Equivariant transfer for finite G -coverings

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(Received June 13, 1983)

Introduction

Transfers $t(p): \Sigma^\infty(Y_+) \rightarrow \Sigma^\infty(X_+)$ for finite coverings $p: X \rightarrow Y$ were studied by D. S. Kahn, S. B. Priddy [6], [7] and by F. W. Roush [13]. Roush proved the uniqueness of the transfer satisfying certain axioms.

Let G be a finite group throughout the present work in which we define equivariant transfers for finite G -coverings and prove their uniqueness. Our approach is, in a sense, parallel to that of Roush [13] even though we encounter several problems of different nature coming from G -actions.

In §1 we discuss the equivariant version of the little cubes operads defined by M. Boardman and M. Vogt [4] and studied by P. May [9]. We define G -little cubes operads $\mathcal{C}_\infty(k; G)$ (Definition 1.1) which is used to construct the equivariant transfer. $G \times \mathfrak{S}_k$ acts on $\mathcal{C}_\infty(k; G)$ and for a subgroup $\tilde{H}$ of $G \times \mathfrak{S}_k$, the fixed-point set $\mathcal{C}_\infty(k; G)^{\tilde{H}}$ is contractible if $\tilde{H} = \{(h, \tau(h)) | h \in H\}$ for a homomorphism $\tau: H \rightarrow \mathfrak{S}_k$, $H \leq G$, and is empty otherwise. Therefore $\mathcal{C}_\infty(k; G)/\mathfrak{S}_k$ is a classifying space of $\mathfrak{S}_k$ -bundles with G -action. In §2 we define the equivariant transfer (Definition 2.2) and the equivariant wreath operation (Definition 2.5), and investigate the relation between them. Let Γ be a subgroup of $\mathfrak{S}_k$ and Cov_G^Γ be the category of k -fold G -coverings $p: (X, A) \rightarrow (Y, B)$ with structure group Γ and pullback diagrams as morphisms. A generalized equivariant transfer t is a correspondence of $p \in \text{Cov}_G^\Gamma$ to $t(p) \in [Y/B, Q(X/A)]^G$ which is natural with respect to pullback diagrams (Definition 2.2). We postulate two axioms (T1) and (T2) for generalized equivariant transfers similar to the axioms of Roush (Definition 2.2 and [13], Chapter VI, Theorem 1B), and define an equivariant transfer as generalized equivariant transfer satisfying these axioms. Our main theorem is

THEOREM 2.3. *The equivariant transfer on Cov_G^Γ exists uniquely.*

To prove this theorem we take a way parallel to the arguments of Roush. We define an equivariant wreath operation w as G -homotopy classes $w(X) \in [\mathcal{C}_\infty(k; G)_+ \wedge_{\Gamma} X^k, Q_G(X)]^G$ which is natural with respect to based G -maps of X . We show that generalized equivariant transfers correspond to

equivariant wreath operations bijectively (Lemma 2.8) and rewrite the axioms (T1) and (T2) on t as the axioms (W1) and (W2) on w (Lemma 2.9, 2.10). In § 3 we show equivariant wreath operations satisfying (W1) and (W2) correspond to equivariant stable wreath operations (Definition 3.1) bijectively and prove that the equivariant stable wreath operation exists uniquely in Proposition 3.5. The proof of this proposition is the most complicated part in this work in comparison with the non equivariant case of Roush.

In § 4 we show that our equivariant transfer $t(p)$ is an adjoint of the equivariant Becker-Gottlieb transfer $\tau(p)$ for a finite G -covering over a finite G -complex (Nishida [12]), and also the induced homomorphism in K -theory is equal to the equivariant Atiyah transfer.

The author is grateful to Professors Shôro Araki and Yasumasa Hirashima for their kind advices and encouragements.

§ 1. G -little cubes operads

We denote by ρ the complex regular representation of G , by $D\rho$ its unit disk, by $S\rho$ its unit sphere and put $\dot{D}\rho = D\rho - S\rho$.

DEFINITION 1.1. A G -little $n\rho$ -cube is a linear embedding f of $(\dot{D}\rho)^n$ in $(\dot{D}\rho)^n$ such that $f = f_1 \times \cdots \times f_n$ where $f_i: \dot{D}\rho \rightarrow \dot{D}\rho$ is a map $f_i(v) = x_i + \varepsilon_i v$, $x_i \in \dot{D}\rho$, $\varepsilon_i > 0$, $v \in \dot{D}\rho$.

Define $\mathcal{C}_n(k; G)$ to be the set of those k -tuples $(c_1, \dots, c_k)$ of G -little $n\rho$ -cubes such that the images of c_i are pairwise disjoint. We regard $(c_1, \dots, c_k)$ as a map from $(\dot{D}\rho)^n \amalg \cdots \amalg (\dot{D}\rho)^n$, k -times, to $(\dot{D}\rho)^n$, and topologise $\mathcal{C}_n(k; G)$ as a subspace of the space of all continuous functions $(\dot{D}\rho)^n \amalg \cdots \amalg (\dot{D}\rho)^n \rightarrow (\dot{D}\rho)^n$. Define the left G -action and the right $\mathfrak{S}_k$ -action on $\mathcal{C}_n(k; G)$ as follows:

$$\begin{aligned} (c_1, \dots, c_k)\sigma &= (c_{\sigma(1)}, \dots, c_{\sigma(k)}), \quad \sigma \in \mathfrak{S}_k, \\ g(c_1, \dots, c_k) &= (gc_1, \dots, gc_k), \quad g \in G, \end{aligned}$$

where $\mathfrak{S}_k$ is the k -th symmetric group and $(gc_i)(v) = g(c_i(g^{-1}v))$. Then $(gc)\sigma = g(c\sigma)$ and we regard $\mathcal{C}_n(k; G)$ as a left $G \times \mathfrak{S}_k$ -space by $(g, \sigma)c = gc\sigma^{-1}$.

Let us choose a non zero element $a \in (\dot{D}\rho)^G$. There is an injection $\mathcal{C}_n(k; G) \rightarrow (\dot{D}\rho)^{2nk}$ defined by

$$(c_1, \dots, c_k) \rightarrow (c_1(0, \dots, 0), c_1(a, \dots, a), \dots, c_k(0, \dots, 0), c_k(a, \dots, a)).$$

This is a G -map and the space $\mathcal{C}_n(k; G)$ can be regarded as a G -subspace of $(\dot{D}\rho)^{2nk}$ by this injection.

Let $F(X; k)$ be the configuration space $F(X; k) = \{(x_1, \dots, x_k) \in X^k \mid x_i \neq$

x_j ($i \neq j$) for a G -space X . $\mathfrak{S}_k$ acts on $F(X; k)$ from the right by permutations of coordinates and G acts on it from the left by the diagonal action. We regard $F(X; k)$ as a $G \times \mathfrak{S}_k$ -space by $(g, \sigma)y = gy\sigma^{-1}$ for $g \in G$, $\sigma \in \mathfrak{S}_k$ and $y \in F(X; k)$.

LEMMA 1.2. *The space $\mathcal{C}_n(k; G)$ has the same $G \times \mathfrak{S}_k$ -homotopy type as $F(C[G]^n; k)$, where $C[G]$ is the complex group ring of G .*

PROOF. Because of $C[G] \approx {}_o \dot{D}\rho$ we can replace $C[G]^n$ by $(\dot{D}\rho)^n$. Let $\phi: \mathcal{C}_n(k; G) \rightarrow F((\dot{D}\rho)^n; k)$ be the $G \times \mathfrak{S}_k$ -map $\phi((c_1, \dots, c_k)) = (c_1(0), \dots, c_k(0))$. We define the $G \times \mathfrak{S}_k$ -homotopy inverse ψ of ϕ as follows. For $c = (c_1, \dots, c_k) \in \mathcal{C}_n(k; G)$, write $c_i = c_{i1} \times \dots \times c_{in}$, where $c_{ij}: \dot{D}\rho \rightarrow \dot{D}\rho$ is given by $c_{ij}(v) = x_{ij} + \varepsilon_{ij}v$. We say this c is the equidiameter of radius d if $\varepsilon_{ij} = d$ for all i and j . Let $\psi(b) = c$ for $b \in F((\dot{D}\rho)^n; k)$ such that c is the equidiameter element of $\mathcal{C}_n(k; G)$ with the maximal radius subject to the condition $\phi(c) = b$. By definition, $\phi \circ \psi = 1$. We define the $G \times \mathfrak{S}_k$ -homotopy $h: \psi \circ \phi \simeq_{G \times \mathfrak{S}_k} 1$ as follows. Let $c \in \mathcal{C}_n(k; G)$ be described as above and d be the radius of $\psi \circ \phi(c)$. Let d' be the radius of the maximal equidiameter element c' subject to the condition $\text{Im } c' \subset \text{Im } c$. Then define

$$h(c, t) = (c_{11}(t) \times \dots \times c_{1n}(t), \dots, c_{k1}(t) \times \dots \times c_{kn}(t)), \quad 0 \leq t \leq 1,$$

where

$$c_{ij}(t)(v) = \begin{cases} x_{ij} + ((1-2t)\varepsilon_{ij} + 2td')v, & 0 \leq t \leq \frac{1}{2}, \\ x_{ij} + ((2-2t)d' + (2t-1)d)v, & \frac{1}{2} \leq t \leq 1. \end{cases}$$

