Internal Kinematics of Tumbler and Planetary Ball Mills:
A Mathematical Model for the Parameter Setting

Jürgen Schilz

German Aerospace Center (DLR), Institute of Materials Research, Linder Höhe, D-51147 Köln, Germany

This paper discusses the internal mechanics of tumbler and planetary ball mills. On the basis of classical mechanics, trajectories of mass particles under different acceleration conditions in these mills are calculated. This allows the derivation of impact velocities and impact powers of the grinding masses—important quantities which characterize milling conditions. It is shown that, under the assumption of single particle movements, the trajectories and thus the impact energies of mass particles depend only on the ratio of planetary and system wheel radii and on the ratio of their angular velocities. As functions of those proportions, movements of particles are calculated and recorded in order to enable the reader to find the (theoretical) optimum mill setting for a desired purpose. For example, the mechanical alloying of brittle materials requires near-normal collisions with a minimum of shear component. The applicability of the developed model is discussed.

(Received April 27, 1998; In Final Form July 27, 1998)

Keywords: planetary ball mill, kinematics, energy input, milling parameters, slip, mechanical alloying

I. Introduction

Depending on the mill configuration, a ball milling device can be employed for (i) mixing and blending, (ii) mechanical comminution and milling, (iii) for the mechanical alloying (4-6), and—still merely for research purposes—the amorphization of materials (40-45). One of the distinguishing parameters between these performance states is the impact (kinetic) energy of the milling balls.

Blending of substances can be achieved with a minimum of energy input. Therefore tumbler mills are often used for this task. Communion and shape modification requires higher intensity settings, i.e. higher impact energies of the grinding balls on the load. For this purpose—and also for a mechanically induced alloy formation—planetary ball mills are of advantage, since the high acceleration of the system wheel in principle allows a large impact.

The employment of a planetary ball mill, however, requires a thorough adjustment of several parameters, such as system wheel and vial sizes (geometry) together with their spinning frequencies. One approach towards the description of milling behavior is through so-called parameter diagrams, which more or less empirically correlates the parameter setting with the derived milling product (40-45).

In this paper, we present a mathematical treatment based on classical mechanics that describes the motion of particles (balls) inside the vial of a tumbler and a planetary ball mill. The tumbler mill has been treated thoroughly in the literature (e.g. (9)). However, its mechanics is an easy access to the more complicated approach needed for a planetary mill. For that reason, a short introduction is given in paragraph II. This analysis is then extended to the planetary type of mill, where gravity is replaced by centrifugal acceleration (paragraph III). Here, the motions of particles and balls are more complicated, which makes an analytical expression of mass trajectories impossible. Instead, the equation of motion is solved numerically. The results are presented in convenient normalized coordinates which allows an easy transfer to any size of mill and any operating condition. Special emphasis is given to those settings, which deliver maxima in impact velocity or milling power. Paragraph IV finally discusses the derived results in terms of practical applicability.

II. Tumbler Mill

In a horizontal tumbler ball or rod mill comminution is achieved by heavy masses inside a rotating cylinder. Those masses either roll ("cascade") down the surface or fall ("cataract") through free space onto the material. The driving force in a tumbler mill is gravity, which also poses a limit for the energy input. The cylindrical vessel rotates around its symmetry axis with angular velocity \( \omega \). This axis is oriented horizontally with respect to the ground plane. Figure 1 shows the accelerations acting on a mass particle which is dragged by the drum wall without slip. This non-slip condition is an idealization of the real situation, which will be used throughout this treatise. Its validity has to be discussed when comparing experimental results with the models received.

