fracture of larger sized particles was found to be preferred to that of smaller sized ones for a given plastic strain. The Young's modulus (E_c) of plastically strained MMC was reduced with increase in plastic strain (ϵ_p). The analytical model based on Eshelby's equivalent inclusion method was constructed to explain the relationship between E_c and ϵ_p , resulting in a good agreement with the experimental results.

(Received April 16, 1991)

Keywords: metal matrix composites, particle fracture, Young's modulus, Weibull distribution, Eshelby's method

I. Introduction

Particle metal matrix composites (MMCs) are attractive due to the fact that the properties are isotropic and its processing into complicated shapes is relatively easy and less costly compared with continuous fiber MMCs, and the thermo-mechanical behavior of particle MMCs has been reviewed by Taya and Arsenault⁽¹⁾. It has been believed that pre-plastic straining of a particle MMC would not influence the elastic properties of the MMC-like dispersion hardened alloys. Recently Lloyd reported the reduction in the Young's modulus of SiC particle/6061 Al composites that were plastically strained to various levels⁽²⁾, and found that larger sized SiC particles are fractured first during plastic straining, in agreement with the Weibull model in that the probability of fracture of particles is governed by a so-called Weibull distribution⁽³⁾.

With the above background in mind, we attempt in this paper to construct analytical models to explain the particle failure mode observed in a particle MMC and also its effect on the Young's modulus of the MMC. To this end, we shall first examine experimentally the particle fracture process in Al_2O_3 particle/6061 Al composites by *in-situ* SEM tensile testing. Based on this SEM results and the data on particle size distribution measured, two analytical models are constructed: (1) calculation of stresses induced in particles in a MMC under plastic straining, and (2) calculation of the Young's modulus of the MMC with fractured particles. The first model will be applied to the cracking process of particles in a particle MMC subjected to plastic strain, while the second model

will be used to quantify the reduction in E_c of the MMC that was plastically strained.

First the experimental set-up and results are stated in Section II. Analytical models are developed in Section III, followed by the discussion in Section IV. Finally, concluding remarks are given in Section V.

II. Experiment

1. MMC specimen

The MMC used in this study is 15 vol% Al_2O_3 particle/6061 Al($\text{Al}_2\text{O}_3/\text{Al}$) composite. $\text{Al}_2\text{O}_3/\text{Al}$ composite was melt-casted and then extruded at 550°C with ratio 40:1, formed into a rod of diameter 9.5 mm. The material properties of Al_2O_3 particles and 6061 Al are given in Table 1. Figure 1 shows a typical optical micrograph of the longitudinal section of the extruded $\text{Al}_2\text{O}_3/\text{Al}$ composite, indicating uniform distribution of Al_2O_3 particles.

2. Young's modulus measurement

Tensile specimens of bar type were machined from the

Table 1 Mechanical and geometrical properties of 6061 aluminum and Al_2O_3 particle

Properties	unit	6061 Al	Al_2O_3 particle
Young's modulus	GPa	68.3	450
Yield stress	MPa	97.0	—
(0.2% off set strain)			
Poisson's ratio	1	0.33	0.25
Coefficient of thermal expansion (CTE)	$\times 10^{-6}/^\circ\text{C}$	23.6	7.7
Burgers vector	nm	0.283	—
Average size	μm	—	8
Average aspect ratio	μm	—	1.8
Volume fraction	1%	balance	14.7

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Fig. 1 A micrograph showing the microstructure of 15 V% Al_2O_3 /6061 Al composite.

extruded Al_2O_3 /Al composite rod, then T4-treated (1 h holding at 550°C , followed by quenching and aging at room temperature for 48 h or more). The tensile testing was conducted at room temperature with cross head speed of 0.042 mm/min. After tensile testing to a specific plastic strain (ε_p), the tensile specimen is unloaded. Then its Young's modulus (E_c) is measured from the slope of the unloading stress-strain curve obtained. The results of $E_c - \varepsilon_p$ relation is shown in Fig. 2, which indicates that E_c is reduced as ε_p increases. The trend of $E_c - \varepsilon_p$ relation in Fig. 2 is the same as in the case of SiC particle/6061 Al composited reported by Lloyd⁽²⁾.

Fig. 2 Experimental results of Young's modulus E_c of Al_2O_3 /Al composite as a function of applied plastic strain ε_p .

3. Particles size distributions

In order to examine the effect of plastic strain ε_p on the Young's modulus E_c , first one must have a clear picture of the distributions of particles size before and after the tensile test. Thus, we conducted a series of observations by optical microscope and scanning electron microscope (SEM) on the longitudinal sections of the composites that were subjected to different plastic straining ε_p . The size distributions of fractured particles in composites with ε_p are obtained from the photographs taken by an optical microscope and SEM. The size (d) of a particle is defined as the maximum diameter, as measured by the Kontron image analyzer.