This gives the $G \times \mathfrak{S}_k$ -homotopy from 1 to $\psi \circ \phi$. □

There is an inclusion $i_n: \mathcal{C}_n(k; G) \rightarrow \mathcal{C}_{n+1}(k; G)$ defined by $i_n((c_1, \dots, c_k)) = (c_1 \times 1, \dots, c_k \times 1)$, which is a $G \times \mathfrak{S}_k$ -map. We define $\mathcal{C}_\infty(k; G) = \text{colim}_n \mathcal{C}_n(k; G)$. $\mathcal{C}_\infty(k; G)$ receives the $G \times \mathfrak{S}_k$ -action which is $\mathfrak{S}_k$ -free. Let $i'_n: F((\dot{D}\rho)^n; k) \rightarrow F((\dot{D}\rho)^{n+1}; k)$ be the inclusion map induced from the map $(\dot{D}\rho)^n \rightarrow (\dot{D}\rho)^{n+1}$, $(v_1, \dots, v_n) \rightarrow (v_1, \dots, v_n, 0)$. Let $\phi_n: \mathcal{C}_n(k; G) \rightarrow F((\dot{D}\rho)^n; k)$ be the $G \times \mathfrak{S}_k$ -homotopy equivalence defined in the proof of (1.2). As the following diagram

$$\begin{array}{ccc} \mathcal{C}_n(k; G) & \xrightarrow{i_n} & \mathcal{C}_{n+1}(k; G) \\ \downarrow \phi_n & & \downarrow \phi_{n+1} \\ F((\dot{D}\rho)^n; k) & \xrightarrow{i'_n} & F((\dot{D}\rho)^{n+1}; k) \end{array}$$

is commutative and $F((\dot{D}\rho)^n; k)$ has the $G \times \mathfrak{S}_k$ -homotopy type of a $G \times \mathfrak{S}_k$ -complex, $\mathcal{C}_\infty(k; G)$ has the $G \times \mathfrak{S}_k$ -homotopy type of a $G \times \mathfrak{S}_k$ -complex and $\text{colim}_n \phi_n: \mathcal{C}_\infty(k; G) \rightarrow \text{colim}_n F((\dot{D}\rho)^n; k)$ is a $G \times \mathfrak{S}_k$ -homotopy equivalence

(Waner [15], Lemma 4.8).

PROPOSITION 1.3. *Let $\tilde{H}$ be a subgroup of $G \times \mathfrak{S}_k$. The fixed-point set $\mathcal{C}_\infty(k; G)^\#$ is contractible if $\tilde{H} = \{(h, \tau(h)) \in G \times \mathfrak{S}_k \mid h \in H\}$ for a homomorphism $\tau: H \rightarrow \mathfrak{S}_k$, where H is a subgroup of G , and is empty otherwise.*

PROOF. Assume $\mathcal{C}_\infty(k; G)^\#$ is not empty. Put $H = pr_1(\tilde{H})$, $pr_1: G \times \mathfrak{S}_k \rightarrow G$. The map $\tau: H \rightarrow \mathfrak{S}_k$ defined by $(h, \tau(h)) \in \tilde{H}$, $h \in H$, is a well defined homomorphism because $\mathcal{C}_\infty(k; G)$ is $\mathfrak{S}_k$ -free. $F(C[G]; k) = (C[G]^n)^k - \Delta$, where $\Delta = \{(x_1, \dots, x_k) \in (C[G]^n)^k \mid x_i = x_j \text{ for some } i \neq j\}$, and therefore $F(C[G]^n; k)^\# = ((C[G]^n)^k)^\# - \Delta$. Let $G = Hg_1 \amalg \dots \amalg Hg_r$ be the left coset decomposition. We denote a point of $(C[G]^n)^k$ by $(\sum_{g \in G} a_{g,m,l} g)_{m,l}$, $1 \leq m \leq n$, $1 \leq l \leq k$, where $a_{g,m,l} \in C$. Then

$$\begin{aligned} (1.4) \quad & (\sum_{g \in G} a_{g,m,l} g)_{m,l} \in ((C[G]^n)^k)^\# \\ \iff & (\sum_{g \in G} a_{g,m,\tau(h)^{-1}(l)} hg)_{m,l} = (\sum_{g \in G} a_{g,m,l} g)_{m,l} \\ \iff & a_{h^{-1}g_i, m, \tau(h)^{-1}(l)} = a_{g_i, m, l} \quad \text{for all } h \in H, \\ & 1 \leq i \leq |G/H|, 1 \leq m \leq n \text{ and } 1 \leq l \leq k. \end{aligned}$$

Therefore the restriction of the projection map $pr: (C[G]^n)^k \rightarrow C^{|G/H|nk}$, $pr((\sum_{g \in G} a_{g,m,l} g)_{m,l}) = (a_{g_i, m, l})_{i,m,l}$, to $((C[G]^n)^k)^\#$ is a homeomorphism. Then $((C[G]^n)^k)^\# - \Delta \approx C^{|G/H|nk} - pr(((C[G]^n)^k)^\# \cap \Delta)$ and $\Delta' = pr(((C[G]^n)^k)^\# \cap \Delta) \subset pr(\Delta)$. Let $(\sum_{g \in G} a_{g,m,l} g)_{m,l} \in ((C[G]^n)^k)^\#$. Then $(\sum_{g \in G} a_{g,m,l} g)_{m,l} \in ((C[G]^n)^k)^\# \cap \Delta$ iff there are l_1 and l_2 such that $l_1 \neq l_2$ and $a_{g, m, l_1} = a_{g, m, l_2}$ for all $g \in G$ and $1 \leq m \leq n$, which is equivalent to saying that there are l_1 and l_2 such that $l_1 \neq l_2$ and $a_{g_i, m, \tau(h)^{-1}(l_1)} = a_{g_i, m, \tau(h)^{-1}(l_2)}$ for all $h \in H$, $1 \leq i \leq |G/H|$ and $1 \leq m \leq n$. Therefore Δ' is a union of linear subspaces contained in $pr(\Delta)$. Take a triangulation of $C^{|G/H|nk}$ containing Δ' as a subcomplex. Then $\dim C^{|G/H|nk} - \dim \Delta' \geq \dim C^{|G/H|nk} - \dim pr(\Delta) = 2|G/H|n$. Therefore by the usual argument of cellular approximations with respect to the dual cell complex of this polyhedron, we see that $C^{|G/H|nk} - \Delta'$ is $(2|G/H|n - 2)$ -connected. Therefore

$$\text{colim}_n F(C[G]^n; k)^\# \quad \text{and} \quad \mathcal{C}_\infty(k; G)^\#$$

are contractible.

