Mathematical Modeling of Centrifugal Casting of Metal Matrix Composites

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Centrifugal casting of Aluminum melts containing suspended ceramic particles has been studied. One dimensional heat transfer model coupled with equations for force balance on particles is formulated and analysed to predict the particles distribution in the casting region, the temperature distribution in the casting and mold regions, and solidification time of the casting. The model takes into consideration propagation of solid-liquid interface and movement of particles due to centrifugal acceleration. The solution of the model equations has been obtained by pure implicit finite difference technique with modified variable time step approach. The effect of various parameters like particle size, mold rotational speed, relative density difference between melt and particle, *etc.* on segregation of particles and solidification time have been examined. For a given set of operating conditions, the thickness of the particle rich region decreases with increase in rotational speed of the mold, particle size, relative density difference between the particle and melt, and the melt superheat. Solidification time increases as the heat transfer coefficient at the metal-mold interface decreases.

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1. Introduction

Centrifugal casting provides an alternate route for producing functional gradient composite (FGC) by controlling various process parameters. The technique involves pouring a liquid melt containing reinforcement ceramic particles into a rapidly rotating mold which is mounted either vertically or horizontally, and continuing the rotation until solidification is complete. During rotation segregation of particles occurs due to the centrifugal force either to the inner or the outer part of the casting, depending on the relative densities of the particles and the melt; resulting in particles reinforced FGC. The extent of segregation depends on process parameters like pouring temperature, solidification time, thermo-physical properties, *etc.*

Kang and Rohatgi¹⁾ have presented a mathematical formulation based on one dimensional heat transfer analysis incorporating variation in thermophysical properties due to particles movement in the matrix. The present investigation is an attempt to further improve that analysis. Although the basic heat transfer equation is the same, the following improvements are incorporated:

1. While Kang and Rohatgi¹⁾ have defined volume fraction variation as a function of moving distance of particles, in the proposed formulation variation of volume fraction of particles with time is considered by ensuring conservation of total mass of particles at all times.
2. Unlike their formulation in which the heat transfer coefficient between the casting and the mold is assumed to be a constant, a more realistic characterization based on variation in air gap thickness between the casting and the mold surface, is incorporated.
3. Variation in latent heat of the melt with variation in fraction solid in the melt is taken into consideration.
4. The equation representing force balance on moving reinforcement particles incorporates a repulsive force term to account for the interaction of moving particles with

the solid-liquid interface.

A schematic representation of the mold of centrifugal casting of a MMC cylinder is shown in Fig. 1. The heat is withdrawn from the liquid region of the casting to the surroundings through the solidified composite region, the graphite mold and the steel shell. Heat is also radiated away from the inner surface of the casting. As the solidification proceeds, the solid-liquid interface moves away from the graphite mold.

2. Model Formulation

The principal model equations are: (i) heat conduction equations for various regions comprising the process, and (ii) force balance equation for the particulates to quantify the segregation of particles in the liquid region. The model is based on the following assumptions.

1. Heat flow is one dimensional and perpendicular to the mold wall.
2. Mold is filled with liquid melt instantaneously.
3. Thermal properties of solid particulates and metal matrix are temperature invariant.
4. Heat transfer coefficient between casting and graphite mold decreases as solidification proceeds due to increase in air gap because of contraction of casting.
5. Natural convection and movement of particles due to buoyancy are neglected.
6. There is no thermal resistance between particles and liquid metal.
7. Interface position between solid and liquid regions is calculated by assuming it to be planar.
8. Reinforcement particles are spherical in shape.

2.1 Heat conduction formulation

The heat transfer processes in various regions of casting and mold are governed by one dimensional unsteady state heat conduction equation written in cylindrical coordinates:

Fig. 1 Schematic representation of complete system of centrifugally cast MMCs with solidification interfaces.

$$\frac{\partial T_i}{\partial t} = \alpha_i \left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r} \frac{\partial T_i}{\partial r} \right) \quad (1)$$

where

$$\alpha_i = \frac{k_i}{\rho_i C_i} \quad (2)$$

and $i = lc, sc, g, m$ for the liquid composite, solid composite, graphite mold and steel shell regions, respectively. The α_{lc} and α_{sc} are estimated by determining the volume fractions of the solid particulates in the liquid and solid composite regions which vary with time due to the movement of particles owing to the density difference between liquid metal matrix and solid particulates. Thermal conductivities, densities, specific heats of composites in either liquid or solid regions are determined using the rule of mixtures described as follows as a function of volume fraction of particulate at any time t , $V_f(t)$

$$P_c = (1 - V_f(t))P_m + V_f(t)P_p \quad (3)$$

where P_c refers to the property of composite under consideration, P_m and P_p are the corresponding properties of the matrix and particulates, respectively.

