Effects of Elastic Modulus, Shape and Volume Fraction of an Elastically Inhomogeneous Second Phase on Stress States in a Loaded Composite

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Regarding a second phase as elastic inhomogeneities, disturbed stresses in a composite under uniform external stresses containing aligned spheroidal inhomogeneities are evaluated analytically using the Eshelby method and the Mori-Tanaka theory. The average stresses in the inhomogeneities and surrounding matrix are graphically indicated when the external stresses applied to the composite are axisymmetrical. The effects of elastic modulus, shape and volume fraction of the second phase on the variations of the average stresses in the composite are discussed. The results obtained are useful to understand the stress states in composites and give us design principles of composites with desirable mechanical properties.

(Received June 4, 1999; In Final Form August 4, 1999)

Keywords: composite, elastic deformation, stress state, inhomogeneity, ellipsoidal inclusion, Eshelby's method, Mori-Tanaka's theory, materials design

I. Introduction

When a second phase of a solid material has elastic moduli different from those of the remainder (matrix), a uniform external stress is disturbed in the material. This effect is considered to be one of the reasons of composite strengthening. **Figure 1** schematically shows this effect. When the material consists of only a uniform matrix M, the stress in M is naturally the same as the externally applied stress (Fig. 1(a)). On the other hand, when elastically harder second phase particles Ω are dispersed in elastically deformed M, the applied stress is disturbed and the stress σ^M in M is lowered as schematically shown in Fig. 1(b). This may cause the suppression of possible plastic deformation of M. However, it is not necessarily an easy task to quantitatively evaluate σ^M and σ^Ω in Fig. 1 and, therefore, systematic studies on the variation of stress components as the second-order tensors are still limited.

The Eshelby method in micromechanics has shown that the disturbed stress in an ellipsoidal inhomogeneity embedded in an infinitely extended matrix becomes uniform⁽¹⁾⁻⁽³⁾. Using the Mori-Tanaka method⁽⁴⁾ of "average stress" together with Eshelby's, the average stresses in the inhomogeneities can be evaluated including elastic interactions between inhomogeneities. Adopting these methods, we will discuss and graphically show the effects of elastic modulus, shape and volume fraction of the second phase on the stress states in a composite under external stresses. Different from simplified experiments in laboratories, multi-axial stresses may be loaded to actual composites in real applications. This case is also included in the present analysis. As will be seen, the results obtained are useful to find the stress

Fig. 1 Schematic illustration showing the stress distribution of an applied external stress in (a) a material consisting only of a matrix M and (b) a composite containing many elastically harder second phase particles Ω embedded in M.

states in composites and give us design principles of composites with desirable mechanical properties.

II. Stress States in Composites Containing Ellipsoidal Inhomogeneities

When an isolated inhomogeneity Ω is embedded in the matrix M, the Eshelby method⁽¹⁾⁻⁽³⁾ has shown that the stress σ_{ij}^Ω in Ω caused by the application of a uniform external stress σ_{ij}^0 to M+ Ω become uniform. The

stress σ_{ij}^{Ω} in Ω is given by solving the following simultaneous equations⁽¹⁾⁻⁽³⁾:

$$\begin{aligned}\sigma_{ij}^{\Omega} &= C_{ijkl}(e_{kl}^0 + S_{klmn}\varepsilon_{mn}^{**} - \varepsilon_{kl}^{**}) \\ &= C_{ijkl}^*(e_{kl}^0 + S_{klmn}\varepsilon_{mn}^{**}),\end{aligned}\quad (1)$$

and

$$\sigma_{ij}^0 = C_{ijkl}e_{kl}^0. \quad (2)$$

Here, C_{ijkl} and C_{ijkl}^* are the elastic moduli of M and Ω , respectively, S_{ijkl} the Eshelby tensor and ε_{ij}^{**} the equivalent eigenstrain of the fictitious homogeneous inclusion which reproduces σ_{ij}^{Ω} in Ω . From eqs. (1) and (2), we find that the relationship between σ_{ij}^{Ω} and σ_{ij}^0 is written as⁽⁵⁾⁻⁽⁷⁾

$$\sigma_{ij}^{\Omega} = G_{ijkl}\sigma_{kl}^0, \quad (3)$$

where G_{ijkl} is the fourth-order tensor which satisfies $G_{ijkl} = G_{jikl} = G_{ijlk}$ and written as a function of elastic moduli of M and Ω and the shape of Ω .

