Elasticity Analysis of Texture for Titanium and Niobium Plate

Chen Hongsun*, You Shiwu*, Ma Lin*,
Han Lubin* and Wang Shuqian*

A new method for texture analysis has been put forward in this paper. According to measurements of the elastic modulus distribution of the plate, the essential information on the texture can be inferred by means of elastic anisotropy pattern of the crystal. The textures of titanium (hcp) and niobium (bcc) plates have been analyzed by this method. The results show no differences from those by the X-ray method.

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I. Introduction

Texture (preferred orientation of crystallite) is a conventional form of the microstructure in metals. It results in the anisotropy of macroscopic properties. So, it is an interesting subject for the metal physicists⁽¹⁾⁻⁽³⁾. The current method for texture research is the X-ray diffraction method by which the distribution probability of the crystal face (i.e. pole figure) can be directly determined. A different method for texture analysis has been put forward in the present paper. According to the measurement of the elastic modulus orientation distribution of the plate, the essential information of the texture can be inferred by means of elastic anisotropy pattern of the crystal. Macro-anisotropy and micro-texture can be obtained simultaneously by this method, therefore, it is very interesting and useful. In this paper titanium (hcp) and niobium (bcc) plates were used as examples for the description of our method.

II. Experiment and Results

1. Measurement of Young's modulus

Commercial pure titanium and niobium plate were used as material under experiment. The thickness is 2.9 mm and 1.0 mm respectively. The original plate is cut into pieces with 10 mm width and 150 mm length every 15° degrees from the rolling direction (R.D.) for Young's modulus measurement. Young's modulus was measured with the bend resonance method. The obtained data were analyzed with a pc-1211 pocket computer and an IBM pc/xt computer.

2. Expression of elasticity anisotropy

(1) Pattern expression

The Young's modulus distribution curve is drawn in

polar coordinates in such a way that the length of the radius vector presents modulus magnitude and its direction is consistent with the angle from the rolling direction. In order to display only anisotropy, smooth treatment is carried out and jump-away point is cast out while the modulus distribution curve is drawn. Elastic anisotropy patterns of titanium and niobium plates are shown in Figs. 1 and 2, respectively.

(2) Fourier series expression

The modulus distribution may be described by Fourier series in the following form⁽⁴⁾:

$$\begin{aligned} E(\theta) &= \sum_{n=0}^5 (a_n \cos 2n\theta + b_n \sin 2n\theta) \\ &= \sum_{n=0}^5 C_n \cos 2n(\theta + \phi_n) \end{aligned} \quad (1)$$

where θ is the direction angle in the plate plane. The coefficients can be calculated according to the following equations:

Fig. 1 Elasticity anisotropy image of Ti-plate.

* Northwest Institute for Nonferrous Metal Research, P.O. Box 71, Baoji, Shaanxi, China.

Fig. 2 Elasticity anisotropy image of Nb-plate.

$$\begin{cases} a_0 = \frac{1}{12} \sum_{m=0}^{11} E_m \\ b_0 = 0 \\ a_n = \frac{1}{6} \sum_{m=0}^{11} E_m \cos \frac{\pi}{6} nm \\ b_n = \frac{1}{6} \sum_{m=0}^{11} E_m \sin \frac{\pi}{6} nm \quad (n=1, 2, 3, 4, 5) \end{cases} \quad (1-1)$$

$$\begin{cases} C_n = \sqrt{a_n^2 + b_n^2} \\ \phi = -\frac{1}{2n} \tan^{-1} (b_n/a_n). \end{cases} \quad (1-2)$$

Where E_m is a measured Young's modulus with an interval of 15° from R.D. to 165° . Usually, it is sufficient to take three terms in the series. The Fourier series expressions of modulus distributions of the titanium and niobium plates obtained by the above method are given as follows:

Ti—

$$E(\theta) = 111.2 + 2.5 \cos 2(\theta - 86^\circ) + 0.6 \cos 4(\theta + 7^\circ)$$

Nb—

$$E(\theta) = 106.9 + 2.8 \cos 2(\theta + 40^\circ) + 11.7 \cos 4(\theta + 42^\circ)$$

(3) Characteristic parameter expression

Some characteristic parameters are defined as follows:

(i) Average modulus (E_0)

$$E_0 = C_0 = a_0 = \frac{1}{12} \sum_{m=0}^{11} E_m$$

(ii) n -order anisotropy coefficient (An)

$$An = Cn/C_0$$

An indicates the level of the anisotropy with π/n cycle

(iii) n -order deviation angle (ϕ_n)

ϕ_n is the clockwise deviation angle of n -order anisotropy pattern with π/n cycle from R.D.