The rate of solidification of the liquid composite is significantly dependent upon air gap formed at the casting-graphite mold interface due to contraction of the casting and thermal expansion of the mold during solidification. It is assumed that the heat transfer coefficient between casting and graphite mold due to air gap varies as.²⁾

$$h_i = h_i \left(\frac{h_f}{h_i} \right)^{\frac{s(t)}{r_i}} \quad (4)$$

where h_i is initial heat transfer coefficient, h_f is final heat transfer coefficient, $s(t)$ is solidified thickness and r_i is total thickness of the casting.

2.1.1 Initial conditions

Before pouring the melt into the mold, the latter is pre-heated to a certain temperature to avoid thermal damage to it. Therefore the initial temperature distributions in the casting, mold and shell regions are taken as

$$T_{lc} = T_p \quad (5)$$

$$T_g = T_m = T_M. \quad (6)$$

As soon as the melt comes in contact with the mold wall, temperature of the metal-mold interface increases suddenly. Initial interface temperature is approximated by considering thermal energy conservation within the very thin layer of the metal and the mold in an adiabatic system.^{3,4)} The detailed description is presented in Appendix A.

2.1.2 Boundary conditions

The boundary conditions in different regions of the casting and the mold are as follows:

1. At the inner surface of the casting, i.e., at $r = R_{ic}$

$$k_{lc} \frac{\partial T_{lc}}{\partial r} = h_2(T_{ci} - T_\beta). \quad (7)$$

2. At the outer surface of the casting, i.e., at $r = R_{oc}$

$$-k_{sc} \frac{\partial T_{sc}}{\partial r} = h_1(T_{co} - T_{gi}). \quad (8)$$

3. At the inner surface of the graphite mold, i.e., at $r = R_{ig}$

$$-k_g \frac{\partial T_g}{\partial r} = h_1(T_{co} - T_{gi}). \quad (9)$$

4. At the outer surface of the graphite mold, i.e., at $r = R_{og}$

$$-k_g \frac{\partial T_g}{\partial r} = h_4(T_{go} - T_{mi}). \quad (10)$$

5. At the inner surface of the steel shell, i.e., at $r = R_{im}$

$$-k_m \frac{\partial T_m}{\partial r} = h_4(T_{go} - T_{mi}). \quad (11)$$

6. At the outer surface of the steel shell, i.e., at $r = R_{om}$

$$-k_m \frac{\partial T_m}{\partial r} = h_3(T_{mo} - T_a). \quad (12)$$

7. At the solid-liquid interface, i.e., at $r = R_{s(t)}$

$$T_{sc} = T_{lc} = T_f. \quad (13)$$

8. Energy balance at the solid-liquid interface, i.e., at $r = R_{s(t)}$ is obtained by equating the rate of heat removed from the solid phase in the positive r direction to the sum of rate of heat supplied to the interface from the liquid phase in the positive r direction and rate of heat liberated at the interface during solidification, i.e.

$$-k_{sc} \frac{\partial T_{sc}}{\partial r} = -k_{lc} \frac{\partial T_{lc}}{\partial r} + \rho_{sc} H \frac{\partial s(t)}{\partial t}. \quad (14)$$

2.2 Problem formulation in terms of dimensionless variables

To facilitate solutions via a finite difference technique, it is advantageous to work in a domain of unchanging size in which the grid can be fixed once and for all. This requires

Equation (27) is valid only when an isolated particle of radius

$$R_p \leq R_{pc} = \left(\frac{9\nu^2}{4\Delta\rho\omega^2 r \rho_l} \right)^{\frac{1}{3}} \quad (28)$$

is under movement in the liquid.⁸⁾ But several phenomena have to be considered when a liquid containing suspended particles is solidified in a mold rotating at high velocity.

- The interaction between the moving particles and the solidification front.
- The variation of viscosity of the liquid due to movement of particles.
- The interaction between the particles in the liquid.