When a material contains many inhomogeneities, average stresses in the matrix and inhomogeneities are evaluated by using Mori and Tanaka's concept⁽⁴⁾. The average stress in the matrix $\langle\sigma_{ij}\rangle_M$ and that in the inhomogeneities Ω , $\langle\sigma_{ij}\rangle_{\Omega}$ satisfy

$$V_f\langle\sigma_{ij}\rangle_{\Omega} + (1 - V_f)\langle\sigma_{ij}\rangle_M = \sigma_{ij}^0. \quad (4)$$

This is because the average of the internal stress in the material is always equal to zero. For simplicity, we consider aligned inhomogeneities Ω with an identical shape. The spatial distribution of Ω is assumed to be random. The volume fraction of Ω is assigned as V_f . Then, using G_{ijkl} in eq. (3) which relates σ_{ij}^{Ω} to σ_{ij}^0 for an isolated Ω , the average stress in the matrix $\langle\sigma_{ij}\rangle_M$ and that in the inhomogeneities $\langle\sigma_{ij}\rangle_{\Omega}$ are obtained by solving the following simultaneous equations⁽⁵⁾⁻⁽⁷⁾:

$$V_f(G_{ijkl}\langle\sigma_{kl}\rangle_M) + (1 - V_f)\langle\sigma_{ij}\rangle_M = \sigma_{ij}^0 \quad (5)$$

and

$$\langle\sigma_{ij}\rangle_{\Omega} = G_{ijkl}\langle\sigma_{kl}\rangle_M. \quad (6)$$

As shown in eqs. (5) and (6), we can evaluate the average stresses $\langle\sigma_{ij}\rangle_M$ and $\langle\sigma_{ij}\rangle_{\Omega}$ from the external stress σ_{ij}^0 if we know G_{ijkl} .

III. Composite Containing Spheroidal Inhomogeneities under Axisymmetrical External Stresses

We consider composites containing aligned spheroidal Ω particles with an identical aspect ratio α . As shown in Fig. 2, the aspect ratio is defined as $\alpha = c/a$ for the spheroidal Ω described as $(x_1^2 + x_2^2)/a^2 + x_3^2/c^2 \leq 1$ on the x_1 - x_2 - x_3 orthogonal coordinate system. For isotropic C_{ijkl} and C_{ijkl}^* , G_{ijkl} for the spheroidal Ω has been obtained analytically. The explicit expressions of the components of G_{ijkl} are shown in Ref. (6) as a function of V_f , α , the shear modulus μ and the Poisson ratio ν of M and the shear modulus μ^* and the Poisson ratio ν^* of Ω .

In the present paper, the axisymmetrical stresses with

Fig. 2 A composite under an axisymmetrical external stress containing spheroidal elastic inhomogeneities Ω with an identical shape and orientation.

respect to the x_3 axis are considered as external stresses applied to the composite. The axisymmetrical external stress is generally written as

$$\begin{aligned}\begin{pmatrix} \sigma_1^0 & 0 & 0 \\ 0 & \sigma_1^0 & 0 \\ 0 & 0 & \sigma_{III}^0 \end{pmatrix} &= \sigma_{\text{hydro}}^0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sigma_{\text{dev}}^0 \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix} \\ &= \sigma_C^0 \left\{ \cos \theta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right. \\ &\quad \left. + \sin \theta \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix} \right\}. \quad (7)\end{aligned}$$