The size distribution of particles prior to plastic straining is shown in Fig. 3 where particle sizes are sorted out in 20 groups with group identification number being i , and the ratio of the total number of the i -th group (N'_i) to the total number of all particles (N') is plotted as a function of particle size. The size distributions of particles after straining to different plastic strains ε_p (2, 5, 7, 9, 11 and 13%) are shown in Fig. 4 where (a) and (b) denote the results for $\varepsilon_p = 9 \sim 13\%$ and $\varepsilon_p = 0 \sim 7\%$, respectively. The vertical axis in Fig. 4 is the ratio of the number of broken (or fractured) particles in the i -th group (N''_i) to the total number of all particles (N'). Accumulated number of broken particles can be calculated from Fig. 4 and the results are plotted as a function of plastic strain, ε_p (%) in Fig. 5. It is clear from Fig. 5 that the accumulated number of broken particles increases with increase in ε_p . From Figs. 3 and 4, one can calculate the ratio of the total number of broken particle in the i -th group (N''_i) to the total number particles of the i -th group (N'_i), the results of which are plotted as a function of

Fig. 3 Distribution of particle sizes (d_i) prior to plastic straining where N'_i and N' denote the total number of particles in the i -th group and the total number of all particles, respectively.

particle size in Fig. 6 where (a) and (b) denote the cases of $\varepsilon_p = 9 \sim 13\%$ and $0 \sim 7\%$, respectively. Figure 6 indicates that the fraction of broken particles becomes larger

toward the larger sized groups, suggesting that the larger sized particles would presumably be broken first at a given plastic strain as compared the smaller sized ones. The results of Fig. 6 provide the key data in determining the Weibull parameters, for the ratio N_i^b/N_i^t is equal to $F_i(x)$, the accumulative probability of fracture of particles in the i -th group that would be broken at stress (x). This will be discussed in more details in Section IV.

4. In-situ SEM observation

The size distributions of fractured particles for a given ε_p are measured by examining the MMC specimen strained to ε_p after it is unloaded. In order to examine the particle fracture process in a MMC while it is straining, one must conduct an *in-situ* SEM observation by setting a tensile MMC specimen in a SEM chamber. Figure 7 shows a sequence of photographs taken on $\text{Al}_2\text{O}_3/\text{Al}$ composite while being tensile loaded (to plastic strain ε_p) in a SEM chamber, where (a) to (e) correspond to plastic strain = 0 to 8%, respectively. Figure 7 supports our hypothesis that larger sized particles are fractured first at smaller straining, then followed by smaller sized ones at increasing strain. It appears from Fig. 7 that the fracture plane of most particles is perpendicular to the loading axis, indicating that the particles are fractured by tensile stress induced in the particle. Based on numerous SEM photographs taken on $\text{Al}_2\text{O}_3/\text{Al}$ composite specimens while plastically straining, one can group them into two types

Fig. 4 Ratio of the number of the broken particles in the i -th group to the total number of all particles, N_i^b/N_i^t as a function of particle size d_i for various applied plastic strains: (a) ε_p : 9 ~ 13%, and (b) 0 ~ 7%.

Fig. 5 Number fraction of broken particles ($\sum_i N_i^b/N_i^t$) as a function of ε_p .

Fig. 6 Ratio of the number of broken particles in the i -th group (N_i^b) to the total number particles in the i -th group (N_i^t) as a function of particle size d_i for various plastic strains: (a) $\varepsilon_p = 9 \sim 13\%$, and (b) $\varepsilon_p = 0 \sim 7\%$.

Fig. 7 Sequence of micrographs taken on a $\text{Al}_2\text{O}_3/\text{Al}$ composite specimen subjected to *in-situ* SEM tensile test: (a) $\epsilon_p=0\%$ (prior to loading), (b) $\epsilon_p=2\%$, (c) $\epsilon_p=4\%$, (d) $\epsilon_p=6\%$, and (e) $\epsilon_p=8\%$.

of failure mode: cracking mode and shattering mode, which are respectively shown in Fig. 8(a) and (b). These two modes will be used in the analytical modeling for the reduced Young's modulus in subsection III.2.

III. Analytical Modeling

In this section the dependence of the Young's modulus of a particle MMC upon plastic deformation will be studied by using Eshelby's method. To explain the reduc-

tion in Young's modulus by particle fracture, we have to establish a criteria for particle fracture. To this end the internal stress field in a particle MMC must be calculated. Thus, we will first calculate the stress in particles in subsection III.1, and then predict the Young's modulus of MMC containing broken particles in subsection III.2.

1. Stress in particles

In this model, we assume that the stress induced in particles when a particle MMC is subjected to a plastic

Fig. 8 Examples of particle failure mode: (a) cracking mode, and (b) shattering mode.

Fig. 9 Dislocation punching model used in the present model to relax the otherwise high stress field due to CTE mismatch strain: (a) before punching, (b) after punching to c' , and (c) rearrangement of dislocation loops.

deformation, is given by

$$\sigma_{ij}^p = \sigma_{ij}^r + \sigma_{ij}^s \quad (1)$$

where σ_{ij}^r is the thermal residual stress in particles that exists initially and is induced due to the CTE mismatch strain between the matrix and particles after the heat treatment (T4 Temper), and σ_{ij}^s is the stress induced by the plastic strain in the matrix during tensile loading, respectively. Computations of σ_{ij}^r and σ_{ij}^s will be shown in what follows.