In this regard, the following assumptions are made:

1. The motion of the particles is stopped by the liquidus front, *i.e.* the particles do not move in the mushy zone of the casting and they are not rejected by the solidification front.
2. Equation (27) is valid only for the motion of an isolated particle in a liquid. But for large volume fractions, interaction between particles occurs thus reducing their velocity. It is assumed that the reduction in velocity is characterised by an increase of apparent viscosity of the liquid.⁹⁾

$$\nu_c = \nu[1 + 2.5V_f(t) + 10.05V_f^2(t)] \quad (29)$$

3. Segregation of particles occurs only up to a maximum volume fraction taken equal to 52%.¹⁰⁾

2.4 Determination of volume fraction

In order to determine the volume fraction variation in liquid metal matrix, the casting thickness is divided into n equal segments. Nodal points are considered at the extreme positions of each segment. The volume fraction of particulates in each segment is defined as the ratio of volume of particles to the total volume in each segment and is given as

$$V_{fs} = \frac{V_s}{V_s + V_l} = \frac{1}{1 + \frac{\rho_s m_l}{\rho_l m_s}} \quad (30)$$

where V_s is the volume of reinforced particles, and V_l is the volume of the metal matrix in each segment. Since the initial volume fraction of particles is known in each segment, the volumes and masses of the solid particles and matrix melt can be obtained using the following relationships.

$$V_s = V_{fs}V, \quad V_l = V - V_s \quad (31)$$

$$m_s = V_s \rho_s, \quad m_l = V_l \rho_l \quad (32)$$

where V , m_s , and m_l refer to total volume, mass of solid and mass of liquid, respectively.

For simplicity, the particle positions are considered at the nodal points. Since the motion of each particle in the melt is dependent on its position, the particles at different nodal points have different velocities. The volume of particulates in between two consecutive nodal points always remains the same. The new particle positions can be obtained by eq. (27). From the new particle positions, total volume of each segment of unit length can be obtained. The new volume fraction of

particles in each segment at time, $t + \Delta t$ is obtained as follows:

$$\rho_l > \rho_p : V_f(t + \Delta t) = \frac{V_s}{\pi[r_i^2(t + \Delta t) - r_{i-1}^2(t + \Delta t)]} \quad (33)$$

$$\rho_p > \rho_l : V_f(t + \Delta t) = \frac{V_s}{\pi[r_{i+1}^2(t + \Delta t) - r_i^2(t + \Delta t)]} \quad (34)$$

3. Solution Procedure

The model equations have been solved numerically using simple implicit finite difference techniques.¹¹⁾ For this the " $\bar{r} - t$ " domain is subdivided into small intervals of constant $\Delta\bar{r}$ in space and variable Δt in time as shown in Fig. 4. The variable time step approach is used to solve the problem. This approach requires that at each time level t_n the time step Δt_n is so chosen that the interface moves exactly a distance $\Delta\bar{r}$ during the time interval Δt , hence always stays on the node. Therefore, the problem is mainly concerned with the determination of the time step $\Delta t = t_{n+1} - t_n$ such that in the time interval from t_n to t_{n+1} , the interface moves from the position $n\Delta\bar{r}$ to the next position $(n+1)\Delta\bar{r}$.

3.1 Finite difference approximation

The differential equation and the boundary conditions for both the mold regions and casting region are discretized using implicit method with the central difference scheme.¹¹⁾ The thickness of each mold region is subdivided into n equal grids. Similarly the total casting thickness is also subdivided into n equal grids but the number of grids in the solid and liquid composite regions vary with time. It means that for the solid composite region the number of grids (n_i) goes on increasing as the solidification proceeds, and for the liquid composite region the number of grids ($n - n_i$) decreases, but their sum always remains n .

Equations (1), (17), and (20) are solved along with the boundary conditions. The resultant discretized equations are arranged in tridiagonal matrix form and the solution of these equations obtained using Thomas Algorithm (TDMA) which gives temperature distributions in casting and mold regions for a particular time step Δt_n . The detailed description is presented elsewhere.¹²⁾

Fig. 4 Subdivision of ($\bar{r} - t$) domain using constant $\Delta\bar{r}$, variable Δt .

3.2 Determination of time steps

During solidification the interface moves from the position $(n + 1)\Delta\bar{r}$ within a time interval of $\Delta t_i = t_{i+1} - t_i$ (shown in Fig. 4), which is not known a priori. Using modified variable time step (MVTs) method the actual value of " $\bar{r} - t$ " domain is subdivided into small intervals of constant $\Delta\bar{r}$ in space and variable Δt_i is obtained by iteration as follows.¹³⁾

1. An initial guess value for the time step Δt_i chosen as $\Delta t_i = \Delta t_i^{(0)}$.
2. Using $\Delta t_i^{(0)}$, first estimate for the nodal temperatures is obtained by solving the finite difference equations for the mold and casting regions.
3. First estimate for the time step $\Delta t_i^{(1)}$ is obtained by using estimates $[T_{l_{ci}}^{p+1}]^{(0)}$ and $[T_{l_{ci+2}}^{p+1}]^{(0)}$.
4. Using $\Delta t_i^{(1)}$ repeat the steps (2) and (3) to obtain $\Delta t_i^{(2)}$.
5. Iteration is carried out until the difference between the two consecutive time steps satisfies a specified convergence criterion.
6. To obtain Δt_{i+1} assume $\Delta t_{i+1}^{(0)} = \Delta t_i$ and repeat the steps from (2) to (5).
7. Repeat steps from (2) to (6) till the solid-liquid interface reaches the inner surface of the casting.