The characteristic of the modulus distribution are concisely represented by the above parameters. Anisotropy characteristic parameters of titanium and niobium plate under the experiment are as follows:

Ti—

$$E_0 = 111.2 \text{ GPa}$$

$$A_1 = 0.022$$

$$\phi_1 = -86^\circ$$

$$A_2 = 0.005$$

$$\phi_2 = 7^\circ$$

Nb—

$$E_0 = 106.9 \text{ GPa}$$

$$A_1 = 0.026$$

$$\phi_1 = 40^\circ$$

$$A_2 = 0.109$$

$$\phi_2 = 42^\circ$$

It is thus clear from the above parameters that

(i) for titanium-plate

$A_1 \gg A_2$ —The anisotropy of modulus distribution mainly cycles with a π -angle period and the pattern is similar to an ellipse.

$|\phi_1| \sim 90^\circ$ —Maximum of modulus approximately lies in the transverse direction (T.D.).

(ii) for niobium-plate

$A_2 \gg A_1$ —The anisotropy of modulus distribution cycles mainly with a 90° -angle period and the pattern has 4-fold symmetry.

$|\phi_2| \sim 45^\circ$ —The maximum of modulus lies approximately in 45° direction from the rolling direction.

(iii) The niobium and titanium plates possess almost the same average modulus but the anisotropy of niobium is much more intense than that of titanium.

3. Analysis of texture by X-ray method

In order to compare between the elastic and X-ray diffractive methods, the pole figures for the titanium and niobium plates were measured. The measurements were performed by a DS-type diffractometer with $\text{MoK}\alpha$ radiation monochromated with a quartz monocrystal. A scintillation counter detector was used. In the case of the niobium specimen, above plate was thinned to 0.06 mm thick for the transmission measurement. An absorption correction coefficient was determined experimentally and the obtained data were calibrated.

III. Analysis and Discussion

1. Elastic anisotropy of single crystal

The anisotropy of a single phase polycrystalline metal reflects the degree of preferred orientation of crystallite in the polycrystal and the anisotropy of its monocrystal. In order to analyze the texture from the anisotropy of a metal, first of all, we must establish the anisotropy pat-

tern of the single crystal of a metal according to the existent theory and data in reference. follows⁽⁵⁾:

(1) The Young's modulus of designated direction of crystal

It is well known that the resistance for the elastic deformation of materials is completely characterized by an elastic modulus tensor. The tensor can be expressed as given in the following matrix:

$$[E_{ij}] = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} & E_{16} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} & E_{26} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} & E_{36} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} & E_{46} \\ E_{51} & E_{52} & E_{53} & E_{54} & E_{55} & E_{56} \\ E_{61} & E_{62} & E_{63} & E_{64} & E_{65} & E_{66} \end{bmatrix} \quad (2)$$

The numbers of independent matrix elements in $[E_{ij}]$ are reduced with enhancing symmetry of the crystal. For example, the numbers of matrix elements for the hexagonal and cubic systems will become five and three respectively (as shown in eqs. (3) and (4)).

$$[E_{ij}] = \begin{bmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 \\ E_{12} & E_{11} & E_{13} & 0 & 0 & 0 \\ E_{13} & E_{13} & E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} \end{bmatrix} \quad (3)$$

$$E_{66} = \frac{1}{2} (E_{11} - E_{12})$$

$$[E_{ij}] = \begin{bmatrix} E_{11} & E_{12} & E_{12} & 0 & 0 & 0 \\ E_{12} & E_{11} & E_{12} & 0 & 0 & 0 \\ E_{12} & E_{12} & E_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{44} \end{bmatrix} \quad (4)$$

The matrix elements of titanium and niobium are as

$$\text{Ti}—E_{11} = 162.4 \text{ GPa}$$

$$E_{12} = 92.0 \text{ GPa}$$

$$E_{13} = 69.0 \text{ GPa}$$

$$E_{33} = 180.7 \text{ GPa}$$

$$E_{44} = 46.7 \text{ GPa}$$

$$\text{Nb}—E_{11} = 246.5 \text{ GPa}$$

$$E_{12} = 134.5 \text{ GPa}$$

$$E_{44} = 28.73 \text{ GPa}$$

The elastic coefficient matrix $[S_{ij}]$ has a similar form to $[E_{ij}]$ and can be deduced from $[E_{ij}]$.