(1) Prediction of the thermal residual stress σ_{ij}^r by a dislocation punching model

When a particle MMC is quenched from T_0 (550°C in this study) to T_q (room temp. 20°C), resulting in a temperature change $\Delta T = T_q - T_0$, mismatch strain α will be induced in particles and it is given by

$$\alpha = (\alpha_p - \alpha_m) \Delta T \quad (2)$$

where α_p and α_m are CTE of the particle and matrix, respectively, and both the matrix and the particle are assumed to be isotropic in stiffness and CTE. Taya *et al.*⁽⁴⁾ recently proposed an analytical model based on dislocation punching to calculate the internal stress field in a particle MMC due to CTE mismatch strain α between particle and matrix. The procedure to calculate the stress within a particle, σ_{ij}^r consists of: first, determination of punching distance of prismatic dislocation loops into the matrix region, and second, calculation of the stress in the particle surrounded by the domain of the punched-out dislocations. Following the dislocation punching model, this CTE mismatch strain can be represented by arrays of prismatic dislocation loops adhered uniformly to the matrix-particle interface, Fig. 9(a), which will be punched-out into the matrix surrounding the particle with the punching distance c' Fig. 9(b), in order to relax the otherwise high stress concentration in and around a particle. It is assumed in the model that the particle is spherical and

the punching of the prismatic dislocation loops takes place radially (i.e., in a spherically symmetric manner) to simplify the analysis. This spherically symmetric arrangement of prismatic dislocation loops is also assumed after punching, requiring the rearrangement of the loops from Fig. 9(b) to (c).

Referring to Fig. 9(a), the domain of the particle of radius a is denoted by Ω_1 . The envelope of the punched-out region becomes a sphere of radius c'' which is related to the punching distance c' by

$$c'' = \sqrt{a^2 + c'^2} \quad (3)$$

The domain of this spherical region with radius c'' is denoted by Ω_2 . The mismatch strain in Ω_2 , e_{ij}^{2*} , is given by

$$e_{ij}^{2*} = \begin{bmatrix} \alpha \left(\frac{a}{c''} \right)^3 & 0 & 0 \\ 0 & \alpha \left(\frac{a}{c''} \right)^3 & 0 \\ 0 & 0 & \alpha \left(\frac{a}{c''} \right)^3 \end{bmatrix} \quad (4)$$

The punching distance c' can be determined by the following equation⁽⁴⁾⁽⁵⁾:

$$-\frac{\partial U}{\partial c'} = \frac{\partial W}{\partial c'} \quad (5)$$

where U is the total potential energy of Fig. 9(b) and W is the work done by the motion of the dislocation loops over the punching distance c' . The left hand term in eq. (5) can be interpreted as the punching force while the right hand term is the retarding force. Thus, eq. (5) is considered as a force balance equation.

The remaining task is to calculate U and W . In order to analyze the stress in particles, one must calculate the internal stress (strain) field in a particle MMC. Calculation of the internal stress in a MMC can be done rather easily if we use the Eshelby equivalent inclusion method⁽⁶⁾. The details of the calculation procedure for obtaining the punching distance c' (or equivalently the radius of the

punched-out domain c'') and the stress in a particle has been given elsewhere⁽⁴⁾. Hence, we list in the following the final results of the residual stress in a particle which is in a state of hydrostatic compression with its component σ^r given by

$$\sigma^r = -\frac{2(1-f_2)(3\lambda_m + 2\mu_m)(1-2\nu_m) \{1 + A(1-f_1)\} e^{2*}}{3(1-\nu_m)} \quad (6)$$

Where e^{2*} is the dilatational component of e_{ij}^{2*} defined by eq. (4), and A is a function of the elastic constants of the matrix and particle and the volume fraction of particle (f_1), and its explicit expression is given in the appendix of Ref. (4), f_2 in eq. (6) is the volume fraction of Ω_2 and related to f_1 as $f_2 = (c''/a)^3 f_1$. Once the punching distance c' (alternatively c'') is determined from eq. (5), then the stress in a particle σ^r can be calculated explicitly. For 15 vol% Al_2O_3 /6061 Al composite with $\Delta T = -530^\circ\text{C}$, the punching distance c'' normalized by the radius of particle a and the residual stress in a particle are computed as

$$\begin{aligned} c''/a &= 1.451 \\ \sigma^r &= -149.6 \text{ MPa.} \end{aligned} \quad (7)$$

(2) Stress in particles due to plastic deformation

Next we will calculate the stress in particles due to plastic strain in the matrix (σ_{ij}^p) by Eshelby's method. Here, σ_{ij}^p is analyzed for a particle MMC containing broken particles so as to estimate σ_{ij}^p after particle fracture occurred.

To facilitate the analysis, we assume that the broken particles are replaced by voids. Hence the analytical model consists of sound (unbroken) particles (Ω_p) and broken particles (Voids, Ω_v) embedded in the matrix, Fig. 10(a). Denote the domain of the entire composite by D .