3.3 Segregation of particles

During centrifugal casting, segregation of particles takes place in the liquid composite due to the movement of particles resulting from difference in densities of the particles and the melt, and also due to centrifugal acceleration. By solving the eqs. (33) and (34) along with eqs. (27) and (29), the thickness of the particle-rich region is estimated for various rotational speeds of the mold at various times. Particle movement is neglected in the mushy zone of the casting which is characterised by the temperature regime ($T_S \leq T \leq T_L$).

4. Results and Discussion

The model equations are solved to evaluate: (i) thickness of the particle rich region, (ii) solidification time, and (iii) temperature distribution within the casting as well as in the mold regions, for various operating conditions. The variables examined are: (i) rotational speed of the mold, (ii) heat transfer coefficient between cast-mold interface, (iii) initial temperatures of liquid metal and mold, and (iv) thermophysical properties of particles. Thermophysical properties of matrix metal, reinforcement particles, and graphite and steel molds used in the simulation are the same as used by Kang and Rohatgi.¹⁾ Various design and operating parameters, like geometric constants for the casting and the mold, the heat transfer coefficients at different regions of the casting and the mold, and initial temperatures of the mold and metal used in simulation are tabulated in Table 1. The cooling conditions at the inner surface of the casting and at the outer surface of the steel mold are defined in terms of heat transfer coefficients h_2 , h_3 respectively and the heat transfer due to the air gap at the metal-mold interface is characterized by the heat transfer coefficient h_1 . The results of simulation are presented below.

Table 1 Design and operating parameters used in simulation.

Outer diameter of steel mold, D_{om} (mm)	215.0
Outer diameter of graphite mold, D_{og} (mm)	150.0
Outer diameter of casting, D_{oc} (mm)	100.00
Inner diameter of casting, D_{ic} (mm)	80.8
Initial heat transfer coefficient due to air gap at the metal-mold interface, h_i ($\text{W m}^{-2} \text{K}^{-1}$)	1000 to 10000
Heat transfer coefficient at the inner surface of casting, h_2 ($\text{W m}^{-2} \text{K}^{-1}$)	8.4
Heat transfer coefficient at the outer surface of steel mold, h_3 ($\text{W m}^{-2} \text{K}^{-1}$)	8.4
Heat transfer coefficient between the graphite-steel mold interface, h_4 ($\text{W m}^{-2} \text{K}^{-1}$)	10000
Ratio of initial to final heat transfer coefficient, h_i/h_f	10
Initial pouring temperature of liquid metal, T_p ($^{\circ}\text{C}$)	650, 730, 800
Initial mold temperature, T_M ($^{\circ}\text{C}$)	250
Speed of rotation, N (min^{-1})	1000–1500
Particle size, R_p (μm)	1–3

4.1 Particle segregation and solidification time calculations

For various sets of operating conditions, the thickness of particles rich region and solidification time have been calculated. Typical results of these calculations are presented in the form of (i) volume fraction, and (ii) thickness of particle rich region for the entire region of the casting, (iii) solidified thickness of the shell, and (iv) total solidification time. The main parameters which influence the displacement velocity of the particles are centrifugal acceleration, size of the particle, and relative density difference between the particle and the melt. Similarly, the main parameters affecting the solidification rate are the pouring temperature, the melt temperature, and the heat transfer coefficient at the metal-mold interface. In the following sub-sections effect of various parameters is presented.

4.1.1 Effect of rotational speed of casting

Figure 5 shows the effect of speed of rotation of casting on the thickness of particle rich region, for three different values of rotational speeds, namely 1000, 1250, and 1500 min^{-1} . Figure 5(a) is for the case where the reinforcement particles are heavier than the matrix melt, while Fig. 5(b) represents, the case when the matrix melt is heavier than the reinforcement particles. It is evident that with increasing speed of rotation, the thickness of the particle rich region decreases due to increased centrifugal acceleration. Depending upon the relative density difference between the particles and the matrix melt, and whether it is positive or negative, the particulates move with greater velocity towards the outer or inner periphery of the casting. The thickness of the Al_2O_3 rich region varies from 0.008 to 0.0053 m, while that of the graphite-rich region varies from 0.0086 to 0.007 m as the speed of rotation increases from 1000 to 1500 min^{-1} , after 10 seconds from the start of solidification.