If $\tilde{H} = \{(h, \tau(h)) \in G \times \mathfrak{S}_k \mid h \in H\}$ for a homomorphism $\tau: H \rightarrow \mathfrak{S}_k$, $H \leq G$, it is easily checked that $\mathcal{C}_\infty(k; G)^\#$ is non empty by (1.4). $\square$

Let X be a based G -space. Define the map

$$\theta'_{n,k}: \mathcal{C}_n(k; G) \times (\Omega^{n\rho} X)^k \rightarrow \Omega^{n\rho} X$$

by

$$\theta'_{n,k}(c, f_1, \dots, f_k)(v) = \begin{cases} f_i \circ c_i^{-1}(v), & v \in \text{Im } c_i, \\ *, & v \notin \text{Im } c_i, \end{cases}$$

where $c = (c_1, \dots, c_k) \in \mathcal{C}_n(k; G)$, $f_i \in \Omega^{n\rho} X$ and $v \in (\dot{D}\rho)^n$. $\theta'_{n,k}$ is $G \times \mathfrak{S}_k$ -equivariant and induces the G -map

$$\theta_{n,k}: \mathcal{C}_n(k; G)_+ \wedge_{\mathfrak{S}_k} (\Omega^{n\rho} X)^k \rightarrow \Omega^{n\rho} X.$$

Let Y_0 be an infinite loop G -space. There is a sequence $\{Y_i\}_{i \geq 0}$ of G -spaces such that $Y_n \simeq_{G\Omega^\rho} Y_{n+1}$. Then $\theta_{n,k}$ induces the G -map

$$\theta_{\infty,k}: \mathcal{C}_\infty(k; G)_+ \wedge_{\mathfrak{S}_k} (Y_0)^k \rightarrow Y_0.$$

§ 2. Equivariant transfers and wreath operations

We call $p: X \rightarrow Y$ a k -fold G -covering if X and Y are G -spaces and p is a k -fold covering G -map. Let $Z = \{z: [k] \rightarrow X \mid z([k]) = p^{-1}(y) \text{ for some } y \in Y\}$, $[k] = \{1, \dots, k\}$, and $\tilde{p}: Z \rightarrow Y$ be $\tilde{p}(z) = y$ if $z([k]) = p^{-1}(y)$. G acts on Z by $(gz)(i) = g(z(i))$. Then $\tilde{p}$ is a principal $(G, \mathfrak{S}_k)$ -bundle in the sense of tom Dieck [14]. p is the G -bundle with fiber $[k]$ associated with $\tilde{p}$. Let Γ be a subgroup of $\mathfrak{S}_k$ and Cov_G^Γ be the category whose objects are k -fold G -coverings $p: (X, A) \rightarrow (Y, B)$, $A = p^{-1}(B)$, with structure group Γ such that (Y, B) have the G -homotopy type of pairs of G -complexes and morphisms are pullback diagrams

$$(2.1) \quad \begin{array}{ccc} (X, A) & \xrightarrow{\tilde{f}} & (X', A') \\ \downarrow p & & \downarrow p' \\ (Y, B) & \xrightarrow{f} & (Y', B'). \end{array}$$

Let $\mathcal{T}_{G*}$ be the category of based G -spaces and G -homotopy classes of based G -maps as morphisms and $\mathcal{W}_{G*}$ be the full subcategory of $\mathcal{T}_{G*}$ consisting of based G -spaces having the G -homotopy types of based G -complexes. Let Q_G be the functor from $\mathcal{W}_{G*}$ to $\mathcal{T}_{G*}$ defined by $Q_G(X) = \text{colim}_n \Omega^{n\rho} \Sigma^{n\rho} X$, the colimit of $\Omega^{n\rho}(\text{adj } 1): \Omega^{n\rho} \Sigma^{n\rho} X \rightarrow \Omega^{(n+1)\rho} \Sigma^{(n+1)\rho} X$, where $\text{adj } 1$ is an adjoint map of $1_{\Sigma^{(n+1)\rho} X}$. Let $F_1: \text{Cov}_G^\Gamma \rightarrow \mathcal{T}_{G*}$ be the functor defined by $F_1(p) = Y/B$ for $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^\Gamma$ and $F_1((f, \tilde{f})) = f: Y/B \rightarrow Y'/B'$ for $(f, \tilde{f})$ of (2.1), and $F_2: \text{Cov}_G^\Gamma \rightarrow \mathcal{T}_{G*}$ be the functor defined by $F_2(p) = Q_G(X/A)$ for $p \in \text{Cov}_G^\Gamma$ and $F_2((f, \tilde{f})) = Q_G(\tilde{f}): Q_G(X/A) \rightarrow Q_G(X'/A')$.

DEFINITION 2.2. A *generalized equivariant transfer* is a natural transformation $t: F_1 \rightarrow F_2$. An equivariant transfer t is a generalized equivariant transfer satisfying the following axioms (T1) and (T2).

(T1) The following diagram

$$\begin{array}{ccc} \Sigma^\rho(Y/B) & \xrightarrow{\Sigma^\rho t(p)} & \Sigma^\rho Q_G(X/A) \\ \parallel & & \downarrow \text{adj } 1 \\ \Sigma^\rho(Y/B) & \xrightarrow{t(1 \times p)} & Q_G \Sigma^\rho(X/A) \end{array}$$

is commutative in $\mathcal{T}_{G*}$, where $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^F$, $1 \times p: (D\rho \times Y, D\rho \times A \cup S\rho \times X) \rightarrow (D\rho \times Y, D\rho \times B \cup S\rho \times Y) \in \text{Cov}_G^F$ and $\text{adj } 1$ is an adjoint map of $1: Q_G(X/A) \approx_{G^{\circ} Q_G} Q_G \Sigma^p(X/A)$.

(T2) Let $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^F$, $(X, A) = (X', A') \coprod (Y, B)$ and $p|_{(Y, B)} = 1_{(Y, B)}$. Then the composition

$$Y/B \xrightarrow{t(p)} Q_G(X/A) = Q_G(X'/A' \vee Y/B) \xrightarrow{c} Q_G(Y/B)$$

is equal to the canonical inclusion map $i: Y/B \rightarrow Q_G(Y/B)$ in $\mathcal{T}_{G*}$, where c collapses X'/A' .

We will prove the following theorem in this and the next sections.

THEOREM 2.3. *The equivariant transfer on Cov_G^F exists uniquely.*

Let $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^F$ and $\tilde{p}: (Z, C) \rightarrow (Y, B)$ be the associated principal (G, Γ) -bundle. Let $\tilde{H}$ be a subgroup of $G \times \Gamma$. If $Z^{\tilde{H}} = \emptyset$ then $\tilde{H} = \{(h, \tau(h)) \mid h \in H\}$ for a homomorphism $\tau: H \rightarrow \Gamma$, where H is a subgroup of G , and $\mathcal{C}_{\infty}(k; G)^{\tilde{H}}$ is contractible by Proposition 1.3. Therefore there is a unique $G \times \Gamma$ -map $d: Z \rightarrow \mathcal{C}_{\infty}(k; G)$ up to $G \times \Gamma$ -homotopy (Bredon [4], Chapter II).

In a way parallel to [13], p. 98, we consider the following diagram

$$(2.4) \quad \begin{array}{ccccc} (X, A) \approx (Z \times_{\Gamma} [k], C \times_{\Gamma} [k]) & \xrightarrow{T \times_{\Gamma} 1} & ((\mathcal{C}_{\infty} \times X^k \times [k])/\Gamma, (\mathcal{C}_{\infty} \times A^k \times [k])/\Gamma) \\ \downarrow p & & \downarrow & & \downarrow \\ (Y, B) \approx (Z/\Gamma, C/\Gamma) & \xrightarrow{T/\Gamma} & (\mathcal{C}_{\infty} \times_{\Gamma} X^k, \mathcal{C}_{\infty} \times_{\Gamma} A^k) \\ & & \downarrow c_0 & & \downarrow \pi_1 \\ & & ((\mathcal{C}_{\infty} \times (X/A)^k \times [k])/\Gamma, (\mathcal{C}_{\infty} \times *^k \times [k])/\Gamma) & \xrightarrow{\pi_1} & (X/A, *) \\ & & \downarrow p' & & \\ & & (\mathcal{C}_{\infty} \times_{\Gamma} (X/A)^k, \mathcal{C}_{\infty} \times_{\Gamma} *^k) & & \end{array}$$

where $\mathcal{C}_{\infty} = \mathcal{C}_{\infty}(k; G)$, $T = (d, f_1, \dots, f_k): Z \rightarrow \mathcal{C}_{\infty} \times X^k$, $f_i(z) = z(i)$, c_0 is the map induced by $X \rightarrow X/A$ and $\pi_1((c, x_1, \dots, x_k, i)) = x_i$. Each squares are pullback diagrams. Let $x \in X$ correspond to $[z, i] \in Z \times_{\Gamma} [k]$ by the canonical homeomorphism $X \approx Z \times_{\Gamma} [k]$. Then

$$\pi_1 \circ c_0 \circ (T \times_{\Gamma} 1)([z, i]) = \pi_1 \circ c_0([d(z), z(1), \dots, z(k), i]) = z(i) = x.$$

Therefore $\pi_1 \circ c_0 \circ (T \times_{\Gamma} 1): (X, A) \rightarrow (X/A, *)$ is the canonical collapsing map.