The uniform plastic strain ε_{ij}^p by uniaxial tensile loading along the x_3 -axis is introduced in the matrix ($D - \Omega_p - \Omega_v$), and is given by

Fig. 10 Analytical model based on the Eshelby's equivalent inclusion method to calculate the stress in particles in a particle MMC with damaged particles(voids): (a) matrix subjected to plastic strain, and (b) the equivalent model where the matrix is free of e_{ij}^p , but inhomogeneities are given $-e_{ij}^p$.

$$\varepsilon_{ij}^p = \begin{Bmatrix} -\varepsilon_p/2 \\ -\varepsilon_p/2 \\ \varepsilon_p \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (8)$$

The stiffness tensor of the matrix ($D - \Omega_p - \Omega_v$) and particle (Ω_p) are denoted by C_{ijkl}^m and C_{ijkl}^p , respectively. As far as the stress field is concerned, the model of Fig. 10(a) is equivalent to that of Fig. 10(b) where the uniform plastic strain ε_{ij}^p in the matrix is removed and added with minus sign to particles and voids. Thus, the present problem is reduced to “the inhomogeneous inclusions problem”. The model of Fig. 10(b) will be used to calculate σ_{ij}^s .

Following Mori and Tanaka⁽⁷⁾, the averaged stress in the matrix $\langle \sigma_{ij} \rangle_m$ is defined by

$$\langle \sigma_{ij} \rangle_m = C_{ijkl}^m \bar{e}_{kl} \quad (9)$$

where $\bar{e}_{kl}$ is the averaged strain disturbance in the matrix due to all Ω_p and Ω_v . According to Eshelby, the stress in particles, σ_{ij}^s can be expressed as

$$\begin{aligned} \sigma_{ij}^s &= C_{ijkl}^m (\bar{e}_{kl} + e_{kl}^p - e_{kl}^{p*}) \\ &= C_{ijkl}^p (\bar{e}_{kl} + e_{kl}^p + e_{kl}^v) \end{aligned} \quad (10)$$

where e_{kl}^p is the strain disturbance in Ω_p , and e_{kl}^{p*} is the fictitious eigenstrain which was introduced to reduce the present problem to “inclusion problem”. e_{kl}^p is related to e_{kl}^{p*} as

$$e_{kl}^p = S_{klmn}^p e_{mn}^{p*} \quad (11)$$

where S_{klmn}^p is Eshelby tensor for Ω_p .

On the other hand, the stress inside voids (Ω_v), σ_{ij}^v , is given by

$$\sigma_{ij}^v = C_{ijkl}^m (\bar{e}_{kl} + e_{kl}^v - e_{kl}^{v*}) = 0 \quad (12)$$

where e_{kl}^v is the strain disturbance in Ω_v and e_{kl}^{v*} is the corresponding eigenstrain in Ω_v . e_{kl}^v is related to e_{kl}^{v*} as

$$e_{kl}^v = S_{klmn}^v e_{mn}^{v*} \quad (13)$$

where S_{klmn}^v is the Eshelby tensor for Ω_v which coincides with S_{klmn}^p . Noting that the integration of the internal stress field σ_{ij} over the entire composite domain (D) vanishes:

$$\int_D \sigma_{ij} dV = \int_{D-\Omega_p-\Omega_v} \langle \sigma_{ij} \rangle_m dV + \int_{\Omega_p} \sigma_{ij}^s dV + \int_{\Omega_v} \sigma_{ij}^v dV = 0 \quad (14)$$

we obtain

$$(1 - f_p - f_v) \langle \sigma_{ij} \rangle_m + f_p \langle \sigma_{ij}^s \rangle + f_v \langle \sigma_{ij}^v \rangle = 0 \quad (15)$$

where f_p and f_v are the volume fractions of Ω_p and Ω_v , respectively. Eliminating e_{ij}^p and e_{ij}^v through eqs. (11) and (13), we have three unknowns, i.e., $\bar{e}_{ij}$, e_{ij}^{p*} and e_{ij}^{v*} , which can be solved by three equations (eqs. (10), (12) and (15)). General formulation for two kinds of ellipsoidal inhomogeneities has been given elsewhere⁽⁸⁾. After

Fig. 11 Predicted stress in a particle, σ_{ij}^p as a function of applied plastic strain, ε_p for several volume fractions of broken particles, $f_b = 0, 0.15$ and 0.30 : (a) σ_{33}^p along the loading axis (x_3 -axis), and (b) σ_{11}^p along the x_1 -axis.

solving for $\bar{e}_{ij}$ and e_{ij}^{p*} , we can compute the stress in a particle due to plastic deformation σ_{ij}^s from eq. (10) as

$$\begin{aligned} \sigma_{11}^s &= \sigma_{22}^s = \varepsilon_p \cdot X_1 \\ \sigma_{33}^s &= \varepsilon_p \cdot X_2 \end{aligned} \quad (16)$$

where X_1 and X_2 are functions of the elastic constants of the matrix and particle, and the volume fraction of particle (Ω_p) and voids (Ω_v), and their explicit expressions are given elsewhere⁽⁹⁾.

In the above model, it is postulated that particles which were originally sound are progressively fractured by plastic deformation. Therefore, we note that the current volume fractions f_p and f_v are related to $(f_p)_0$ and f_b as

$$f_p = (1 - f_b)(f_p)_0 \quad (17)$$

$$f_v = f_b(f_p)_0 \quad (18)$$

where $(f_p)_0$ and f_b denote the initial volume fraction of particles and the fraction of broken particles, respectively.