Rotation of the vessel causes a centrifugal acceleration \( a_c = \omega^2 r \). Gravity (with acceleration \( g \)) acts downwards, i.e. in the direction of the \( x \)-axis. In the case of negligible slip, only the radial component of the acceleration is relevant and the equation of motion can simply be written as

\[
\mathbf{a} = a_c + g = \omega^2 r \mathbf{e} + g \cos (\varphi),
\]

with \( \varphi \) denoting the polar angle with respect to the \( x \)-axis (cf. Fig. 1).
1. Detachment angle, flight time, and impact velocity

Equation (1) immediately allows the calculation of the polar angle \( \phi_0 \), where the mass particle leaves the wall. The condition for separation is \( a_c = 0 \). One obtains

\[
\phi_0 = \arccos \left( -\frac{\omega r^2}{g} \right),
\]

with

\[
\frac{\pi}{2} \leq \phi_0 \leq \pi \quad \text{for} \quad 0 \leq \omega \leq \frac{g}{r}.
\]

\( \sqrt{g/r} = \omega_{crit} \) is the so-called critical angular velocity. Above this value pinning of the matter to the internal wall occurs.

The trajectory of the free falling particle, its impact point at angle \( \phi_1 \), and impact velocity \( v_1 \) are of further interest. At the time \( t_0 \) the particle leaves the wall with a velocity \( v_1 = g \cdot t \). The gravity acceleration adds a component \( v_1 = g \cdot t \). This is the familiar problem of projectile motion. The particle path \((x(t), y(t))\) can be parameterized as a function of detachment angle \( \phi_0 \) and time \( t \):

\[
x = r \cdot \cos \phi_0 - v_1 \cdot t \cdot \sin \phi_0 + \frac{1}{2} gt^2
\]

\[
y = r \cdot \sin \phi_0 + v_1 \cdot t \cdot \cos \phi_0.
\]

The polar angle of impact, \( \phi_1 \), can be calculated via the flight time \( t_1 \geq 0 \), which is derived from the condition \( x^2(t) + y^2(t) = r^2 \). One finds

\[
t_1 = \frac{4ar}{g} \sqrt{1 - \frac{\omega^2 r^2}{g^2}} = \frac{4}{\omega_{crit}} \sin \phi_0 \cdot \sqrt{1 - \cos \phi_0}.
\]

It is convenient to introduce a dimensionless variable \( \varpi = \omega / \omega_{crit} \), which is independent of the individual ball mill size. Eq. (4) then reads:

\[
t_1(\varpi) = \frac{4\varpi}{\omega_{crit}} \sqrt{1 - \varpi^2}.
\]

By replacing the parameter time, \( t_1 \) in eq. (3) with the flight time, \( t_1 \), and utilizing \( x(t_1) = r \cdot \cos \phi_1 \), the following relations between separation angle \( \phi_0 \) and impact angle \( \phi_1 \) are obtained:

\[
\cos \phi_1 = \cos (3\phi_0) \quad \text{and} \quad \sin \phi_1 = -\sin (3\phi_0).
\]

Fig. 2. Tumbler mill: Flight time \( t_1 \) and magnitude of impact velocity \( v_1 \) of a particle as a function of normalized frequency \( \varpi = \omega / \omega_{crit} \).

Here we note, that this is the impact velocity in the fixed frame of reference \((x, y)\) and not in the system of the rotating vessel. In the next section, where we deal with the planetary type of mill, a correction is taken into account.

**Figure 2** shows the dependence of flight time \( t_1 \) and impact velocity \( v_1 \) on \( \varpi \). As expected, they both exhibit a maximum, however at different angular velocities. Differentiating the equations for \( t_1 \) and \( v_1 \), we get a maximum in \( t_1 \) at \( \varpi = 0.76 \), which means at a separation angle \( \phi_0 = 125^\circ \). The impact velocity \( v_1 \) is at maximum when

Fig. 3. Tumbler mill: Trajectories of particles for different normalized frequencies \( \varpi = \omega / \omega_{crit} \). The two paths with maximum flight time and maximum impact velocity are marked by \( t_{1,max} \) and \( v_{1,max} \), respectively.
\( \varpi \equiv 0.783 \), which corresponds to \( \varphi_0 = 128^\circ \).