Let $F_s: \mathcal{W}_{G*} \rightarrow \mathcal{T}_{G*}$ be the functor defined by $F_s(X) = \mathcal{C}_{\infty}(k; G) \wedge_{\Gamma} X^k$ for $X \in \mathcal{W}_{G*}$ and $F_s([f]) = [1 \wedge_{\Gamma} f]$ for $[f] \in [X, X']^G$.

DEFINITION 2.5. An *equivariant wreath operation* is a natural transformation $w: F_s \rightarrow Q_G$.

Let w be an equivariant wreath operation. We construct a generalized

equivariant transfer t from w as follows. For $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^r, c_0 \circ T/\Gamma$ in (2.4) induces a G -map $Y/B \rightarrow \mathcal{C}_\infty(k; G)_+ \wedge_r (X/A)^k$. We define $t(p)$ to be the following composition.

$$(2.6) \quad t(p): Y/B \rightarrow \mathcal{C}_\infty(k; G)_+ \wedge_r (X/A)^k \xrightarrow{w(X/A)} Q_G(X/A).$$

For a pullback diagram (2.1), let $\tilde{p}: (Z, C) \rightarrow (Y, B)$ and $\tilde{p}': (Z', C') \rightarrow (Y', B')$ be the principal Γ -bundles associated with p and p' . The following diagram is commutative.

$$\begin{array}{ccc} (Z, C) & \xrightarrow{\tilde{f}} & (Z', C') \\ \downarrow \tilde{p} & & \downarrow \tilde{p}' \\ (Y, B) & \xrightarrow{f} & (Y', B') \end{array} \quad \tilde{f}(z)(i) = \bar{f}(z(i)).$$

Let $d': Z' \rightarrow \mathcal{C}_\infty(k; G)$ be a $G \times \Gamma$ -map and put $d = d' \circ f$. Then $d: Z \rightarrow \mathcal{C}_\infty(k; G)$ is a $G \times \Gamma$ -map and

$$\begin{array}{ccc} Z & \xrightarrow{T} & \mathcal{C}_\infty(k; G) \times X^k \\ \downarrow \tilde{f} & & \downarrow 1 \times f^k \\ Z' & \xrightarrow{T'} & \mathcal{C}_\infty(k; G) \times (X')^k \end{array}$$

is commutative, where

$$T(z) = (d(z), z(1), \dots, z(k)) \quad \text{and} \quad T'(z') = (d'(z'), z'(1), \dots, z'(k)).$$

Therefore the left square of the following diagram is commutative.

$$\begin{array}{ccccc} Y/B & \longrightarrow & \mathcal{C}_\infty(k; G)_+ \wedge_r (X/A)^k & \xrightarrow{w(X/A)} & Q_G(X/A) \\ \downarrow f & & \downarrow 1_+ \wedge_r f^k & & \downarrow Q_G(f) \\ Y'/B' & \longrightarrow & \mathcal{C}_\infty(k; G)_+ \wedge_r (X'/A')^k & \xrightarrow{w(X'/A')} & Q_G(X'/A'). \end{array}$$

The right square is commutative by the naturality of w . Therefore t defined by (2.6) is a generalized equivariant transfer.

Let t be a generalized equivariant transfer. Then we construct an equivariant wreath operation w from this t as follows. Let $p': ((\mathcal{C}_\infty \times X^k \times [k])/\Gamma, (\mathcal{C}_\infty \times *^k \times [k])/\Gamma) \rightarrow (\mathcal{C}_\infty \times_r X^k, \mathcal{C}_\infty \times_r *^k)$, $\mathcal{C}_\infty = \mathcal{C}_\infty(k; G)$, be the k -fold G -covering defined by the projection. For a based G -space X , define $w(X)$ to be the following composition.

$$(2.7) \quad w(X): \mathcal{C}_\infty(k; G)_+ \wedge_r X^k \xrightarrow{t(p')} Q_G((\mathcal{C}_\infty \times_r X^k \wedge [k])_+)/\Gamma \xrightarrow{Q_G(\pi_1)} Q_G(X).$$

This $w(X)$ is well defined as a G -homotopy class. For a G -map $f: X \rightarrow X'$, the diagram

$$\begin{array}{ccc}
(\mathcal{C}_\infty(k; G) \times X^k \times [k])/\Gamma & \xrightarrow{(1 \times f^k \times 1)/\Gamma} & (\mathcal{C}_\infty(k; G) \times (X')^k \times [k])/\Gamma \\
\downarrow & & \downarrow \\
\mathcal{C}_\infty(k; G) \times_r X^k & \xrightarrow{1 \times_r f^k} & \mathcal{C}_\infty(k; G) \times_r (X')^k
\end{array}$$

is a pullback diagram. Therefore w is an equivariant wreath operation because of the naturality of t .

By an argument parallel to [13], Chapter VII, we get the following lemmas.

LEMMA 2.8. (2.6) and (2.7) give a one to one correspondence between equivariant wreath operations and generalized equivariant transfers.

LEMMA 2.9. A generalized equivariant transfer t satisfies the axiom (T1) if and only if the associated equivariant wreath operation w satisfies the following axiom:

(W1) The diagram

$$\begin{array}{ccc}
\Sigma^\rho(\mathcal{C}_{\infty+} \wedge_r X^k) & \xrightarrow{\Sigma^\rho w(X)} & \Sigma^\rho Q_G(X) \\
\downarrow \phi & & \downarrow \text{adj } 1 \\
\mathcal{C}_{\infty+} \wedge_r (\Sigma^\rho X)^k & \xrightarrow{w(\Sigma^\rho X)} & Q_G(\Sigma^\rho X)
\end{array}$$

is commutative in $\mathcal{T}_{G*}$, where $(v \wedge [c \wedge (x_1, \dots, x_k)]) = [c \wedge (v \wedge x_1, \dots, v \wedge x_k)]$.

LEMMA 2.10. A generalized equivariant transfer t satisfies (T2) iff the associated equivariant wreath operation w satisfies the following axiom:

(W2) The diagram

$$\begin{array}{ccc}
(\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge X & & \\
i_0 \pi \swarrow & & \searrow p' \circ i_1 \\
Q_G(X) & \xleftarrow{w(X)} & \mathcal{C}_{\infty+} \wedge_r X^k
\end{array}$$

is commutative in $\mathcal{T}_{G*}$, where

$$i_1: ((\mathcal{C}_\infty \times_r [k]) \times X, (\mathcal{C}_\infty \times_r [k]) \times *) \rightarrow ((\mathcal{C}_\infty \times X^k \times [k])/\Gamma, (\mathcal{C}_\infty \times *^k \times [k])/\Gamma)$$

is given by $i_1([c, i], x) = [c, *, \dots, *, x, *, \dots, *, i]$ where x is in the i -th place, $\pi: ((\mathcal{C}_\infty \times_r [k]) \times X, (\mathcal{C}_\infty \times_r [k]) \times *) \rightarrow (X, *)$ is the projection to X and $i: X \rightarrow Q_G(X)$ is the canonical inclusion.