Figure 11 shows the stress induced in particles due to plastic deformation, σ_{ij}^s as a function of the plastic strain in the matrix, ε_p . From Fig. 11, it is obvious that stresses in particles along loading axis (x_3 -axis) and that perpendicular to loading axis (x_1 - and x_2 axes) are tensile (Fig. 11(a)) and compressive (Fig. 11(b)), respectively, and

Fig. 12 Two types of failure mode associated with particles: (a) shattered particles, and (b) cracked particles.

both increase with the increase in ε_p . It follows from Fig. 11(a) that the magnitude of the tensile stress in the loading axis, σ_{33} increases with increase in fraction of broken particle, f_b .

2. Young's modulus of a particle MMC containing broken particles

By the experimental results in Section II, it is confirmed that there exist two cases of failure mode associated with particles: the shattering of particles (shattering mode) as shown in Fig. 12(a) and the cracking of particles (cracking mode) as shown in Fig. 12(b). Recently, Mochida *et al.*⁽⁹⁾ proposed a model to predict the Young's modulus E_c of a particle MMC weakened by particles broken by the shattering and cracking modes, where particles broken by the shattering mode and cracking mode are regarded as complete voids and penny-shaped cracks within particles, respectively. In the following, the analytical model of Mochida *et al.* will be stated briefly.

(1) Analytical model and formulation

Model for shattering mode

Consider an infinite elastic body (D) which contains spherical particles (Ω_p) and voids (Ω_v) in the matrix, subjected to the applied stress σ_{ij}^0 ($\sigma_{33}^0 = \sigma_0$ in this model), as shown in Fig. 13(a). It is assumed in this model, that the particles shattered by plastic deformation are converted to spherical voids. Denote the stiffness tensors of the matrix ($D - \Omega_p - \Omega_v$) and particles (Ω_p) by C_{ijkl}^m and C_{ijkl}^p , respectively, which are assumed to be linearly isotropic.

Under the applied stress σ_{ij}^0 , the average total stress in the matrix is given by $\sigma_{ij}^0 + \langle \sigma_{ij} \rangle_m$ where

$$\langle \sigma_{ij} \rangle_m = C_{ijkl}^m \bar{e}_{kl} \quad (19)$$

where $\bar{e}_{kl}$ is the averaged strain disturbance in the matrix due to all Ω_p and Ω_v . The stress inside the particles can be express as

$$\begin{aligned} \sigma_{ij}^0 + \sigma_{ij}^p &= C_{ijkl}^m (e_{kl}^0 + \bar{e}_{kl} + e_{kl}^p - e_{kl}^{p*}) \\ &= C_{ijkl}^p (e_{kl}^0 + \bar{e}_{kl} + e_{kl}^p) \end{aligned} \quad (20)$$

where σ_{ij}^p and e_{kl}^p are the disturbances of the stress and strain in Ω_p , respectively. e_{kl}^{p*} is the eigenstrain which has non-vanishing components in the domain Ω_p and becomes zero outside Ω_p . For the entire composite domain D , the following relation always holds:

$$\sigma_{ij}^0 = C_{ijkl}^m e_{kl}^0 \quad (21)$$

With eq. (21), eq. (20) yields:

$$\sigma_{ij}^p = C_{ijkl}^m (\bar{e}_{kl} + e_{kl}^p - e_{kl}^{p*}) \quad (22)$$

Following Eshelby we have

$$e_{kl}^p = S_{klmn}^p e_{mn}^{p*} \quad (23)$$

where S_{klmn}^p is the Eshelby tensor for Ω_p .

Next the total stress in Ω_v can be express as

$$\sigma_{ij}^0 + \sigma_{ij}^v = C_{ijkl}^m (e_{kl}^0 + \bar{e}_{kl} + e_{kl}^v - e_{kl}^{v*}) = 0 \quad (24)$$

Fig. 13 Analytical model to predict the Young's modulus of a particle MMC: (a) shattering mode (voids replacing shattered particles), and (b) cracking mode (penny-shaped crack replacing cracked particles).

where e_{kl}^v is again related to e_{kl}^{v*}

$$e_{kl}^v = S_{klmn}^v e_{mn}^{v*} \quad (25)$$

and where e_{kl}^{v*} is the eigenstrain defined in Ω_v , and S_{klmn}^v is the Eshelby's tensor for Ω_v which is equal to S_{klmn}^p . Since the disturbed stress σ_{ij} must satisfy $\int_D \sigma_{ij} dV = 0$, we obtain

$$(1 - f_p - f_v) \langle \sigma_{ij} \rangle_m + f_p \langle \sigma_{ij}^p \rangle + f_v \langle \sigma_{ij}^v \rangle = 0 \quad (26)$$

where f_p and f_v are the volume fractions of Ω_p and Ω_v , respectively. Eliminating e_{ij}^p and e_{ij}^v through eqs. (23) and (25), we have three unknowns, i.e., e_{ij} , e_{ij}^{p*} and e_{ij}^{v*} , which can be solved by the three equations, eqs. (20), (24) and (26). Once e_{ij}^{p*} and e_{ij}^{v*} are solved, we can compute the overall stiffness of the composite by using the equivalence of the strain energies,