$$\text{Ti}—S_{11} = 0.009581 / \text{GPa}$$

$$S_{12} = -0.004623 / \text{GPa}$$

$$S_{13} = -0.001893 / \text{GPa}$$

$$S_{33} = 0.00698 / \text{GPa}$$

$$S_{44} = 0.02141 / \text{GPa}$$

$$\text{Nb}—S_{11} = 0.006599 / \text{GPa}$$

$$S_{12} = -0.00233 / \text{GPa}$$

$$S_{44} = 0.03481 / \text{GPa}$$

The Young's modulus for designated direction of the hexagonal and cubic single crystals can be calculated as follows⁽⁶⁾:

for hexagonal:

$$\frac{1}{E} = S_{11}(1 - R_3^2)^2 + S_{33}R_3^4 + (2S_{13} + S_{44})R_3^2(1 - R_3^2) \quad (5)$$

for cubic:

$$\frac{1}{E} = S_{11} - 2 \left[(S_{11} - S_{12}) - \frac{1}{2} S_{44} \right] (R_1^2 R_2^2 + R_2^2 R_3^2 + R_3^2 R_1^2) \quad (6)$$

where R_1, R_2, R_3 are the cosines of the angles between designated direction and crystallographic axis a, b, c . For the hexagonal and cubic systems the cosine of the angle between crystal face (hkl) and (HKL) can be calculated by the following equations⁽⁷⁾:

for hexagonal:

$$R = \cos \delta = \frac{\frac{4}{3a^2} \left[Hh + Kk + \frac{1}{2} (Hk + hK) \right] + Ll / C^2}{\sqrt{\left[\frac{4}{3a^2} (H^2 + K^2 + HK) + L^2 / C^2 \right] \left[\frac{4}{3a^2} (h^2 + k^2 + hk) + l^2 / C^2 \right]}} \quad (7)$$

for cubic:

$$R = \cos \delta = \frac{Hh + Kk + Ll}{\sqrt{(H^2 + K^2 + L^2)(h^2 + k^2 + l^2)}} \quad (8)$$

It follows from eq. (5) that Young's modulus of the designated direction in hexagonal crystal is only related to R_3 i.e. it is of rotational symmetry around c axis.

(2) Anisotropy stereogram of the monocrystal (ASC)

From eqs. (5) and (6), we can easily get the Young's modulus for every direction indicated by the pole points on the standard projection, and then mark them by the side of the pole point. Connecting those points with the same modulus to lines (may call them a contour line), the

Fig. 3 Anisotropy stereogram of Ti-crystal.

distribution density of contour lines would qualitatively present the level of Young's modulus at every area. Thus, we have obtained the elastic anisotropy stereographic projection which can audiovisually display the elastic anisotropy of the monocrystal. It is an important means for elastic analysis of the texture. The ASC of Ti and Nb are shown in Figs. 3 and 4. The rotational symmetry of modulus distribution for Ti (the symmetry axis is $[0001]$) is appeared clearly in Fig. 3. It is seen from the modulus distribution of Nb crystal (Fig. 4) that the $\langle 100 \rangle$ direction which possesses the highest modulus, 151.5 GPa is of four-fold axis, $\langle 111 \rangle$ direction which possesses the lowest modulus 81.64 GPa is of the three-fold axis and $\langle 110 \rangle$ direction which possesses a "saddle-mountain" shape and Young's modulus of 92.28 GPa is of two-fold axis.

(3) Anisotropy diagram of crystallographic plane (ADCP)

It is necessary to plot the modulus orientation distribution diagram of some crystallographic plane of the metal for analyzing conveniently the plate texture. By finding out the crystallographic direction, which lie on the present plane and belong to same zone, on the ASC, it can be achieved without difficulty. The angle between two directions can be calculated from eqs. (7) and (8). ADCP of (0001) and $(11\bar{2}0)$ for Ti and (010) , (011) , (111) and (112) for Nb are shown in Figs. 5 and 6.

2. Elasticity analysis of the titanium plate texture

(1) Comparison between statistical and measured value

Although the anisotropy of Young's modulus rises from the texture, Young's modulus of a polycrystal is

Fig. 4 Anisotropy stereogram of Nb-crystal.

$$P_{hkl} = n \frac{I_{hkl}}{I_{ohkl}} \left/ \sum \frac{I_{hkl}}{I_{ohkl}} \right. . \quad (9)$$

Table 1 Statistical and measured Young's modulus of Ti plate.