§ 3. Proof of Theorem 2.3

Define

$$\phi: \Sigma^\rho(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) \rightarrow \mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k$$

by

$$\phi(v \wedge [c \wedge (v_1 \wedge x_1, \dots, v_k \wedge x_k)]) = [c \wedge (v \wedge v_1 \wedge x_1, \dots, v \wedge v_k \wedge x_k)].$$

Let $\text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k)$ be the telescope of the sequence

$$\mathcal{C}_{\infty+} \wedge_r X^k \rightarrow \dots \rightarrow \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) \xrightarrow{\Omega^{n\rho} \text{adj } \phi} \Omega^{(n+1)\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k) \rightarrow \dots.$$

Define

$$\psi: \Sigma^\rho((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \rightarrow (\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{(n+1)\rho} X$$

by

$$\psi(v \wedge [c \wedge i] \wedge v' \wedge x) = [c \wedge i] \wedge v \wedge v' \wedge x.$$

Let $\text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X)$ be the telescope of the sequence

$$\begin{aligned} (\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge X &\rightarrow \dots \rightarrow \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \\ &\xrightarrow{\Omega^{n\rho} \text{adj } \psi} \Omega^{(n+1)\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{(n+1)\rho} X) \rightarrow \dots. \end{aligned}$$

The following diagram is commutative.

$$\begin{array}{ccc} \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) & \xrightarrow{\Omega^{n\rho}(p' \circ i_1)} & \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) \\ \downarrow \Omega^{n\rho}(\text{adj } \psi) & & \downarrow \Omega^{n\rho}(\text{adj } \phi) \\ \Omega^{(n+1)\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{(n+1)\rho} X) & \xrightarrow{\Omega^{(n+1)\rho}(p' \circ i_1)} & \Omega^{(n+1)\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k), \end{array}$$

which induces the G -map

$$\widetilde{p' \circ i_1}: \text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \longrightarrow \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k).$$

A G -map

$$\widetilde{i \circ \pi}: \text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \longrightarrow Q_G(X)$$

is defined from the following commutative diagram

$$\begin{array}{ccc} \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) & \xrightarrow{\Omega^{n\rho}(i \circ \pi)} & \Omega^{n\rho} Q_G(\Sigma^{n\rho} X) \\ \downarrow \Omega^{n\rho} \text{adj } \psi & & \downarrow Q_G(X) \\ \Omega^{(n+1)\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{(n+1)\rho} X) & \xrightarrow{\Omega^{(n+1)\rho}(i \circ \pi)} & \Omega^{(n+1)\rho} Q_G(\Sigma^{(n+1)\rho} X). \end{array}$$

DEFINITION 3.1. An *equivariant stable wreath operation* $\tilde{w}$ is a natural transformation from the functor $\text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho}(\))^k)$ to Q_G on $\mathcal{W}_{G*}$ such that the following diagram

$$(3.2) \quad \begin{array}{ccc} \text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) & & \\ \swarrow \widetilde{i \circ \pi} & & \searrow \widetilde{p' \circ i_1} \\ Q_G(X) & \xleftarrow{\tilde{w}(X)} & \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) \end{array}$$

is commutative in $\mathcal{T}_{G*}$.

Let w be an equivariant wreath operation satisfying (W1) and (W2). We construct an equivariant stable wreath operation $\tilde{w}$ from w . Though $w(X)$ is defined as a G -homotopy class, we denote a representative G -map also by the same notation with an abuse of notations. We will show later that the G -homotopy class $\tilde{w}(X)$ is independent of choice of representatives of $w(X)$.

Consider the diagram

$$\begin{array}{ccc} \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{n\rho} X)^k) & \xrightarrow{\Omega^{n\rho} w(\Sigma^{n\rho} X)} & \Omega^{n\rho} Q_G(\Sigma^{n\rho} X) \\ \downarrow \Omega^{n\rho} \text{adj } \phi & & \downarrow 1 \\ \Omega^{(n+1)\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{(n+1)\rho} X)^k) & \xrightarrow{\Omega^{(n+1)\rho} w(\Sigma^{(n+1)\rho} X)} & \Omega^{(n+1)\rho} Q_G(\Sigma^{(n+1)\rho} X). \end{array}$$

By (W1), there is a G -homotopy $H^{(n)'}: \Sigma^{\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{n\rho} X)^k) \wedge I_+ \rightarrow Q_G(\Sigma^{(n+1)\rho} X)$, $I=[0, 1]$, from $(\text{adj } 1) \circ \Sigma^{\rho} w(\Sigma^{n\rho} X)$ to $w(\Sigma^{(n+1)\rho} X)$. Then a G -homotopy $H_t^{(n)}$, $0 \leq t \leq 1$, from $\Omega^{n\rho} w(\Sigma^{n\rho} X)$ to $\Omega^{(n+1)\rho} w(\Sigma^{(n+1)\rho} X) \circ (\Omega^{n\rho} \text{adj } \phi)$ is defined by $H_t^{(n)} = \Omega^{n\rho}(\text{adj } H_t^{(n)'})$, where $H_t^{(n)'}(y) = H^{(n)'}(y \wedge t)$. Define

$$(3.3) \quad \tilde{w}(X): \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{n\rho} X)^k) \longrightarrow Q_G(X)$$

by

$$w(X)|_{\Omega^{(n+1)\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{(n+1)\rho} X)^k)} = \Omega^{(n+1)\rho} w(\Sigma^{(n+1)\rho} X)$$

and

$$w(X)|_{\Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{n\rho} X)^k) \wedge I_+} = H^{(n)},$$

where $H^{(n)}(y \wedge t) = H_t^{(n)}(y)$. We show that (3.2) is G -homotopy commutative for $\tilde{w}(X)$ defined in this way and prove in (3.5) that $\widetilde{p' \circ i_1}$ is a G -homotopy equivalence. Then $\tilde{w}(X)$ is G -homotopic to $(\widetilde{i \circ \pi}) \circ (\widetilde{p' \circ i_1})^{-1}$, where $(\widetilde{p' \circ i_1})^{-1}$ is a G -homotopy inverse of $\widetilde{p' \circ i_1}$ and the G -homotopy class of $\tilde{w}(X)$ is independent of representatives of $w(X)$ and of choice of G -homotopy $H^{(n)'}$.

There is a G -homotopy $F^{(0)'}: (\mathcal{C}_{\infty+} \wedge_{\Gamma}[k]_+) \wedge X \wedge I_+ \rightarrow Q_G(X)$ from $i \circ \pi$ to $w(X) \circ p' \circ i_1$ by (W2). Assume that we get a G -homotopy

$$F^{(n)'}: (\mathcal{C}_{\infty+} \wedge_{\Gamma}[k]_+) \wedge \Sigma^{n\rho} X \wedge I_+ \longrightarrow Q_G(\Sigma^{n\rho} X)$$

from $i \circ \pi$ to $w(\Sigma^{n\rho} X) \circ p' \circ i_1$ in the following diagram

$$(3.4) \quad \begin{array}{ccc} & (\mathcal{C}_{\infty+} \wedge_{\Gamma}[k]_+) \wedge \Sigma^{n\rho} X & \\ \swarrow i \circ \pi & & \searrow p' \circ i_1 \\ Q_G(\Sigma^{n\rho} X) & \xleftarrow{w(\Sigma^{n\rho} X)} & \mathcal{C}_{\infty+} \wedge_{\Gamma}(\Sigma^{n\rho} X)^k. \end{array}$$

Then we define $F^{(n+1)'}$ as follows:

$$F^{(n+1)'}(y \wedge t) = \begin{cases} (adj\ 1)(\Sigma^\rho F^{(n)'}(\psi^{-1}(y) \wedge 2t)), & 0 \leq t \leq 1/2, \\ H^{(n)'}(\Sigma^\rho(p' \circ i_1) \circ \psi^{-1}(y) \wedge (2t-1)), & 1/2 \leq t \leq 1. \end{cases}$$

There is a G -homotopy $F^{(n)}: \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+ \rightarrow Q_G(X)$ defined by $F_t^{(n)} = \Omega^{n\rho} F_t^{(n)'}$, $0 \leq t \leq 1$. Consider the following diagram

$$\begin{array}{ccccc} & \Omega^{(n+1)\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{(n+1)\rho} X) & & & \\ & \uparrow \Omega^{n\rho} adj\ \psi & & & \\ & \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) & & & \\ \Omega^{(n+1)\rho}(i \circ \pi) \swarrow & \downarrow \Omega^{n\rho}(i \circ \pi) & \xleftarrow{\Omega^{n\rho} w(\Sigma^{n\rho} X)} & \downarrow \Omega^{n\rho}(p' \circ i_1) & \searrow \Omega^{(n+1)\rho}(p' \circ i_1) \\ \Omega^{n\rho} Q_G(\Sigma^{n\rho} X) & & \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) & & \\ \swarrow = & \xleftarrow{\Omega^{(n+1)\rho} w(\Sigma^{(n+1)\rho} X)} & \searrow \Omega^{n\rho} adj\ \phi & & \\ \Omega^{(n+1)\rho} Q_G(\Sigma^{(n+1)\rho} X) & & \Omega^{(n+1)\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k) & & \end{array}$$