$$\frac{1}{2} C_{ijkl}^{c-1} \sigma_{ij}^0 \sigma_{kl}^0 = \frac{1}{2} C_{ijkl}^{m-1} \sigma_{ij}^0 \sigma_{kl}^0 + \frac{1}{2} f_p \sigma_{ij}^0 e_{ij}^{p*} + \frac{1}{2} f_v \sigma_{ij}^0 e_{ij}^{v*} \quad (27)$$

where C_{ijkl}^{m-1} and C_{ijkl}^{c-1} are the compliances of the matrix and the composite, respectively. The details of derivation of eq. (27) are given elsewhere⁽⁸⁾. Noting that the non-vanishing component of σ_{ij}^0 is $\sigma_{33}^0 = \sigma_0$, eq. (27) is reduced to

$$\frac{\sigma_0^2}{2E_c} = \frac{\sigma_0^2}{2E_m} + \frac{\sigma_0 e_{33}^{p*} f_p}{2} + \frac{\sigma_0 e_{33}^{v*} f_v}{2} \quad (28)$$

where E_m and E_c are Young's moduli of the matrix and the composite, respectively. We substitute e_{33}^{p*} and e_{33}^{v*} into eq. (28) to obtain the weakened Young's modulus E_c for the shattering mode as

$$\frac{E_c}{E_m} = \frac{1}{1 + \eta_p^1 f_p + \eta_v f_v} \quad (29)$$

where η_p^1 and η_v are also function of f_p and f_v and their explicit expressions are given elsewhere⁽⁹⁾⁽¹⁰⁾. We note that f_p and f_v can be express as

$$\begin{aligned} f_p &= (1 - f_b)(f_p)_0 \\ f_v &= f_b(f_p)_0 \end{aligned} \quad (30)$$

Model for cracking mode

In order to compute E_c for cracking mode, the theoretical model shown in Fig. 13(b) is considered. It is assumed in the cracking mode model where cracked particles by plastic deformation are converted to penny-shaped cracks located perpendicular to the loading direction (x_3 -axis) and which are oblate spheroids with the major and minor axes being c (equal to the radius of particles) and t , respectively. In Fig. 13(b), the domain of entire composite, particles and cracks are denoted by D , Ω_p and Ω_{cr} , respectively.

The solution procedure of E_c for the cracking mode is the same as in the foregoing steps of the shattering mode model except that the superscript v and v^* and the subscript v are replaced by cr , cr^* and cr , respectively, which are associated with Ω_{cr} . Therefore, we obtain E_c

for the cracking mode model as

$$\frac{E_c}{E_m} = \frac{1}{1 + \eta_p^{II} f_p + \eta_{cr} \bar{f}_{cr}} \quad (31)$$

where η_p^{II} and η_v are given elsewhere⁽⁹⁾⁽¹⁰⁾ and where $\bar{f}_{cr}$ is related to f_{cr} (the volume fraction Ω_{cr}), $(f_p)_0$ and f_b as

$$\begin{aligned} f_{cr} &= \bar{f}_{cr} \left(\frac{t}{c} \right) \\ \bar{f}_{cr} &= f_b(f_p)_0 \end{aligned} \quad (32)$$

(2) Analytical results

We have computed E_c for two failure modes of particles, the shattering mode and the cracking mode, by using eqs. (29) and (31), respectively. To see the reduction in E_c clearly, we have plotted the analytical results of $E_c/(E_c)_0$ vs f_b for $(f_p)_0 = 0.15$ in Fig. 14 where the shattering mode and the cracking mode correspond to solid and dash curves, respectively and where $(E_c)_0$ is the Young's modulus of the MMC with no broken particles. It follows from Fig. 14 that the reduction in E_c for the shattering mode is larger than that for the cracking mode.

IV. Discussion

In Section II, the relation between the Young's modulus (E_c) of Al_2O_3 particle/6061 Al and applied plastic strain (ε_p) was obtained experimentally, while in Section III, the analytical model to predict E_c of the composite subjected to ε_p was constructed. The comparison between the experimental and analytical results so obtained cannot be made, however, unless the relation between the volume fraction of broken particles (f_p) and applied plastic strain (ε_p) is given. Thus, in this Section,

Fig. 14 Predicted Young's moduli of the composite with broken particles, E_c normalized by that with non-broken particles, $(E_c)_0$ as a function of volume fraction of broken particles, f_b , where the initial volume fraction of particles $(f_p)_0$ was set equal to 0.15.

we shall employ a Weibull model⁽³⁾ by which the probability of fracture of a particle, $F(x)$ is related to the stress within the particle (x) which in turn can be expressed in terms of ε_p according to the analytical results in Section III.1. Here, we shall state briefly a two-parameter Weibull model associated with particle fracture in Section IV.1, then discuss the comparison between the analytical and experimental results in Section IV.2.