2. Trajectories of particles in a tumbler mill

In Fig. 3 trajectories of the falling mass particles for different normalized angular velocities are plotted. Each trajectory has the shape of a parabola. Since the kinetic energy of the impacting mass is \( E_{\text{kin}} = me^2/2 \), the maximum energy available for impact and compression is reached when \( \varphi_1 \) has its peak value. Conclusively, a speed \( \varpi \equiv 0.8 \) of a tumbler mill is best for crushing hard and brittle materials by impact. In this case the inclination angle of the impact velocity is also nearly normal.

III. Planetary Mill

The derived concept can now be adopted to planetary mills. Here, the vial is fixed on a system wheel with radius \( R \). This wheel rotates at an angular velocity \( \Omega \). The vial itself has a radius \( r \) and spins around its symmetry axis with \( \omega \) (Fig. 4). The analysis is performed in a coordinate system which is fixed to the vial with the \( x \)-axis always pointing radial to the system wheel, and thus rotating with \( \Omega \). Three acceleration components are acting on the particle dragged by the vial wall: First, the centrifugal acceleration caused by the vial rotation \( a_c = \varpi \cdot r \). This is similar to the case of the tumbler ball mill. Second, there is the centrifugal acceleration from the system wheel \( a_{\text{CF}} = \Omega^2 (R + r) \). The third term is the Coriolis acceleration \( a_o = 2 \omega \times \Omega \), which acts on the particle moving with velocity \( v \) in a frame of reference rotating with angular velocity \( \Omega \). For the particle dragged by the wall is \( v = \omega \times r \). The Coriolis acceleration \( a_o \) points inward if \( \Omega \) and \( \omega \) are antiparallel, and outward in the parallel case.

If \( R \gg r \), i.e. for a planetary mill with a long arm of rotation, this mill is equivalent to a tumbler ball mill with \( g \) replaced by \( a_{\text{CF}} = \Omega^2 R \). The Coriolis acceleration can be neglected in this case and all results which were derived in the previous chapter can immediately be transferred to planetary mills with small \( r/R \) simply by replacing the gravity acceleration with the centrifugal acceleration of the system wheel, \( a_{\text{CF}} \).

The value of \( a_{\text{CF}} \) (usually given in multiples of the earth's gravitation, \( g \)), is often used to compare different types of planetary ball mills. As explained, this comparison is only justified for a planetary mill with long arm, i.e. \( r/R \) smaller than 0.1. In practice, values \( r/R \) are found between 0.1 and 0.3, which makes an exact treatment of the internal mechanics necessary.

1. Detachment angle

In order to calculate the detachment angle \( \varphi_0 \) as done for the tumbler ball mill, the equation of motion has to be devised. For this purpose one first has to set up the correct radial component of the system wheel's centrifugal acceleration \( a_{\text{CF}} \).

From Fig. 5 it is seen, that the magnitude of \( a_{\text{CF}} \) amounts to \( \Omega^2 \hat{R} \). Further we have

\[
\hat{R} = R \cdot \sqrt{\frac{r^2}{R^2} \sin^2 \varphi + \left( 1 + \frac{r}{R} \cos \varphi \right)^2}.
\]

The desired radial component of \( a_{\text{CF}} \) is

\[
a_{\text{CF},r} = a_{\text{CF}} \cdot \cos \beta = \Omega^2 \hat{R} \cdot \cos (\pi - (\varphi - \alpha))
\]

with

\[
\alpha = \arccos \left( \frac{R + r \cos \varphi}{\hat{R}} \right).
\]

Because all equations depend only on the radii ratio \( r/R \), we introduce a normalized radius \( \hat{r} = r/R \). By employing this parameter, a (cumbersome) equation for the condition of separation from the vial wall is obtained:

\[
0 = \omega^2 \hat{r} + 2 \omega \hat{r} \Omega + \Omega^2 \cdot f
\]

with

\[
f = \sqrt{\hat{r}^2 \sin^2 \varphi + (1 + \hat{r} \cos \varphi)^2} \times \cos \left[ \varphi - \arccos \left( \frac{1 + \hat{r} \cos \varphi}{\hat{r}^2 \sin^2 \varphi + (1 + \hat{r} \cos \varphi)^2} \right) \right].
\]

The Coriolis acceleration \( a_c = 2 \omega \varphi \hat{r} \) is negative when \( \Omega \) and \( \omega \) have opposite directions and it is positive in the other case.