The inner triangle is G -homotopy commutative by $F^{(n)}$ and the outer one is so by $F^{(n+1)}$. A G -homotopy

$$K^{(n)}: \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+ \wedge I_+ \longrightarrow Q_G(X)$$

from $F^{(n)}$ to $F^{(n+1)} \circ (\Omega^{n\rho} adj\ \psi \wedge 1)$ is defined as follows. Let

$$K^{(n)'}: \Sigma^\rho((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+ \wedge I_+ \longrightarrow Q_G(\Sigma^{(n+1)\rho} X)$$

be a G -homotopy from $(adj\ 1) \circ \Sigma^\rho F^{(n)'} to $F^{(n+1)'} \circ (\psi \wedge 1)$ defined by$

$$K^{(n)'}(y \wedge t \wedge u) = \begin{cases} (adj\ 1)(\Sigma^\rho F^{(n)'}(y \wedge (2t/(2-u))))), & 0 \leq t \leq 1-(u/2), \\ H^{(n)'}(\Sigma^\rho(p' \circ i_1)(y) \wedge (2t+u-2)), & 1-(u/2) \leq t \leq 1. \end{cases}$$

Put $K_{t,u}^{(n)} = \Omega^{n\rho} adj\ K_{t,u}^{(n)'}$ and $K^{(n)}(y \wedge t \wedge u) = K_{t,u}^{(n)}(y)$. Define

$$K: \text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+ \longrightarrow Q_G(X)$$

by $K([y \wedge u] \wedge t) = K^{(n)}(y \wedge t \wedge u)$ for $[y \wedge u] \wedge t \in \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+ \wedge I_+ \subset \text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n\rho} X) \wedge I_+$. Because of

$$K^{(n)}(y \wedge t \wedge 1) = F^{(n+1)}((\Omega^{n\rho} adj\ \psi)(y) \wedge t) = K^{(n+1)}((\Omega^{n\rho} adj\ \psi)(y) \wedge t \wedge 0),$$

K is well defined on the telescope.

$$K([y \wedge u] \wedge 0) = K^{(n)}(y \wedge 0 \wedge u) = F^{(n)}(y \wedge 0) = \Omega^{n\rho}(i \circ \pi)(y)$$

and

$$K([y \wedge u] \wedge 1) = K^{(n)}(y \wedge 1 \wedge u) = H^{(n)}(\Omega^{n\rho}(p' \circ i_1)(y) \wedge u).$$

Therefore K is a G -homotopy from $\widetilde{i \circ \pi}$ to $w(X) \circ (\widetilde{p' \circ i_1})$.

PROPOSITION 3.5. $\widetilde{p' \circ i_1}$ is a G -homotopy equivalence.

PROOF. It is sufficient to prove that

$$(\widetilde{p' \circ i_1})^H: (\text{Tel } \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X))^H \longrightarrow (\text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k))^H$$

gives a one to one correspondence of components and homotopy equivalence between each component for any subgroup H of G (Bredon [5], Chapter II, (5.5)).

By the equivariant suspension theorem (Murayama [10], Theorem 2.9; Kakutani-Murayama [8], Theorem 3.6; Namboodiri [11], Theorem 2.3),

$$\Sigma_*^{\rho}: [\Sigma^{n\rho}, (\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X]^H \longrightarrow [\Sigma^{(n+1)\rho}, \Sigma^{\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X)]^H$$

is an isomorphism for large n . Then

$$(\Omega^{n\rho} \text{adj } \psi)^H: (\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X))^H \longrightarrow (\Omega^{(n+1)\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{(n+1)\rho} X))^H$$

gives a one to one correspondence of components for large n . Therefore

$$(\text{Tel } (\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X)))_{[f,t]}^H \simeq \text{Tel } ((\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X)))_{[f]}^H,$$

where $f \in \Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X)$, $t \in I$, and $(\text{Tel } (\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X)))_{[f,t]}^H$ and $(\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X))_{[f]}^H$ are connected components containing $[f, t]$ and $[f]$. Then

$$\begin{aligned} (\widetilde{p' \circ i_1})_*^H: [\Sigma^l, \text{Tel } (\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X))]^H \\ \longrightarrow [\Sigma^l, \text{Tel } (\Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k))]^H \end{aligned}$$

is an isomorphism iff

$$\begin{aligned} \text{colim}_n (p' \circ i_1)_*^H: \text{colim}_n [\Sigma^l, (\Omega^{n\rho}((\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X))]^H \\ \longrightarrow \text{colim}_n [\Sigma^l, (\Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k))]^H \end{aligned}$$

is so, which is equivalent to saying that

$$\begin{aligned} \text{colim}_n (p' \circ i_1)_*: \text{colim}_n [\Sigma^{n\rho} \wedge \Sigma^l \wedge G/H_+, (\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X]^G \\ \longrightarrow \text{colim}_n [\Sigma^{n\rho} \wedge \Sigma^l \wedge G/H_+, \mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k]^G \end{aligned}$$

is an isomorphism. We will prove that

$$(3.6) \quad \begin{aligned} (p' \circ i_1)_*: [\Sigma^{n\rho} \wedge \Sigma^l \wedge G/H_+, (\mathcal{C}_{\infty+} \wedge_{\Gamma} [k]_+) \wedge \Sigma^{n\rho} X]^G \\ \longrightarrow [\Sigma^{n\rho} \wedge \Sigma^l \wedge G/H_+, \mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k]^G \end{aligned}$$

is an isomorphism for large n . To show this it is sufficient to prove that

$$(p' \circ i_!)^K: ((\mathcal{C}_{\infty+} \wedge_r [k]_+) \wedge \Sigma^{n_p} X)^K \rightarrow (\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n_p} X)^k)^K$$

is a $(\dim(\Sigma^{n_p} \wedge \Sigma^l \wedge G/H_+)^K + 1)$ -equivalence for all subgroups K of G (Mura-
yama [10], Proposition 2.3). Before proving this we prepare a number of
lemmas.

We say $\tau' \in \text{Hom}(K, \Gamma)$ is conjugate to $\tau \in \text{Hom}(K, \Gamma)$ if $\tau'(a) = \gamma^{-1} \tau(a) \gamma$
for a suitable $\gamma \in \Gamma$ and for all $a \in K$. Then we denote $\tau \sim \tau'$. Let $(\tau) \in$
 $\text{Hom}(K, \Gamma)/\sim$ be the conjugate class of τ .

LEMMA 3.7. *Let $X \in \mathcal{W}_{G*}$ and K be a subgroup of G .*

- i) $(\mathcal{C}_{\infty} \times_r ([k]_+ \wedge X))^K = \coprod_{(\tau)} (\coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')} \times ([k]_+^{(\tau')} \wedge X^K)) / \Gamma,$
- ii) $(\mathcal{C}_{\infty} \times_r X^K)^K = \coprod_{(\tau)} (\coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')} \times (X^K)^{K(\tau')}) / \Gamma,$
- iii) $(\mathcal{C}_{\infty} / \Gamma)^K = \coprod_{(\tau)} (\coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')}) / \Gamma,$

where (τ) runs over the conjugate classes $\text{Hom}(K, \Gamma)/\sim$ and $K(\tau') = \{(a, \tau'(a)) \in K \times \Gamma \mid a \in K\}$.

PROOF. We will give the proof only for i) as the proofs of ii) and iii)
are similar and easier. Let $\pi: \mathcal{C}_{\infty} \times ([k]_+ \wedge X) \rightarrow \mathcal{C}_{\infty} \times_r ([k]_+ \wedge X)$ be the pro-
jection. It is sufficient to prove the following two facts:

- (i) $\pi^{-1}((\mathcal{C}_{\infty} \times_r ([k]_+ \wedge X))^K) = \coprod_{\tau \in \text{Hom}(K, \Gamma)} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+^{(\tau)} \wedge X^K),$
- (ii) $(\coprod_{\tau \in \text{Hom}(K, \Gamma)} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+^{(\tau)} \wedge X^K))$ is closed under the Γ -action.