1. Weibull model

In this study, the following two-parameter Weibull cumulative distribution function (CDF), $F(x)$ is used to obtain the probability of particle fracture for a given stress x :

$$F(x) = 1 - \exp \left\{ - \left(\frac{x}{s} \right)^m \right\} \quad (33)$$

where m and s are shape and scale parameters, respectively. A derivative of $F(x)$, $f(x)$ is called the probability density function. $F(x)$ denotes the probability of fracture of a particle with known size d under a given stress (x), while $f(x)$ is its relative frequency. Examples of $F(x)$ and $f(x)$ are shown in Fig. 15(a) and (b), respectively. It is noted here that for larger values of s , the graph of $f(x)$ is shifted toward the right, whereas for larger values of m the graph of $f(x)$ becomes narrower. For a given size group (say the i -th group), the corresponding CDF, $F(x)$ is given by

Fig. 15 Typical Weibull two-point failure probability functions as a function particle stress x : (a) cumulative probability distribution function, $F(x)$, and (b) probability density function, $f(x)$.

$$F_i(x) = 1 - \exp \left\{ - \frac{V_i}{V_0} \left(\frac{x}{s} \right)^m \right\} \quad (34)$$

where V_i and V_0 are respectively, the volumes of the i -th group and that of the reference group for which the Weibull parameters s and m are determined. Determination of s and m is normally done by using the following relation obtained from double ln-ln plot of eq. (33):

$$\ln \ln \left\{ \frac{1}{1 - F(x)} \right\} = m \ln x - m \ln s \quad (35)$$

If the 17-th group is taken as a reference, then referred to Fig. 6 and noting that $F_i(x)$ is equal to N_i^b / N_i^t with $i=17$, one can construct the graph of eq. (35), the results of which are shown in Fig. 16. Please note in the figure that the fracture stress, x , is taken as the stress component 33 in a particle (σ_{33}^p), and σ_{33}^p was computed from eq. (1). One of the input data for the analytical model to compute σ_{33}^p , is the volume fraction of broken particles (f_b) and it is obtained by

$$f_b = \frac{\sum_{i=1}^{20} d_i^3 N_i^b}{\sum_{i=1}^{20} d_i^3 N_i^t} \quad (36)$$

where d_i is the particle size of the i -th group. The mean square fit for the experimental data points (filled circles) is obtained as

$$Y = 1.435X - 14.21 \quad (37)$$

where $Y = \ln \ln (1 / (1 - F(x)))$ and $X = \ln x$. Comparing eq. (35) with eq. (37), one can obtain the Weibull parameters m and s for the 17th group

$$\begin{aligned} m &= 1.43 \\ s &= 1.99 \text{ GPa} \end{aligned} \quad (38)$$

Once the Weibull CDF is obtained for the reference group ($i=17$), then one can determine the CDF for the other groups from eq. (34), i.e.,

$$F_i(x) = 1 - \exp \left\{ - \left(\frac{d_i}{d_{17}} \right)^3 \left(\frac{x}{s} \right)^m \right\} \quad (39)$$

Fig. 16 Weibull plot of the experimental data where $x = \sigma_{33}^p$, X and Y denote the values of the vertical and horizontal axes.

where d_{17} is the size of the 17th (reference) group.

2. Comparison between the experimental and analytical results

The analytical results of E_c of a particle MMC subjected to ε_p are obtained by taking the following steps:

- (1) compute the stress σ_{33}^p in a particle for a given ε_p , eq. (1).
- (2) determine f_b from eqs. (36) and (39) with $x = \sigma_{33}^p$.
- (3) calculate E_c from eq. (29).

The analytical results of E_c so calculated are plotted as a function of ε_p , Fig. 17 where the solid and dash lines denote the case of shattering and cracking modes, respectively, and the experiment is shown by filled circles taken from Fig. 2. Apart from the gap for $\varepsilon_p = 0$, the dependence of E_c on ε_p observed in the experiment is the

Fig. 17 A comparison of E_c between the experiment and the prediction for two types of particle failure modes.

Fig. 18 Effect of aspect ratio of short fiber (particulate), l/d , on the composite Young's modulus E_c normalized by the matrix Young's modulus $E_m^{(11)}$, $f_p = 0.2$.

same as the prediction based on the present model. The initial gap between the experiment and the prediction is mainly due to the assumption used in the present model: spherical shape was used for particles in the model while the actual Al_2O_3 particles in the $\text{Al}_2\text{O}_3/6061$ Al composite used in the experiment are of particulate shape with aspect ratio 1.8, Table 1. It is known that the E_c of a MMC with particulate aspect ratio higher than 1.0 is larger than that of a particle MMC where the aspect ratio is 1.0⁽¹¹⁾. Figure 18 demonstrates the effect of fiber or particulate aspect ratio (l/d) on the composite Young's modulus E_c normalized by the matrix Young's modulus E_m for the case of SiC short fiber (or particle)/1100 Al composite with $f_p = 0.2$ ⁽¹¹⁾. If one attempts to calculate the ratio of E_c of a composite with aspect ratio 1.8 to that of a particle composite (aspect ratio 1) from Fig. 18 with $f_p = 0.15$, it is about 1.05, resulting in a good agreement with the corresponding ratio of Fig. 17. In order to examine the E_c dependence on ε_p more clearly, we have plotted the results of $E_c/(E_c)_0$ as a function of ε_p in Fig. 19 where $(E_c)_0$ is the Young's modulus of the composite with $\varepsilon_p = 0$. It can be concluded from Fig. 19 that the present analytical model explains well the experimental results.