Direction angle (θ)	Young's modulus (GPa)	
	Statistical value	Measured value
0°	107.2	109.3
45°	111.8	111.0
60°	115.3	112.7
105°	118.7	113.2

It is seen from Table 1 that both the absolute values and the tendency of directional variation are similar between statistical and measured results. It follows that Young's modulus of the metal is basically determined by that of the monocrystal and the orientation distribution of the crystallite in the polycrystal. Therefore, we can statistically analyze the anisotropy of polycrystal and its cause.

Assuming that orientation of crystallite is entirely ran-

Fig. 5 Anisotropy diagram of crystallographic plane for Ti.

Fig. 6 Anisotropy diagram of crystallographic plane for Nb.

Fig. 7 Inverse pole figure of several direction for titanium plate.

dom and there are not any preferred orientation, the plate will yield isotropic properties and its Young's modulus can be statistically calculated by the following equation:

$$\bar{E} = \frac{\int E d\omega}{\int d\omega} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} E \cdot \sin \theta d\theta d\phi \quad (10)^\dagger$$

where

- θ —latitude angle of spheroidal coordinates
- ϕ —longitude angle
- E —Young's moduli determined by eq. (5).

[†] θ, ϕ here are direction angles of the spherical coordinate system but not the those used in eq. (1).

Substituting S_{ij} of Ti crystal into eq. (5) and exchanging R_3 for $\cos \theta$, then substituting E into eq. (10) and integrating, the isotropic modulus of Ti is obtained.

$$\bar{E} = 113.8 \text{ GPa}$$

Young's modulus orientation distribution of titanium plate is mainly first anisotropy and there is the lowest modulus in R.D. ($E_R = 109.3$ GPa) and the highest modulus in T.D. ($E_T = 114.2$ GPa). Comparing to $\bar{E}$, there are $E_R < \bar{E}$ and $E_T \approx \bar{E}$. As mentioned above, the anisotropy pattern of Ti crystal is of rotational symmetry around c axis and the highest modulus lies in c direction ($E_c = 143.3$ GPa) but the basal plane is isotropic ($E_b = 104.4$ GPa). Therefore, the anisotropy of the plate means that the c axis of crystallites has a different distribution tendency in different direction of plate.

Fig. 8 Partial (0001) pole figure of Ti-plate (Thick line-EDD).

Figure 1 shows that there is the certain distribution tendency of c in R.D. so E_R becomes lower than $\bar{E}$. In T.D., $E_T \approx \bar{E}$ means that the orientation of the crystallites is elastically equivalent to random state. If distribution tendency in any direction are same, the Young's modulus distribution on plate plane will be isotropic even though there is some texture. Above analysis for titanium plate texture is consistent with that of the X-ray pole figure (Fig. 8).

(3) Quantitative analysis of texture

Quantitative analysis of Ti plate texture can be further carried out on the basis of qualitative analysis. Several parameters have been defined to describe characteristic and the level of the texture.

(i) Equivalent inclination angle (α) and equivalent divergence diagram (EDD)

Equivalent inclination angle means a inclination angle of the c axis to the normal of plate. In the case, modulus of monocrystal will be just equal to measured modulus in designated direction on the plate plane. It represents the monocrystal equivalent inclination degree in the direction of the plate. Marking α on a polar net and connecting them to line, it would express the equivalent divergence of the c axis to normal of plate i.e. divergence level of (0001) texture. Equivalent inclination angle (α) can be obtained according to the following two methods.

(a) Graphic method: This is to find out an angle which correspond to the measured modulus of the designated direction from any ADCP of longitudinal section of Ti crystal (such as (1120) in Fig. 5).

(b) Computation method: Substituting the designated direction angle (θ) into the Fourier series expression of the plate to compute Young's modulus (E), and then substituting the computed E and S_{ij} of Ti crystal into the following four-order equation transformed from eq. (5):

$$aR_3^4 + bR_3^2 + C = 0 \quad (11)$$

$$a = (S_{11} + S_{33} - 2S_{13} - S_{44})E$$

$$b = (2S_{13} - 2S_{11} + S_{44})E$$

$$c = (ES_{11} - 1).$$

Solving eq. (11), we may obtain R_3 . Thus

$$\alpha = 90^\circ - \arccos R_3. \quad (12)$$

For comparing, EDD is plotted on X-ray pole figure of the Ti plate (Fig. 8, thicker line). It is thus clear that EDD reflects the tendency and level of the texture.