In eq. (7), $\sigma_{\text{hydro}}^0 = (2\sigma_1^0 + \sigma_{III}^0)/3$ and $\sigma_{\text{dev}}^0 = (\sigma_{III}^0 - \sigma_1^0)$ respectively show the hydrostatic and deviatoric parts of the external stress. Since the axisymmetrical situation is discussed in the present study, σ_{dev}^0 coincides with the effective stress for both the von-Mises and Tresca yield criteria. As shown in eq. (7), any axisymmetrical external stress can be expressed by choosing appropriate values of σ_C^0 and θ ($0 \leq \theta < 2\pi$). In other words, any axisymmetrical external stress state can be expressed as a point on a unit circle in the two-dimensional $(\sigma_{\text{hydro}}^0/\sigma_C^0) - (\sigma_{\text{dev}}^0/\sigma_C^0)$ plane in Fig. 3. For example, the external stress becomes purely hydrostatic when $\theta=0$ (point H($\diamond$)) or $\theta=\pi$, purely deviatoric when $\theta=\pi/2$ (point D($\square$)) or $\theta=3\pi/2$, uniaxial ($\sigma_{11}^0 = \sigma_{22}^0 = 0$ and $\sigma_{33}^0 \neq 0$) when $\theta = \arctan 3$ (point U($\circ$)) or $\theta = \pi + \arctan 3$, and biaxial ($\sigma_{11}^0 = \sigma_{22}^0 \neq 0$ and $\sigma_{33}^0 = 0$) when $\theta = \pi + \arctan(-3/2)$ (point B($\triangle$)) or $\theta = 2\pi + \arctan(-3/2)$. Components of $(\sigma_{\text{hydro}}^0/\sigma_C^0)$ and $(\sigma_{\text{dev}}^0/\sigma_C^0)$ for the biaxial stress (point B($\triangle$)) are shown by arrows in Fig. 3 as an example. The average stresses $\langle\sigma_{ij}\rangle_M$ in M and $\langle\sigma_{ij}\rangle_{\Omega}$ in Ω due to σ_{ij}^0 also become axisymmetrical on the symmetrical requirements adopted in the present study. Hence the variations of $\langle\sigma_{ij}\rangle_M$ and $\langle\sigma_{ij}\rangle_{\Omega}$ can also be indicated by equations

Fig. 3 The diagram showing the various axisymmetrical external stress states. The $(\sigma_{\text{hydro}}^0 / \sigma_c^0)$ and $(\sigma_{\text{dev}}^0 / \sigma_c^0)$ axes respectively show the normalized magnitudes and signs of hydrostatic and deviatoric parts of the axisymmetrical stresses. The points H($\diamond$), D($\square$), U($\circ$) and B($\triangle$) in this figure respectively show purely hydrostatic, purely deviatoric, uniaxial and biaxial stress states.

similar to eq. (7) and by diagrams similar to Fig. 3.

When the axisymmetrical external stress is applied to a material consisting of only M with μ and ν , the elastic-strain energy density E_{elastic} of the material is written as

$$E_{\text{elastic}} = (\sigma_{\text{hydro}}^0)^2 / (2K) + (\sigma_{\text{dev}}^0)^2 / (6\mu), \quad (8)$$

where $K = \{2\mu(1+\nu)\} / \{3(1-2\nu)\}$ is the bulk modulus of M. When $\nu = 0.35$ and hence $K = 3\mu$, from eq. (8), we have

$$(\sigma_{\text{hydro}}^0)^2 + (\sigma_{\text{dev}}^0)^2 = 6\mu E_{\text{elastic}}. \quad (9)$$

Therefore, it is interesting to know that various axisymmetrical external stresses shown by different points on the circle in Fig. 3 results in the same elastic-strain energy density E_{elastic} of the uniform M with $\nu = 0.35$.

IV. Results and Discussion

1. Effects of the elastic modulus of Ω on the average stress in Ω

Figures 4 show the effects of elastic modulus of Ω on the average stresses in Ω with (a) spherical ($\alpha = 1$), (b) prolate spheroidal ($\alpha = 5$) and (c) oblate spheroidal ($\alpha = 0.2$) shapes when $V_f = 0.3$ and $\nu = \nu^* = 0.35$. The stresses $\langle \sigma_{\text{hydro}}^{\Omega} \rangle$ and $\langle \sigma_{\text{dev}}^{\Omega} \rangle$ respectively mean the hydrostatic and deviatoric parts of the average stress in Ω .

For elastically harder inhomogeneities ($\mu^* / \mu > 1$), stresses larger than the external stresses generate in Ω as shown by $\langle \sigma_{\text{hydro}}^{\Omega} \rangle / \sigma_c^0 > 1$ and $\langle \sigma_{\text{dev}}^{\Omega} \rangle / \sigma_c^0 > 1$ in Figs. 4(a) to (c). However, even if μ^* / μ reaches infinity, the stresses in Ω converge to certain values as for $\mu^* / \mu = 10^3$ in Fig. 4(a).