$\coprod_{\tau \in \text{Hom}(K, \Gamma)} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+^{(\tau)} \wedge X^K) = \coprod_{\tau \in \text{Hom}(K, \Gamma)} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+ \wedge X^K)$ because $\mathcal{C}_{\infty}$
is Γ -free. $\pi((c, i \wedge x)) \in (\mathcal{C}_{\infty} \times_r ([k]_+ \wedge X))^K$ iff there is a homomorphism $\tau: K$
 $\rightarrow \Gamma$ such that $(ac, i \wedge ax) = (c\tau(a), \tau(a)^{-1}(i) \wedge x)$ for all $a \in K$, which is equiva-
lent to saying that $c \in \mathcal{C}_{\infty}^{K(\tau)}$, $i \in [k]_+^{(\tau)}$ and $x \in X^K$. Therefore

$$\pi^{-1}((\mathcal{C}_{\infty} \times_r ([k]_+ \wedge X))^K) \subset \coprod_{\tau \in \text{Hom}(K, \Gamma)} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+^{(\tau)} \wedge X^K).$$

The converse inclusion is obvious and (i) is proved. Let $(c, i \wedge x) \in \mathcal{C}_{\infty}^{K(\tau)} \times$
 $([k]_+^{(\tau)} \wedge X^K)$ and $\gamma \in \Gamma$. $(c, i \wedge x)\gamma \in \mathcal{C}_{\infty}^{K(\tau')} \times ([k]_+^{(\tau')} \wedge X^K)$ for a $\tau' \in \text{Hom}(K, \Gamma)$
by (i). Then $a(c\gamma) = c\tau(a)\gamma = (c\gamma)(\gamma^{-1}\tau(a)\gamma)$ and $\tau'(a) = \gamma^{-1}\tau(a)\gamma$ for all $a \in K$.
Therefore $\tau \sim \tau'$ and (ii) is proved. $\square$

Let $r = |[k]_+^{(\tau)}|$, the number of the fixed points, for $\tau \in \text{Hom}(K, \Gamma)$ and a
subgroup K of G . We assume $[k]_+^{(\tau)} = \{k-r+1, \dots, k\}$ for simplicity. Then
 $\tau = \iota \circ \tilde{\tau}$ for a homomorphism $\tilde{\tau}: K \rightarrow \mathfrak{S}_{k-r}$ and $\iota: \mathfrak{S}_{k-r} \rightarrow \mathfrak{S}_k$, where an element
 $\sigma \in \mathfrak{S}_{k-r}$ is considered as a permutation of k letters which leaves the last r
letters fixed. Put $\Gamma' = \tau(K)$. Let $[k-r]/\Gamma' = \{\Gamma' \cdot i_1, \dots, \Gamma' \cdot i_s\}$ be the orbits
and Γ'_l the isotropy group at i_l , $1 \leq l \leq s$.

LEMMA 3.8. *Let $X \in \mathcal{W}_{G*}$. Then*

$$(X^K)^{K(\tau)} \approx X^{\tau^{-1}(\Gamma'_1)} \times \dots \times X^{\tau^{-1}(\Gamma'_s)} \times (X^K)^r.$$

PROOF. $(X^k)^{K(\tau)} \approx (X^{k-r})^{K(\tau)} \times (X^r)^K$ and we prove

$$(X^{k-r})^{K(\tau)} = X^{\tau^{-1}(\Gamma)\hat{\imath}} \times \cdots \times X^{\tau^{-1}(\Gamma)\hat{\imath}}.$$

$K(\hat{\tau})$ is isomorphic to K by $a \mapsto (a, \hat{\tau}(a))$, $a \in K$, and $\text{Ker } \tau$ is considered as a subgroup of $K(\hat{\tau})$. There are isomorphisms $\alpha: K(\hat{\tau})/\text{Ker } \tau \rightarrow \Gamma'$ and $\beta: K/\text{Ker } \tau \rightarrow \Gamma'$ defined by $\alpha([a, \hat{\tau}(a)]) = \hat{\tau}(a)$ and $\beta([a]) = \hat{\tau}(a)$. We identify Γ' with $K(\hat{\tau})/\text{Ker } \tau$ by α . Then $(X^{k-r})^{K(\tau)} = ((X^{k-r})^{\text{Ker } \tau})^{\Gamma'} = ((X^{\text{Ker } \tau})^{k-r})^{\Gamma'}$, where Γ' acts on $(X^{k-r})^{\text{Ker } \tau}$ by $\gamma(x_1, \dots, x_{k-r}) = (\beta^{-1}(\gamma)x_{\gamma^{-1}(1)}, \dots, \beta^{-1}(\gamma)x_{\gamma^{-1}(k-r)})$ for $\gamma \in \Gamma'$. There is a homeomorphism

$$((X^{\text{Ker } \tau})^{k-r})^{\Gamma'} \approx ((X^{\text{Ker } \tau})^{|\Gamma' \cdot i_1|})^{\Gamma'} \times \cdots \times ((X^{\text{Ker } \tau})^{|\Gamma' \cdot i_s|})^{\Gamma'}$$

by a permutation. Let $\Gamma'/\Gamma'_i = \gamma_{i,1} \cdot \Gamma'_i \amalg \cdots \amalg \gamma_{i,n(i)} \cdot \Gamma'_i$, $\gamma_{i,1} = 1$. A homeomorphism $f: X^{\tau^{-1}(\Gamma)\hat{\imath}} \rightarrow ((X^{\text{Ker } \tau})^{|\Gamma' \cdot i_1|})^{\Gamma'}$ is defined by $f(x) = (\gamma_{i,1}x, \dots, \gamma_{i,n(i)}x)$. The inverse is given by the first projection. Therefore the desired homeomorphism is obtained. $\square$

LEMMA 3.9. *The inclusion map*

$$\begin{aligned} \mathcal{C}_{\infty}^{K(\tau)} \times ([k]_+^{\tau(K)} \wedge (\Sigma^{n\rho} X)^K) &\rightarrow \mathcal{C}_{\infty}^{K(\tau)} \times ((\Sigma^{n\rho} X)^k)^{K(\tau)}, \\ (c, i \wedge y) &\rightarrow (c, *, \dots, *, y, *, \dots, *) \end{aligned}$$

with y in the i -th place, is a $(2n(|G/H|+1)-1)$ -equivalence.

PROOF. It is sufficient to prove that $[k]_+^{\tau(K)} \wedge (\Sigma^{n\rho} X)^K \rightarrow ((\Sigma^{n\rho} X)^k)^{K(\tau)}$ is a $(2n(|G/H|+1)-1)$ -equivalence because $\mathcal{C}_{\infty}^{K(\tau)}$ is contractible. The composition of this inclusion and the homeomorphism of Lemma 3.8 is the map

$$(*, i): [k]_+^{\tau(K)} \wedge (\Sigma^{n\rho} X)^K \rightarrow ((\Sigma^{n\rho} X)^{\tau^{-1}(\Gamma)\hat{\imath}} \times \cdots \times (\Sigma^{n\rho} X)^{\tau^{-1}(\Gamma)\hat{\imath}}) \times ((\Sigma^{n\rho} X)^K)^r,$$

where $*$ is the constant map to the base point, $r = |[k]^{\tau(K)}|$ and

$$i: [k]_+^{\tau(K)} \wedge (\Sigma^{n\rho} X)^K \approx (\Sigma^{n\rho} X)^K \vee \cdots \vee (\Sigma^{n\rho} X)^K \subset ((\Sigma^{n\rho} X)^K)^r$$

is the canonical inclusion. i is a $(4n|G/K|-1)$ -equivalence because $\dim(\Sigma^{n\rho})^K = 2n|G/K|$. $\tau^{-1}(\Gamma)\hat{\imath} \not\subseteq K$ because $[k-r]^{\tau(K)} = \emptyset$ and

$$\dim(\Sigma^{n\rho})^{\tau^{-1}(\Gamma)\hat{\imath}} = 2n|G/\tau^{-1}(\Gamma)| \geq 2n(|G/K|+1).$$

Therefore $(\Sigma^{n\rho} X)^{\tau^{-1}(\Gamma)\hat{\imath}} \times \cdots \times (\Sigma^{n\rho} X)^{\tau^{-1}(\Gamma)\hat{\imath}}$ is $(2n(|G/K|+1)-1)$ -connected and $(*, i)$ is a $(2n(|G/K|+1)-1)$ -equivalence. $\square$

Put

$$\begin{aligned} L(\tau) &= \coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')} \times ((\Sigma^{n\rho} X)^k)^{K(\tau')}, \\ M(\tau) &= \coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')} \times ([k]_+^{\tau'(K)} \wedge (\Sigma^{n\rho} X)^K), \\ N(\tau) &= \coprod_{\tau' \in (\tau)} \mathcal{C}_{\infty}^{K(\tau')} \end{aligned}$$

for $(\tau) \in \text{Hom}(K, \Gamma)/\sim$. Let $\text{Hom}(K, \Gamma)/\sim = \{(\tau_1), \dots, (\tau_s)\}$. Then

$$\begin{aligned} (\mathcal{C}_{\infty+} \wedge_{\Gamma} ([k]_+ \wedge \Sigma^{n\rho} X))^K &= \bigvee_{i=1}^s (M(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma), \\ (\mathcal{C}_{\infty+} \wedge_{\Gamma} (\Sigma^{n\rho} X)^k)^K &= \bigvee_{i=1}^s (L(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma), \end{aligned}$$

and $L(\tau)/\Gamma$, $M(\tau)/\Gamma$, $N(\tau)/\Gamma$, $(M(\tau)/\Gamma)/(N(\tau)/\Gamma)$ and $(L(\tau)/\Gamma)/(N(\tau)/\Gamma)$ are connected.