(ii) Equivalent ordered level (γ)

The equivalent ordered level (γ) means a level of the preferred orientation which finds the expression in the modulus distribution. If orientation of the crystallite is completely random, the modulus distribution will be isotropic and $\bar{E} = 113.8$ GPa. On the contrary, if the plate has a ideal (0001) texture i.e. the (0001) planes of every crystallite are all parallel to rolling surface of the plate, Young's modulus of the plate plane is also isotropic and it will be equal to modulus value of basal plane ($E_b = 104.4$ GPa). The measured average modulus of the titanium plate is $E_0 = 111.2$ GPa which is between $\bar{E}$ and E_b . It can be thought that the position of E_0 between $\bar{E}$ and E_b would reflect the level of the preferred orientation. The equivalent ordered level (γ) may be defined as follows:

$$\gamma = \frac{\bar{E} - E_0}{\bar{E} - E_b}. \quad (13)$$

If the plate has a ideal (0001) texture, $E_0 = E_b$ and $\gamma = 1$. On the contrary, the orientations of crystallites are completely random, then $E_0 = \bar{E}$ and $\gamma = 0$. The γ -value of Ti plate is equal to 0.28. The equivalent ordered level is the reflection of the ordered level in modulus but not a direct expression, therefore it is only as a reference.

(iii) Anisotropy level of divergence of C (ζ_c)

If α -values of all direction are the same i.e. EDD is circular, Young's modulus will be isotropic on rolling plane of plate. The anisotropy is only caused in such case that α is different for different direction. The deviation of EDD from a circular form is more serious and the anisotropy will become more intense. The anisotropy level of α may be defined by the following equation:

$$\zeta_c = \frac{\alpha_{\max} - \alpha_{\min}}{2\alpha_{\text{ave}}} \quad (14)$$

where

$\alpha_{\max}$ —maximum of α

$\alpha_{\min}$ —minimum of α

α_{ave} —average of $\alpha_{\max}$ and $\alpha_{\min}$

ζ_c of the titanium plate under experiment is equal to 0.17. In the same way, anisotropy level of Young's modulus can be computed as follows:

$$\zeta_c = \frac{E_{\max} - E_{\min}}{2E_{\text{ave}}} \quad (15)$$

where

$E_{\max}$ —maximum of E
 $E_{\min}$ —minimum of E
 E_{ave} —average of $E_{\max}$ and $E_{\min}$

ζ_E of this plate is equal to 0.02. The very large discrepancy between ζ_E and ζ_c is determined by the elasticity anisotropy diagram of longitudinal section of Ti crystal. It follows that although the anisotropy of metal is results from the texture, the level of the anisotropy depends on the anisotropy characteristic of the crystal. Therefore, it is necessary to consider the anisotropy of the corresponding crystal besides the texture when we investigate the anisotropy of a metal plate.

3. Elasticity analysis of the niobium plate texture

It is seen from Fig. 4 and Fig. 6 that the anisotropy pattern of Nb crystal is more complex than that of Ti, therefore the elastic analysis for the texture of the Nb plate would become more complex.

(1) Qualitative analysis of texture

Substituting S_{ij} of Nb crystal into eq. (6) and exchanging R_1 , R_2 and R_3 by the following relationship

$$\begin{cases} R_1 = \cos \phi \cdot \sin \theta \\ R_2 = \sin \phi \cdot \sin \theta \\ R_3 = \cos \theta \end{cases} \quad (16)$$

and then substituting eq. (6) into eq. (10) to integrate, we can obtained isotropic statistical Young's modulus: $\bar{E} = 102.6$ GPa. The second anisotropy is the major factor of Young's modulus of niobium plate under experiment and the maximum of modulus lies in the 45° direc-