4.1.2 Effect of particle size

As shown in Fig. 6, the thickness of the particle rich region decreases with increasing particle size leading to more intense segregation in both cases, i.e. (a) when $\rho_p > \rho_l$, and (b) when $\rho_p < \rho_l$. While in the former case particles are

segregated towards the outer periphery, in the latter case the segregation is towards the inner periphery. During the particle motion in the liquid melt, the centrifugal and the viscous

forces on the particle act opposite to each other. Both the centrifugal and viscous forces increase with increase in particle size, but the increase in centrifugal force is more than that in viscous force. Hence, larger particles move rapidly towards the outer or inner periphery of the casting depending upon the relative density difference between the particles and the liquid melt. After 10 seconds of solidification time, the thickness of the particle rich region for Al–Al₂O₃ and Al–Gr composites varies from 0.0095 to 0.0053 m, and from 0.0096 to 0.007 m respectively, as the particle size increases from 1 to 3 μm .

Repeating these calculations for SiC ($\rho = 3200 \text{ kg m}^{-3}$) as the reinforcement particles in Al matrix, the thickness of the particle rich region is found to be more than that for Al₂O₃ particles ($\rho = 4000 \text{ kg m}^{-3}$). Larger density difference between particles and matrix in case of Al₂O₃–Al system leads to larger centrifugal force on particles, which results in moving the particles to a farther distance causing more intense particle segregation towards the outer periphery.

4.1.3 Effect of initial pouring and initial mold temperatures

Figure 7 shows the effect of initial pouring temperature of the metal on the particle segregation, for three different values of T_p , namely 650, 730 and 800°C for three particle sizes, namely 1, 2, and 3 μm . With increase in initial pouring temperature (T_p), the thickness of the particle rich region decreases, or in other words, the thickness of the particle free region increases for all three particle sizes considered. Higher initial pouring temperature essentially means that more heat is to be removed from the MMC melt before the solidification begins. Hence, it takes longer for the MMC melt to solidify. During this extra time period, the reinforcement particles are able to segregate more resulting in increased thickness of the particle free region.

With increasing initial mold temperature also, the thickness of the particle rich region decreases as in the previous case. Solidification time increases with increase in mold temperature because of the reduced heat transfer between the melt and the mold owing to the reduced thermal gradient between the two, leading to more intense segregation.

4.1.4 Effect of heat transfer coefficient at the mold-metal interface

Figure 8 shows the effect of the heat transfer coefficient between the mold-metal interface, h_i . As the heat transfer rate is directly proportional to the heat transfer coefficient between

Fig. 5 Effect of rotational speed of the casting on particles segregation at $t = 10 \text{ s}$ for (a) $\rho_p > \rho_l$ (b) $\rho_p < \rho_l$.

Fig. 6 Effect of particle size on particles segregation at $t = 10 \text{ s}$ for (a) $\rho_p > \rho_l$ (b) $\rho_p < \rho_l$.

Fig. 7 Effect of initial pouring temperature on variation of thickness of particle free region of Al–Al₂O₃ composite.

Fig. 8 Effect of heat transfer coefficient between metal-mold interface (h_i) on variation of thickness of particle free region of Al-Al₂O₃ composite.

the metal-mold interface, the heat removal rate decreases with reduction in heat transfer coefficient. Hence, the total time required for solidification increases, resulting in more intense segregation of the particles, *i.e.* a reduction in the thickness of the particle rich region. In other words, the thickness of the particle free region increases with decreasing heat transfer coefficient. It is to be noted that the true instantaneous heat transfer coefficient at the mold-metal interface, h_i , itself changes with time according to the eq. (4). Hence, as the value of h_i decreases with increasing solidification, the interface offers more resistance to heat transfer from casting side to the mold. This reduces the rate of solidification requiring more time for complete solidification as compared to a situation in which no air gap is formed between the casting and the mold.

4.1.5 Effect of particulate volume fraction

Figure 9 shows the effect of initial volume fraction of the particulates on the final distribution of the volume fraction of particulates and also on the thickness of the particulate rich region of the casting. Increased initial volume fraction of solid particulates increases the final thickness of the particulate rich region. With increasing initial volume fraction of particulates the viscosity of the liquid composite increases which in turn, reduces the mobility of the particles. This results in a larger particle rich region. Figure 10 shows the variation of total solidification time as a function of the volume fraction of reinforcement particulates. As the initial volume fraction of particulates increases the effective thermal diffusivity of composite reduces. This results in longer time duration for complete solidification of the casting.