Although the applied external stress is represented by a circle, the average stresses in Ω are in general shown by

Fig. 4 The diagrams showing the effect of the ratio of elastic moduli μ^* / μ of Ω on the average stresses in Ω with (a) spherical ($\alpha = 1$), (b) prolate spheroidal ($\alpha = 5$) and (c) oblate spheroidal ($\alpha = 0.2$) shapes when $V_f = 0.3$ and $\nu = \nu^* = 0.35$. The $\langle \sigma_{\text{hydro}}^{\Omega} \rangle / \sigma_c^0$ and $\langle \sigma_{\text{dev}}^{\Omega} \rangle / \sigma_c^0$ axes respectively show the magnitudes and signs of hydrostatic and deviatoric parts of the average stresses in Ω normalized by the σ_c^0 component of the external stress defined in eq. (7). The points H($\diamond$), D($\square$), U($\circ$) and B($\triangle$) in this figure respectively show the resultant stress states in Ω when the external stresses applied to the composite M + Ω are purely hydrostatic, purely deviatoric, uniaxial and biaxial.

ellipses due to the inhomogeneity effect. Since the assumption of $\nu=\nu^*$ is adopted in Figs. 4, the ratio of shear moduli μ^*/μ is identical with that of bulk moduli K^*/K . However, as shown in Fig. 4(a), the ratios of the maximum values of $\langle\sigma_{\text{hydro}}^{\Omega}\rangle/\sigma_c^0$ to $\langle\sigma_{\text{dev}}^{\Omega}\rangle/\sigma_c^0$ are not unity even for the spherical inhomogeneities.

The points H($\diamond$), D($\square$), U($\circ$) and B($\triangle$) in Figs. 4(a) to (c) show resultant stress states in Ω when external stresses are hydrostatic, deviatoric, uniaxial and biaxial, respectively. Modes of stress states in Ω are generally different from those of the external stresses. However, when the shape of Ω is spherical (Fig. 4(a)), hydrostatic and deviatoric external stresses cause the identical modes of the stress states in Ω . Uniaxial and biaxial external stresses respectively cause similar modes of the stress states in Ω for fibrous and flat inhomogeneities (Figs. 4(b) and (c)).

Aspect ratios and orientations of the ellipses in Figs. 4 show the magnitudes and directions of the additional deviatoric or hydrostatic stresses generated in Ω . When we know $\langle\sigma_{\text{hydro}}^{\Omega}\rangle$ and $\langle\sigma_{\text{dev}}^{\Omega}\rangle$, those for the matrix M, $\langle\sigma_{\text{hydro}}^M\rangle$ and $\langle\sigma_{\text{dev}}^M\rangle$, are easily obtained from eq. (4). Hence we conclude that such graphical representation as those in Figs. 4 are useful to easily find and evaluate stress states in various composites containing various shapes of inhomogeneities.

2. Effect of the volume fraction of Ω on the average stresses

Figures 5 show the effect of volume fraction of Ω on the average stresses in (a) Ω and (b) M when $\alpha=5$, $\mu^*/\mu=5$ and $\nu=\nu^*=0.35$. The results in Fig. 5 for the extremely small value of V_f ($V_f=10^{-5}$) correspond to the Eshelby results for an isolated inhomogeneity⁽²⁾⁽³⁾. When V_f is close to zero or unity, the average stresses in M or Ω are naturally the same as those of the external stresses. However, it should be noted that even if the value of V_f is close to unity, it is impossible to decrease the average stresses in M to zero. When the external stress is uniaxial and $V_f=0.9$, we have $\sigma_{\text{dev}}/\sigma_{\text{dev}}^0 \approx 0.3$ from Fig. 6(b). We can hence say that the suppression of the plastic deformation of M by introducing more and more elastically harder Ω has its own limitations.

3. The Ω shape dependence of the average stresses

Figures 6 show the effect of aspect ratio α of Ω on the average stresses in Ω when $\mu^*/\mu=5$ and $\nu=\nu^*=0.35$. The ellipses for oblate spheroids $\alpha=0.2$ and $\alpha=0.001$ cross each other and this indicates that the α dependence is more complicated compared with the μ^* and V_f dependence shown in Figs. 4 and 5.