By further introducing a dimensionless angular velocity \( \hat{\varphi} = \omega / \Omega \), the sense of rotation is absorbed in the sign

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Fig. 4 Principle of a planetary ball mill. \( \Omega \) and \( \omega \) are the angular velocities of the system and planetary wheels, respectively. \( R \) and \( r \) are their respective radii. \( a_c = \varpi \cdot r \) is the centrifugal acceleration due to the vial rotation and \( a_{\text{CF}} = \Omega^2 (R + r) \) that due to the rotation of the system wheel. \( a_o = 2 \omega \times \Omega \) is the Coriolis acceleration acting on a particle moving with velocity \( v \) in a frame of reference rotating with angular velocity \( \Omega \).

Fig. 5 Planetary mill: Deriving the magnitude and direction of the system's wheel centrifugal acceleration \( a_{\text{CF}} \).
of \( \dot{\Omega} \), i.e. negative values of \( \dot{\Omega} \) denote rotation of system and planetary wheel in opposite directions. Equation (8)

\[
\dot{\omega}(\phi_0) = -1 \pm \frac{1}{\sqrt{\sin^2 \phi_0 + \frac{1}{r} + \cos \phi_0}^2} \cos \left( \phi_0 - \arccos \left( \frac{1 + \tilde{r} \cos \phi_0}{\sqrt{\tilde{r}^2 \sin^2 \phi_0 + (1 + \tilde{r} \cos \phi_0)^2}} \right) \right).
\]

(9)

Because the reverse function \( \phi_0(\dot{\omega}) \) is of more practical use, it is this function which is plotted in Fig. 6 for different radii ratios \( \tilde{r} \). For \( \dot{\omega} = \omega / \Omega = -1 \) the angle of detachment \( \phi_0 \) for all radii ratios is \( \pi / 2 \). It will be seen later that in this case the flight path is virtually a point, so that a single particle in the vessel constantly stays at the position of the \( y \)-axis (cf. Fig. 4). This is, however, an unstable equilibrium, so that for a finite number of balls the result will be a chaotic movement with a large number of collisions.

When shifting the value of \( \dot{\omega} \) from \(-1\) in either positive or negative direction, the detachment angle \( \phi_0 \) will rise from \( \pi / 2 \) to \( \pi \). As in the case of the tumbler mill, a value of \( \pi \) denotes that the centrifugal acceleration \( a_d \) does not allow the particles to separate from the wall—the condition of pinning is reached. The spinning speed of the vessel, i.e. the normalized angular velocity \( \dot{\omega}_{\text{crit}} \), above which pinning occurs, is dependent on the mill geometry, which means the radii ratio \( \tilde{r} = r / R \). The longer the arm of rotation, i.e. the smaller \( \tilde{r} \), the larger the value of the speed ratio can be chosen. The exact value of \( \dot{\omega}_{\text{crit}} \) is derived from eq. (9) by setting \( \phi_0 \) to \( \pi \). This leads to

\[
\dot{\omega}_{\text{crit}} = -1 \pm \frac{1}{r}.
\]

(10)

From this expression (or from Fig. 6) it is seen that the absolute value of \( \dot{\omega}_{\text{crit}} \) for opposite \( \Omega \) and \( \omega \) at a given radii ratio \( \tilde{r} \) is a value of 2 higher than that for positive \( \dot{\omega} \).