4.2 Temperature profiles in the casting and mold regions

The temperature profiles in the casting as well as the mold regions have also been calculated at different rotational speeds for different values of h_i solving the energy balance equations along with the boundary conditions. Higher interfacial heat transfer coefficient, signifying better thermal contact between the casting and mold, results in higher temperatures both at the inner as well as outer surfaces of the graphite mold. Due to the reduction in heat transfer coefficient between the metal-mold interface with time, however, the rate of heat transfer from the casting to the mold goes on decreasing.

Fig. 9 Variation of particulate volume fraction profiles for different initial particulate volume fractions.

Fig. 10 Variation of total solidification time with the particulate initial volume fraction.

Since the interface is unable to discharge the amount of heat supplied to it from the liquid region of the casting through the solidified shell, there is a local heating effect on the outer periphery of the casting. In the mold region, however, there is a cooling effect. This is because while the heat transfer from the casting to the mold is reduced, that from the mold to the ambient remains more or less unchanged.

5. Discussion

The salient feature of the present formulation is a more realistic representation of the variation of the volume fraction of the particles due to particle movement in the liquid composite region. Unlike the previous work of Kang and Rohatgi¹⁾ in which the variation of the volume fraction is considered as a function of moving distance only and the effect of presence of particle on the latent heat of solidification is neglected, the variation of volume fraction in the present formulation is estimated ensuring that the total mass of particles in the casting is conserved at all times and the latent heat of composite melt is modified depending upon the volume fraction of the liquid metal matrix in the melt. Analysis of particle segregation and solidification times for the various operating conditions clearly indicates that while the particle segregation mainly depends on the rotational speed of the casting, density difference between liquid metal and reinforcement particles, and the particle size, the solidification rate depends primarily on the heat transfer coefficient between metal-mold interface and initial temperatures of the melt and the mold.

The force balance on the particles and energy balance equa-

tions are nonlinear and mutually coupled. Solution of such partial differential equations in their original form is often difficult because of the associated problem of convergence and also the boundary is moving with time. It is advantageous to work in the domain of unchanging size in which the grid is fixed once and for all. One way to get around this problem is to transform the governing differential equations and the associated boundary conditions into their equivalent dimensionless forms.

Formulation of the model is based on assumptions several of which are only partially valid. For instance, the assumption of interface between the solid and liquid region to be planar is approximate. Strictly speaking the solidification front can be planar, cellular, or dendritic depending upon the rate of solidification. It is very difficult to predict the either cellular or dendritic morphology of the solidification front, hence the assumption of planar solidification front is made.

The assumption of negligible thermal resistance between the particles and melt can also be questioned. In actual practice there may be some solubility of solid particulates in the liquid matrix, or some chemical reaction between the two. In both cases there may be a heat of reaction associated with the phenomena taking place at the surface of the particle. This heat may appear as a heat source at the particle-melt interface. Further, a chemical reaction at the surface may result in formation of a product layer which offers thermal resistance to the heat flow, resulting in distortion of the temperature field around the particle. This may, in any case, happen also due to the different thermal conductivities of the particle and the melt.

The assumption of temperature invariant thermal properties is just for convenience. If reliable expressions for thermal properties as function of temperature are available, these can be easily incorporated in the model equations.

6. Summary

Using one dimensional heat transfer model coupled with equations for force balance, the effect of process parameters of centrifugal casting of aluminium containing suspended ceramic particles has been analysed to predict: (i) particle distribution in the casting region, (ii) temperature distributions in the casting and mold regions, and (iii) solidification time. Some of the important results of this investigation are as follows:

1. Thickness of particle rich region in the composite decreases with increasing rotational speed, particle size, initial pouring temperature and initial mold temperature.
2. Reduced heat transfer coefficient at the metal-mold interface leads to increased solidification time which, in turn, results in more intense segregation of solid particulates.
3. With increasing initial volume fraction of solid particulates, both the solidification rate as well as the final thickness of the particulate rich region increase.

List of Symbols

C specific heat, $\text{J kg}^{-1} \text{K}^{-1}$

h_1 heat transfer coefficient at $r = 0$, $\text{W m}^{-2} \text{K}^{-1}$

h_2 heat transfer coefficient at $r = -r_i$, $\text{W m}^{-2} \text{K}^{-1}$

h_3 heat transfer coefficient at $r = -r_o$, $\text{W m}^{-2} \text{K}^{-1}$

h_4 heat transfer coefficient at $r = -r_g$, $\text{W m}^{-2} \text{K}^{-1}$

h_i initial heat transfer coefficient at metal-mold interface, $\text{W m}^{-2} \text{K}^{-1}$

h_l instantaneous heat transfer coefficient at metal-mold interface, $\text{W m}^{-2} \text{K}^{-1}$