The complicated Ω shape dependence of the average stresses stems partly from the three-dimensional geometry of inhomogeneities considered in the present study. If we consider stress states in two-dimensional composites containing elliptic inhomogeneities, the aspect-ratio dependence of the average stresses in the elliptic inhomogeneities become more simple. As shown by eq. (5), G_{ijkl} in eq. (3) essentially determines the average

Fig. 5 The diagrams showing the effect of volume fraction of Ω on the average stresses in (a) Ω and (b) M when $\alpha=5$, $\mu^*/\mu=5$ and $\nu=\nu^*=0.35$. The points H($\diamond$), D($\square$), U($\circ$) and B($\triangle$) in this figure respectively show the resultant stress states in Ω or M when the external stresses applied to the composite M + Ω are purely hydrostatic, purely deviatoric, uniaxial and biaxial.

Fig. 6 The diagrams showing the effect of the aspect ratio of Ω on the average stresses in Ω when $V_f=0.3$, $\mu^*/\mu=5$ and $\nu=\nu^*=0.35$.

stresses in M and Ω . For elliptic inhomogeneities described as $(x_1^2) + (x_3^2/\alpha^2) \leq 1$ on the x_1 - x_3 plane, we have relationships among components of two-dimensional G_{ijkl} such as $G_{3333}(\alpha) = G_{1111}(1/\alpha)$ and $G_{1133}(\alpha)$

$=G_{3311}(1/\alpha)$. However, this kind of relationships of G_{ijkl} is not satisfied for the three-dimensional Ω . This is one of the reasons why the Ω shape dependence of the average stresses is complicated for spheroidal Ω .

4. Stress state and deformation behavior of composite

Because of the relationship of eq. (4), values of $\langle\sigma_{\text{dev}}^{\Omega}\rangle$ in Ω larger than those for $\mu^*/\mu=1$ (material consisting of only M) lower the average deviatoric stresses $\langle\sigma_{\text{dev}}^{\text{M}}\rangle$ in M. Elastically harder Ω are effective for the suppression of plastic deformation of M. The opposite can be said for elastically softer inclusions with the values of μ^*/μ smaller than unity. Here we discuss the effects of Ω shape on the hydrostatic stress $\langle\sigma_{\text{hydro}}^{\Omega}\rangle$ in Ω as an example of applications of the present analysis to composite materials design.

Figures 7 show the Ω shape (the aspect ratio α of Ω) dependence of $\langle\sigma_{\text{dev}}^{\text{M}}\rangle$ and $\langle\sigma_{\text{hydro}}^{\Omega}\rangle$ when $\mu^*/\mu=3$, $V_f=0.25$ and $\nu=\nu^*=0.35$. In Fig. 7, these stresses are normalized by the deviatoric stress σ_{dev}^0 of the external stresses as $\langle\sigma_{\text{dev}}^{\text{M}}\rangle/\sigma_{\text{dev}}^0$ and $\langle\sigma_{\text{hydro}}^{\Omega}\rangle/\sigma_{\text{dev}}^0$. The results in Figs. 7(a) and (b) are those when the external stresses are uniaxial (U) and purely deviatoric (D), respectively.

Although the variations of $\langle\sigma_{\text{dev}}^{\text{M}}\rangle/\sigma_{\text{dev}}^0$ with the change in α are not monotonic, both the values of $\langle\sigma_{\text{dev}}^{\text{M}}\rangle/\sigma_{\text{dev}}^0$ in

Figs. 7(a) and (b) become minimum (about 0.7) when the shape of Ω is needle-like with values of large α . However, in Fig. 7(a) where tensile uniaxial external stress is considered, it is found that the decrease of $\langle\sigma_{\text{dev}}^{\text{M}}\rangle/\sigma_{\text{dev}}^0$ in M with large α accompanies significant increase as large as 0.7 of the tensile hydrostatic stress $\langle\sigma_{\text{hydro}}^{\Omega}\rangle/\sigma_{\text{dev}}^0$ in Ω . The tensile hydrostatic stress may causes fracture of the inhomogeneities if they are brittle. However, when the external stress is purely deviatoric (Fig. 5(b)), the value of $\langle\sigma_{\text{hydro}}^{\Omega}\rangle/\sigma_{\text{dev}}^0$ in Ω is at most 0.3.