The physical reason is, that in the case of counter rotation the inward pointing Coriolis acceleration partly compensates the centrifugal one and higher vessel spinning frequencies are allowed. For equal rotating directions, the Coriolis acceleration points outward, which—after a particle has left the wall—leads to weakly curved and thus quite short trajectories of the balls. As already seen in the treatment of the tumbler ball mill, long trajectories are desired for high impact velocities. It has already been experimentally verified, that the impact energy in the latter case is much smaller than for counter rotation\(^{10}\). Therefore we will restrict our examinations in the following sections to the case of negative \( \dot{\omega} \)-values.

2. Trajectories

When a mass particle has detached the vial wall at a time \( t_0 \), there are no physical forces acting. However, in the frame of the rotating vessel, the centrifugal acceleration \( a_{CF} \) with the magnitude \( a_{CF}(t_0) \) and the Coriolis acceleration \( a_c \) perpendicular to the actual velocity \( v \), are still active. Because the magnitude and direction of \( v \) and the direction of \( a_{CF} \) are not constant, an analytical solution for the particle’s trajectory is not possible. Therefore the trajectories were calculated by using a numerical iteration method, which calculates the location \( (x, y) \) of the particle by assuming the accelerations to be constant during a finite but suitably small time interval \( dt \). The shape of a path only depends on the reduced variables \( \dot{\omega} = \omega / \Omega \) and \( \tilde{r} = r / R \). Figures 7(a) to (c) show trajectories for fixed \( \tilde{r} \) with \( \dot{\omega} \) as parameter.

It can be seen from the figures, that with decreasing \( \tilde{r} / R \), the particle paths approach the common parabolic shape. The results are consistent with and complement those published in Ref. (11) where trajectories were obtained by a graphical method.

3. Flight time and impact velocity

The parameters at the time of particle detachment can be found by analytical methods and there are several papers dealing with this problem, e.g.\(^{10}(3)\). The flight time and the motion parameters at the moment of impact, however, can only be found by numerical iteration of the motion equations as performed here.

Figures 8 and 9 show impact velocities \( v_i \) and the radial (normal) component of \( v_i \), i.e. \( v_{i,r} = v_{i} r / r \) (all normalized by \( R \) and \( \Omega \)) as a function of \( \dot{\omega} \). The impact velocity \( v_i \) is measured in the frame of reference of the rotating vessel. Both, \( v_i \) and \( v_{i,r} \) exhibit a maximum at a certain angular frequency \( \dot{\omega} \). For the impact velocity, \( v_i \), this maximum appears shortly before the condition of pinning is reached. Very often the normal component \( v_{i,r} \) is of special interest. This parameter determines the impact energy \( E_i = v_{i,r} m / 2 \) (\( m \) mass of ball). The maximization of \( v_{i,r} \) maximizes the compression force of an impacting mass. On the other hand the tangential component of the impact velocity vector (parallel to the vial surface) results in a frictional interaction between ball and powder.
In Fig. 10 this setting of \( \dot{\omega} \) for the maximum normal impact energy of a given mill, i.e. for a given \( \dot{r} \), is plotted. The graph was empirically fitted to the following function \( \dot{\omega}(\dot{r}) \):

\[
\dot{\omega}_{\text{max}}(\dot{r}) = \frac{1}{3.29 \cdot \dot{r} + 0.2} + 1.37.
\]  

(11)

With the help of eq. (11), for a given mill geometry \( \dot{r} \), one can find the frequency ratio \( \dot{\omega}_{\text{max}} \), at which the radial impact velocity and hence the radial impact energy reaches its maximum.

4. Milling power

From the impact energy \( E_i \) and the flight time \( t_i \), the milling power \( P = E_i / t_i \) can be determined. The normalizing factor of \( t_i \) is \( \Omega \), which means that also the milling power is only dependent on \( \dot{r} \) and \( \dot{\omega} \). Figure 11 shows the resulting plot. The maxima are somewhat more pronounced as in Fig. 9, but it is to note that they are still at the same positions.

Fig. 8 Normalized impact velocities \( v_i \) in a planetary mill for different mill setups.

Fig. 9 Normalized radial (normal) impact velocities \( v_{ir} \), for different mill setups.