H latent heat of solidification, J kg^{-1}

H_e effective latent heat of solidification, J kg^{-1}

k thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$

N rotational speed of mold, rpm

R_{ic} position of casting inside from center, m

R_{ig} position of graphite mold inside from center, m

R_{im} position of steel mold inside from center, m

R_{oc} position of casting outside from center, m

R_{og} position of graphite mold outside from center, m

R_{om} position of steel mold outside from center, m

R_p particle radius, m

R_{pc} critical size of the particle, m

$R_{s(t)}$ position of solid-liquid interface from center, m

$r, \bar{r}$ coordinate system

r_i casting thickness, m

r_g distance from casting outside to graphite mold outside, m

r_o distance from casting outside to steel mold outside, m

$r_0(t)$ position of particle at time $t = 0$, m

$r_i(t)$ position of particle at time t , m

$r_i(t + \Delta t)$ position of particle at time $t + \Delta t$, m

$s(t)$ thickness of solid as a function of time, m

s_f thickness of particle free region, m

s_p thickness of particle rich region, m

T temperature, $^{\circ}\text{C}$

T_f solidification front temperature, $^{\circ}\text{C}$

T_g temperature of graphite mold, $^{\circ}\text{C}$

T_l initial cast-mold interface temperature, $^{\circ}\text{C}$

T_L liquidus temperature of base alloy, $^{\circ}\text{C}$

T_{lc} temperature of liquid region of composite, $^{\circ}\text{C}$

T_M initial temperature of the graphite and steel mold, $^{\circ}\text{C}$

T_m temperature of steel mold, $^{\circ}\text{C}$

T_p pouring temperature, $^{\circ}\text{C}$

T_S solidus temperature of base alloy, $^{\circ}\text{C}$

T_{sc} temperature of solid region of composite, $^{\circ}\text{C}$

T_a temperature related to position of steel mold ambient, $^{\circ}\text{C}$

T_b temperature related to position of cast inner surface ambient, $^{\circ}\text{C}$

t time, s

Δt time increment, s

$V_f(t + \Delta t)$ volume fraction of particle as function of $(t + \Delta t)$

V_r Stokes velocity of the particle, m s^{-1}

V_s volume of solid particles in each segment, m^3

ρ_p, ρ_l density of particle and liquid respectively, kg m^{-3}

α thermal diffusivity, $\text{m}^2 \text{s}^{-1}$

ν viscosity of base alloy, N s m^{-2}

ν_c apparent viscosity, N s m^{-2}

ω angular velocity of mold, rad s^{-1}

- δ_1 thickness of air gap between casting and graphite mold, m
 δ_2 thickness of air gap between graphite mold and steel mold, m

Subscripts

- g graphite mold
 ic position of casting inner surface
 ig position of graphite mold inside
 im position of steel mold inside
 L liquid region of base metal
 lc liquid region of composite
 m steel mold
 n time step
 oc position of casting outer surface
 og position of graphite mold outside
 om position of steel mold outside
 p particle
 S solid region of base metal
 Sc solid region of composite

Superscript

- p present time step
 $p + 1$ previous time step

REFERENCES

- 1) C. G. Kang and P. K. Rohatgi: *Metall. Trans. B.*, **27B** (1996), 277–285.
- 2) L. Lajoie and M. Suery: *Proc. Intl. Symp. on Advances in Cast Reinforced Metal Composites*, Publ. ASM Intl., (1988), pp. 15–20.
- 3) Y. Ebisu: *AFS Trans.*, **77** (1977), 643–654.
- 4) R. D. Pehlke, M. J. Kirt, R. E. Marrone and D. J. Cook: *Numerical Simulation of Casting Solidification*, AFS, Illinois, (1974), pp. 1–7.
- 5) N. Ramachandran, Y. Jaluria and J. P. Gupta: *Letters in Heat and Mass Transfer*, **8** (1981), 69–77.
- 6) T. Saitoh: *Trans. ASME, J. of Heat Transfer*, **100** (1978), 294–299.
- 7) P. K. Mallick: *Composites Engineering Handbook*, Moral Dekkar Inc., USA, (1997), pp. 638–640.
- 8) Quingmin Liu, Yuning Jiao, Yuansheng Yang and Zhuangqi Hu: *Metall. Trans. B.*, **27B** (1996), 1025–1029.
- 9) D. M. Stefanescu, A. Moitra, A. S. Kacar and B. K. Dhindaw: *Metall. Trans. A*, **21A** (1990), 231–239.
- 10) R. Sasikumar and B. C. Pai: *Redistribution of Second-Phase Particles in Melt during Solidification: Computer Model*, Solidification Processing 87, Sheffield 21–24 Sept., (1987), pp. 281–296.
- 11) M. N. Ozisik: *Finite Difference Methods in Heat Transfer*, CRC Press, London, (1994), pp. 134–138 and 281–296.
- 12) P. Samba Siva Raju: *Mathematical Modeling of Centrifugal Casting of Metal Matrix Composites*, M. Tech. Thesis, Dept. Materials and Metallurgical Engineering, I.I.T. Kanpur, India, May, (1999).
- 13) R. S. Gupta and Dharendra Kumar: *Int. J. Heat and Mass Transfer*, **24** (1981), 251–259.
- 14) Y. Nishida, W. Droste and S. Engler: *Metall. Trans. B.*, **17B** (1986), 833–844.