Cu-base *in-situ* composite alloys containing elastically harder but brittle Cr fibers are being developed as high-strength and high-conductivity wires⁽⁸⁾⁻⁽¹¹⁾. Although strength of the composite alloys is more than 900 MPa, tensile deformation of a single wire along the fiber direction sometimes causes severe brittle fracture during their macroscopically elastic region⁽¹²⁾. We believe that this fracture behavior is related to the large tensile hydrostatic stress states in Ω shown in Fig. 5(a). If the mode of the stress applied to the wire is changed from uniaxial to deviatoric, ductility of the wire should increase by lowering the tensile hydrostatic stress. One way to practically achieve this situation is to twist many wires so as to make a cord or wire rope. Transverse forces acting between twisted wires can change the mode of stress state in each wire from uniaxial to deviatoric even if the overall cord or wire rope is under tension. This is one of many possible design principles of composites obtained from the present study to improve their mechanical properties. The composite strengthening by the elastic inhomogeneities are well understood with the present graphical representations of the stress states in composites.

V. Conclusions

Disturbed stresses in a composite under uniform external stresses containing aligned spheroidal inhomogeneities have been analytically evaluated by considering the second phase as elastic inhomogeneities. The Eshelby method and the Mori-Tanaka theory have been used for this purpose. We have shown graphically the average stresses in the inhomogeneities and matrix when the external stresses applied to the composite are axisymmetrical. The effects of elastic modulus, shape and volume fraction of the second phase on the variations of the average stresses in the composite have been discussed. The graphical representations are useful to easily understand the stress states in composites and give us clues to design composites with desirable mechanical properties.

Acknowledgements

This work has been partially supported by the Proposal-Based New Industry Creative Type Technology R&D Promotion Program from the New Energy and Industrial Technology Development Organization (NEDO) of Japan, and a Grant-in-Aid for Scientific Research (C) from the Ministry of Education, Science and

Fig. 7 The Ω shape (the aspect ratio α of Ω) dependence of the normalized deviatoric average stress $\langle\sigma_{\text{dev}}^{\text{M}}\rangle/\sigma_{\text{dev}}^0$ in M and the normalized hydrostatic average stress $\langle\sigma_{\text{hydro}}^{\Omega}\rangle/\sigma_{\text{dev}}^0$ in Ω when $V_f=0.25$, $\mu^*/\mu=3$ and $\nu=\nu^*=0.35$. Results for (a) uniaxial and (b) purely deviatoric external stresses are shown.

Culture under Grant No. 10650687.

REFERENCES

- (1) T. Mura: *Micromechanics of Defects in Solids*, Martinus Nijhoff, Hague, (1987), p. 74–88.
- (2) J. D. Eshelby: Proc. R. Soc., **A241** (1957), 376–396.
- (3) J. D. Eshelby: Prog. Solid Mech., **2** (1961), 89–140.
- (4) T. Mori and K. Tanaka: Acta Metall., **21** (1973), 571–574.
- (5) Y. Benveniste: Mech. Mat., **6** (1987), 147–157.
- (6) M. Kato, T. Fujii and S. Onaka: Mat. Sci. Eng., A, in press (1999).
- (7) S. Onaka, T. Fujii, Y. Suzuki and M. Kato: Mat. Sci. Eng., A, in press (1999).
- (8) K. Mihara, T. Takeuchi and H. G. Suzuki: Mater. Trans., JIM, **39** (1998), 1093–1100.
- (9) K. Adachi, S. Tsubokawa, T. Takeuchi and H. G. Suzuki: J. Japan Inst. Metals, **61** (1997), 397–396.
- (10) K. Adachi, S. Tsubokawa, T. Takeuchi and H. G. Suzuki: J. Japan Inst. Metals, **61** (1997), 397–403.
- (11) P. D. Funkenbusch, T. H. Courtney and D. G. Kubisch: Scripta Metall., **18** (1984), 1099–1103.
- (12) D. L. Zhang, K. Mihara, E. Takahashi and H. G. Suzuki: Mat. Sci. Eng. A, **A266** (1999), 99–108.