Fig. 10 Planetary mill: Setting of \( \dot{\omega} \) to achieve maximum normal impact velocity, i.e. maximum radial impact energy. The graph empirically fits to eq. (11).

IV. Conclusions

The calculations of particle movements in a planetary milling device presented here, allow for the first time the derivation of impact parameters of particles (i.e. milling balls) in terms of normalized milling parameters. Due to the normalization of the kinetic parameters, the results can conveniently be scaled to any size and type of planetary ball mill.

Figure 11 shows that for every mill geometry, i.e. vial radius \( r \) and planetary wheel radius \( R \), a certain frequency ratio \( \dot{\omega} = \omega / \Omega \) of the corresponding angular
velocities exists, where the impact power exhibits a maximum. This operating condition point delivers the highest amount of normal impact force and is thus the preferred setting for e.g. the mechanical alloying of hard and brittle materials. For a typical planetary ball mill which has \( r/R \) between 0.1 and 0.3, the optimum setting in \( \omega/\Omega \) is therefore between \(-3.25\) and \(-2.25\) (opposite sense of rotation).

Shifting the value of \( \omega/\Omega \) towards \(-1\), the flight paths become shorter and, concomitantly, the energy for the single impact declines. Additionally, the impact inclination angle becomes more acute, which means that the milling effect is increasingly based on attrition rather than impact.

A similar tangential, i.e. frictional, effect is observed, when increasing the absolute value of the frequency ratio towards the critical one (lets say for a typical mill to \(-3.5\)) where pinning of the masses occurs. When exceeding this critical value, theoretically no movement of masses takes place. However, under real conditions, relative movements between the balls are still possible, resulting in a pure attritional mode. Under certain conditions this mode may be preferential for the treatment of soft and ductile materials.

As emphasized before, all results presented here are derived by assuming point-like masses, disregarding particle (i.e. grinding ball) interactions, as well as neglecting any slip between moving vial wall and particle. In practice however, those assumptions are not strictly valid, leading to the question of the applicability of the derived models.

If the grinding balls have a finite radius \( \rho \), the simplest correction of the model is to replace the vessel radius \( r \) by \( r - \rho \). This still neglects spinning effects of the balls, but these are thought to be of minor importance. More difficult is to estimate an effect of a finite filling factor of the vessel, which means the influence of particle interactions. Own experiments indicated, that for vessel filling factors below about 0.2 the mutual interactions do not play the major role in performance. As long as the filling factor is below 0.5, the (theoretical) detachment angle is not influenced by the materials in the vessel.

There are some studies of milling performance which include the frequency ratio as milling parameter, e.g.\(^{(8)}\)\(^{(12)}\)\(^{(16)}\) or study the influence of the slip factor\(^{(17)}\). Mostly the milling conditions are correlated with a characteristic development parameter of the milling process—like hardness of powder particles, degree of alloying, or type of end product. It turns out, however, that it is not possible to derive a general conclusion for an arbitrary processing load. The type and nature of the material to be processed (whether hard and brittle, soft and ductile, element or compound) and the desired aim (e.g. alloying, defect introduction, amorphization, comminution) are additional parameters to be considered. For example in Ref. (12) the time evolution of the Vickers hardness during ball milling of copper is investigated with the result, that the ratio setting \( \omega/\Omega = -1 \) refers to the fastest defect introduction, since the chaotic (non-stable) ball movement delivers the highest frequency of shocks. Other experiments find no influence on shock frequency, but indeed on the injected power\(^{(13)}\). In the case of mechanical alloying, a small, but considerable influence on the evolution of alloy formation has been observed\(^{(14)}\)\(^{(16)}\). It was additionally found that the slip shifts the optimum absolute value of the frequency ratio to higher rates than theoretically predicted. This is especially pronounced when one of the components is soft and ductile. Thus, for the design of a planetary ball mill, a frequency ratio setting between \(-2.5\) and \(-3\) is preferential for most applications.

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