tion ($E_{45} = 115.6$ GPa) whereas those in R.D. and T.D. is minimum $E_R \sim E_T \sim 95$ GPa. Comparing the E_R , E_T and E_{45} to $\bar{E}$, obviously, E_{45} is higher than $\bar{E}$, but E_R and E_T are lower than $\bar{E}$. $E_{45} > \bar{E}$ means an accumulation of higher E directions of crystallites in the 45° direction. Consulting ASC of Nb crystal (Fig. 4) and dividing it into area: $\langle 100 \rangle$, $\langle 110 \rangle$, $\langle 111 \rangle$, it must be $\langle 100 \rangle$ area, because $E_{\langle 110 \rangle}$ and $E_{\langle 111 \rangle}$ all are lower than $\bar{E}$. It follows that the crystallographic plane which is parallel to rolling plane must possess four-fold symmetric $\langle 100 \rangle$ direction. Referring to ADCP of Nb (Fig. 4), only $\{100\}$ plane is satisfactory for the above condition. From above, we conclude that the texture of the Nb plate under experiment should be of $\{100\}\langle 110 \rangle$ type. The discrepancy in modulus distribution between the $\{100\}$ and plate planes can be explained by the divergency of the (100) texture or other texture component. It is difficult to determine $E_{\langle 110 \rangle}$ or $E_{\langle 111 \rangle}$ collect to R.D. or T.D. Therefore it cannot become the basis of analysis. The (100)[110] texture show that the $\langle 110 \rangle$ direction lies in R.D. and T.D.

(2) Quantitative analysis of the Nb plate texture

If the Nb plate possesses an ideal $\{100\}$ type texture, the modulus distribution should be the same as ADCP of the $\{100\}$ plane. The Fourier series expression and characteristic parameters can be given as follows:

$$E(\theta) = 117.2 + 29.1 \cos 4(\theta + 45^\circ)$$

$$E_0 = 117.2 \text{ GPa}$$

$$A_1 = 0$$

$$A_2 = 0.248 \quad \phi_2 = 45^\circ.$$

On the contrary, if the orientation of crystallites is completely random, the plate is isotropic and $E_0 = \bar{E} = 102.6$ GPa. The measured E values in the plate plane are situated between those of the ideal (100) texture and the random state as shown in Table 2 and Fig. 10.

Equivalent ordered level can be computed according to the following two equations:

$$\gamma_{E_0} = \frac{E_{02} - E_{01}}{E_{03} - E_{01}} \quad (17)$$

Fig. 9 (110) pole figure of Nb-plate.

Fig. 10. Elasticity anisotropy pattern of Nb-plate for three states.

Table 2 Characteristic parameter of three state.

State	Average (E_0)	Anisotropy coefficient (A_2)
(1) random state	102.6 GPa	0
(2) measured state	106.9 GPa	0.109
(3) (001)[110] texture	117.2 GPa	0.248

or

$$\gamma_{A_2} = \frac{A_{22} - A_{21}}{A_{23} - A_{21}}. \quad (18)$$

Where the subscripts indicate the different state. γ_{E_0} and γ_{A_2} are equal to 0.29 and 0.44 respectively. The disparity between γ_{E_0} and γ_{A_2} results from different equivalence parameter (E_0 or A_2). The γ_{E_0} and γ_{A_1} will obviously possess a different meaning.

The results of texture analysis for Nb plate under experiment are satisfactory. This is caused by intensive anisotropies of the plate under experiment and the Nb crystal.

IV. Conclusion

(1) The established anisotropy expressions of a single crystal (ASC and ADPC) are the fundamental reference to analyze the preferred texture by an elastic method.

(2) By means of polar coordinates diagram, Fourier series and characteristic parameters, the anisotropy of a plate can be numerically characterized.

(3) Some information of the texture can be inferred according to the measured Young's modulus distribution of plate.

(4) This method will be efficient under such a case that the elastic anisotropy of a monocrystals is large.

(5) The principal advantage of an elasticity analysis method is that the texture component can be qualitatively estimated by the elasticity analysis when the Young's modulus distribution is measured.

REFERENCES

- (1) A. A. Bryukhanov and I. A. Moroz: *Fiz. Metallov Metalloved.*, **42** (1976), 664.
- (2) A. A. Bryukhanov, V. S. Ivaniy, I. A. Moroz and Z. A. Bryuhanov: *Izv. Vyssh. Uchebn. Zaved. Tsvorn. Metallurg.*, No. 3 (1976), 106.
- (3) B. R. Tittmann, G. A. Alers and L. J. Graham: *Metall. Trans.*, **7A** (1976), 229.
- (4) Zhang Qi-ren: *Computation Base for Measured Value*, Science Press, (1965), 342.
- (5) *Metal Handbook*, Japan Institute of Metals, MARUZEN CO. Ltd., (1974), 35.
- (6) E. Schmid and W. Boas: *Kristallplastizität*, Chinese Translation, Science Press, (1962), 215.
- (7) Xu Shun-sheng: *Metal X-ray Science*, Shanghai Science-technique Press, (1962), 214.