V. Concluding Remarks

In this paper, the reduction in the Young's modulus of a particle MMC subjected to applied plastic straining was studied both experimentally and analytically. The analytical modeling consists of three parts: (1) Weibull failure model, (2) calculation of particle stress due to the CTE mismatch strain existing initially and tensile loading with plastic strain ε_p , and (3) calculation of E_c of a MMC with sound particles and fractured particles. The main findings are:

- (i) the larger sized particles are more likely to be frac-

Fig. 19 A comparison of $E_c/(E_c)_0$ between the experiment and the prediction for two types of particle failure modes.

tured than the smaller sized ones for a given applied plastic strain ε_p .

(ii) The composite Young's modulus E_c decreases with increase in ε_p .

(iii) Eshelby's model can be used successfully to predict the E_c dependance on ε_p .

Acknowledgments

The present authors are very thankful to Alcan International Limited for its support of the present study and also to Professor S. Hanada and Mr. K. Miura of Tohoku University for their assistance in the experiment.

NOMENCLATURE

A	parameter in Ref. (4)
a	radius of particle
C_{ijkl}^{e-1}	compliance tensor of composite
C_{ijkl}^m	stiffness tensor of matrix
C_{ijkl}^{m-1}	compliance tensor of matrix
C_{ijkl}^p	stiffness tensor of particle
c	radius of penny-shaped crack
c'	punching distance of dislocation loops
c''	radius of the domain of punched-out dislocations
D	domain of composite
d_i	particle diameter of the i -th group
E_c	Young's modulus of composite
E_m	Young's modulus of matrix
e_{ij}^p	strain disturbance in the composite due to particles and voids or cracks
e_{ij}^{p*}	fictitious eigenstrain in Ω_p
e_{ij}^0	the uniform strain corresponding to applied stress σ_{ij}^0
$\bar{e}_{ij}$	averaged strain disturbance in the matrix
e_{ij}^{2*}	dilatational component of e_{ij}^{2*}
e_{ij}^{2*}	eigenstrain in domain Ω_2 equal to the CTE mismatch strain smeared-out in Ω_2
e_{ij}^{v*}	fictitious eigenstrain in Ω_v
e_{ij}^v	strain disturbance in Ω_v
f_1	volume fraction of Ω_1
f_2	volume fraction of Ω_2
f_b	volume fraction of broken particles out of all particles
f_{cr}	volume fraction of cracks
$\bar{f}_{cr}$	volume fraction of cracked particles
f_p	volume fraction of particles (Ω_p)
$(f_p)_0$	initial volume fraction of particles
f_v	volume fraction of voids (Ω_v)
$F(x)$	two-parameter Weibull cumulative distribution function
$f(x)$	two-parameter Weibull probability distribution function
$F_i(x)$	two-parameter Weibull cumulative distribution function of the i -th size group
m	Weibull scale parameter (Weibull modulus)
N_i^b	total number of broken particles in the i -th size group
N^t	total number of all particles
N_i^t	total number of the i -th size group

s	Weibull shape parameter
S_{klmn}^p	Eshelby's tensor of particle
S_{klmn}^v	Eshelby's tensor of void
t	thickness of penny-shaped crack
U	total potential energy of composite
V_i	volume of the particle of the i -th group
v_0	volume of the particle of the reference group
W	total work done by the motion of dislocation loops over the punching distance c'
X	$\ln x$
x	particle stress in Weibull model
Y	$\ln \ln (1./ (1.-F(x)))$

Greek

α	CTE mismatch strain component equal to $(\alpha_p - \alpha_m)\Delta T$
α_p	coefficient of thermal expansion (CTE) of particle
α_m	CTE of matrix
ΔT	temperature difference equal to $T_q - T_0$ where T_0 and T_q are the initial (520°C) and final temperature (20°C) during quenching
ε_{ij}^p	plastic strain of the matrix
ε_p	plastic strain component along the x_3 -axis
λ_m	Lamé constant of matrix
μ_m	shear modulus of matrix
ν_m	Poisson's ratio of matrix
$\eta_p^{\text{II}}, \eta_{cr}$	parameters defined by eq. (31)
$\eta_p^{\text{I}}, \eta_v$	parameters defined by eq. (29)
$\langle \sigma_{ij} \rangle_m$	average stress disturbance in the matrix
σ_0	component of σ_{ij}^0 along the x_3 -axis
σ_{ij}^0	applied stress
σ_{ij}^p	total stress in a particle
σ^r	hydrostatic compression stress component in a particle prior to plastic straining
σ_{ij}^r	residual stress in a particle prior to plastic straining
σ_{ij}^s	stress in a particle due to plastic straining
σ_{ij}^v	stress in a void equal to zero
Ω_1	domain of particles
Ω_2	domain of the punched-out region including Ω_1
Ω_p	domain of particles equal to Ω_1
Ω_v	domain of voids

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