Appendix: Initial Temperature Distribution

In order to find the metal-mold interface temperature at time $t = 0$, the mold is assumed to be at a temperature T_M and the initial metal temperature is assumed to be the temperature of the metal as it enters the mold cavity. To find a reasonable approximation to the initial interface temperature, consider an adiabatic system shown in Fig. A-1.

The thermal energy initially contained in this system is $2\pi R_{oc}\delta r(\rho_g C_g T_m + \rho_{lc} C_{lc} T_p)$. The system is allowed to come to equilibrium adiabatically. The adiabatic assumption is reasonable because as $\delta r \rightarrow 0$ whatever happens to the system, it is assumed to happen instantly. Also, by definition of this condition, no heat flux has yet been established, hence no heat is transferred to or from the system. Depending upon the relative values of ρ_g , ρ_{lc} , C_g , C_{lc} , T_M , T_p and the heat of fusion of the metal (H), there are three possible final states of the system.

Case I: None of the metal solidifies

Equating the thermal energy in the system initially to that in the system at equilibrium yields

$$2\pi R_{oc}\delta r(\rho_g C_g T_m + \rho_{lc} C_{lc} T_p) = 2\pi R_{oc}\delta r(\rho_g C_g + \rho_{lc} C_{lc})T_I \quad (A-1)$$

where T_I is the initial metal-mold interface temperature. On rearranging the terms the interface temperature can be written as

$$T_I = \frac{\rho_g C_g T_M + \rho_{lc} C_{lc} T_p}{\rho_g C_g + \rho_{lc} C_{lc}} \quad (A-2)$$

Case II: All of the metal solidifies

The thermal energy released when metal solidifies is

$$2\pi R_{oc}\delta r(\rho_{lc} H) \quad (A-3)$$

Conservation of energy yields

$$2\pi R_{oc}\delta r(\rho_g C_g T_m + \rho_{lc} C_{lc} T_p) + 2\pi R_{oc}\delta r(\rho_{lc} H) = 2\pi R_{oc}\delta r(\rho_g C_g + \rho_{lc} C_{lc})T_I \quad (A-4)$$

On rearranging terms, T_I can be written as

$$T_I = \frac{\rho_g C_g T_M + \rho_{lc} C_{lc} T_p + \rho_{lc} H}{\rho_g C_g + \rho_{lc} C_{lc}} \quad (A-5)$$

Fig. A-1 Control volume considered for calculating the initial temperature of metal mold interface.

Case III: The metal partially solidifies

If the liquidus and solidus lines of a binary system are approximated by straight lines, the fraction of the liquid solidified as a function of temperature can be expressed as

$$FS = \frac{T_L - T}{T_L - T_f} \quad (\text{A.6})$$

where T_L and T_f are the liquidus and solidification front temperatures, respectively. Equating thermal energy in the system initially to that in the system at equilibrium yields

$$\begin{aligned} 2\pi R_{oc} \delta r \left[(\rho_g C_g T_m + \rho_{lc} C_{lc} T_p) + \rho_{lc} H \frac{T_L - T_I}{T_L - T_f} \right] \\ = 2\pi R_{oc} \delta r (\rho_g C_g + \rho_{lc} C_{lc}) T_I. \end{aligned} \quad (\text{A.7})$$

On rearranging terms, T_I can be written as

$$T_I = \frac{\rho_g C_g T_m + \rho_{lc} C_{lc} T_p + \frac{T_L \rho_{lc} H}{T_L - T_f}}{\rho_g C_g + \rho_{lc} C_{lc} + \frac{\rho_{lc} H}{T_L - T_f}}. \quad (\text{A.8})$$

It is necessary to assume the final state of the metal (liquid, mushy, or solid) in order to determine the initial interface temperature. After estimating T_I using the appropriate equation according to the assumed condition, its value is checked to ensure that the calculated T_I falls within the temperature range initially assumed.