LEMMA 3.10.

- i) $(L(\tau)/\Gamma)/(M(\tau)/\Gamma)$ is $(2n(|G/K|+1)-1)$ -connected,
- ii) $(M(\tau)/\Gamma)/(N(\tau)/\Gamma)$ is $(2n|G/K|-1)$ -connected,
- iii) $(L(\tau)/\Gamma)/(N(\tau)/\Gamma)$ is $(2n|G/K|-1)$ -connected.

PROOF. i) $L(\tau)$ and $M(\tau)$ are Γ -free and the projection maps $L(\tau) \rightarrow L(\tau)/\Gamma$ and $M(\tau) \rightarrow M(\tau)/\Gamma$ are principal Γ -bundles. The inclusion map $M(\tau) \rightarrow L(\tau)$ is a $(2n(|G/K|+1)-1)$ -equivalence by Lemma 3.9 and $M(\tau)/\Gamma \rightarrow L(\tau)/\Gamma$ is so by the long exact sequence of homotopy groups for fibrations and five lemma. Then $\pi_m((L(\tau)/\Gamma)/(M(\tau)/\Gamma)) \cong \pi_m(L(\tau)/\Gamma, M(\tau)/\Gamma) = 0$, $1 \leq m \leq 2n(|G/K|+1)-1$, by Blakers-Massey's theorem.

ii) is obtained by $(M(\tau)/\Gamma)/(N(\tau)/\Gamma) = (\bigvee_{\tau' \in (\tau)} \mathcal{C}_{\infty+} \wedge ([k]_+^{\tau'(K)} \wedge (\Sigma^{n\rho} X)^K))/\Gamma = (\Sigma^{n\rho})^K \wedge (\bigvee_{\tau' \in (\tau)} \mathcal{C}_{\infty+} \wedge ([k]_+^{\tau'(K)} \wedge X^K))/\Gamma$.

iii) Each component of $L(\tau)$ is $(2n|G/K|-1)$ -connected by Lemma 3.8 and the proof is similar to i). $\square$

LEMMA 3.11. *The inclusion map $(M(\tau)/\Gamma)/(N(\tau)/\Gamma) \rightarrow (L(\tau)/\Gamma)/(N(\tau)/\Gamma)$ is a $(2n(|G/K|+1)-2)$ -equivalence.*

PROOF. By Lemma 3.10, ii), iii) and the Blakers-Massey's theorem,

$$\pi_m((L(\tau)/\Gamma)/(N(\tau)/\Gamma), (M(\tau)/\Gamma)/(N(\tau)/\Gamma)) \cong \pi_m((L(\tau)/\Gamma)/(M(\tau)/\Gamma))$$

for $1 \leq m \leq 4n|G/K|-2$ and it is equal to 0 for $1 \leq m \leq 2n(|G/K|+1)-2$ by Lemma 3.10, i). $\square$

LEMMA 3.12. *The inclusion map*

$$\bigvee_{i=1}^s (M(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma) \rightarrow \bigvee_{i=1}^s (L(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma)$$

is a $(2n(|G/K|+1)-2)$ -equivalence.

PROOF. The case $s=1$ is proved in Lemma 3.11. Assume the statement is proved for $s-1$. Put

$$\begin{aligned} X_1 &= \bigvee_{i=1}^{s-1} (M(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma), & X_2 &= (M(\tau_s)/\Gamma)/(N(\tau_s)/\Gamma), \\ Y_1 &= \bigvee_{i=1}^{s-1} (L(\tau_i)/\Gamma)/(N(\tau_i)/\Gamma) & \text{and} & & Y_2 &= (L(\tau_s)/\Gamma)/(N(\tau_s)/\Gamma). \end{aligned}$$

Then X_i and Y_i , $i=1, 2$, are $(2n|G/K|-1)$ -connected and $\pi_m(X_1 \vee X_2) \cong \pi_m(X_1) + \pi_m(X_2)$ and $\pi_m(Y_1 \vee Y_2) \cong \pi_m(Y_1) + \pi_m(Y_2)$ for $1 \leq m \leq 4n|G/K|-2$. Therefore the statement is valid for s by induction. $\square$

By the above lemma, (3.6) is an isomorphism for $2n \geq l$ and this completes the proof of Proposition 3.5.

COROLLARY 3.13. *An equivariant stable wreath operation exists uniquely.*

PROOF OF THEOREM 2.3. We proved that an equivariant wreath operation satisfying (W1) and (W2) defines an equivariant stable wreath operation in (3.3). Conversely an equivariant stable wreath operation $\tilde{w}$ defines an equivariant wreath operation w satisfying (W1) and (W2) by

$$(3.14) \quad w(X): \mathcal{C}_{\infty+} \wedge_r X^k \subset \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k) \xrightarrow{\tilde{w}(X)} Q_G(X).$$

w satisfies (W2) clearly. To prove that it satisfies (W1) we show the diagram

$$\begin{array}{ccc} \Sigma^\rho(\mathcal{C}_{\infty+} \wedge_r X^k) \subset \Sigma^\rho(\text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k)) & \xrightarrow{\Sigma^\rho \tilde{w}(X)} & \Sigma^\rho Q_G(X) \\ \downarrow \phi & \downarrow h & \downarrow \text{adj } 1 \\ \mathcal{C}_{\infty+} \wedge_r (\Sigma^\rho X)^k \subset \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k) & \xrightarrow{\tilde{w}(\Sigma^\rho X)} & Q_G(\Sigma^\rho X) \end{array}$$

is commutative in $\mathcal{T}_{G*}$, where

$$h(v \wedge [f \wedge t]) = [f' \wedge t], \quad v \in \Sigma^\rho, \quad [f \wedge t] \in \text{Tel } \Omega^{n\rho}(\mathcal{C}_{\infty+} \wedge_r (\Sigma^{n\rho} X)^k)$$

and $f': \Sigma^{n\rho} \rightarrow \mathcal{C}_{\infty+} \wedge_r (\Sigma^{(n+1)\rho} X)^k$ is defined by $f'(v) = \phi(v \wedge f(v))$. The left square is commutative by definition. Put $\tilde{w}'(X) = \Omega^\rho \tilde{w}(\Sigma^\rho X) \circ (\text{adj } h)$. It is easily checked that $\tilde{w}'$ is an equivariant stable wreath operation. Therefore $\tilde{w}' = \tilde{w}$ and the right square of the above diagram is commutative in $\mathcal{T}_{G*}$.

The correspondences $w \mapsto \tilde{w}$ of (3.3) and $\tilde{w} \mapsto w$ of (3.14) are inverses to each other and equivariant wreath operation satisfying (W1) and (W2) exists uniquely. Then an equivariant transfer on Cov_G^r exists uniquely by Lemmas 2.8, 2.9 and 2.10. $\square$

The equivariant transfer of $p: (X, A) \rightarrow (Y, B) \in \text{Cov}_G^r$ may be described explicitly as the following composition.

$$t(p): (Y, B) \xrightarrow{T/G} \mathcal{C}_{\infty+} \wedge_r (X/A)^k \subset \mathcal{C}_{\infty+} \wedge_r Q_G(X/A)^k \xrightarrow{\theta_{\infty, k}} Q_G(X/A).$$

By a routine argument we obtain

PROPOSITION 3.15. i) *Let $p: (X, A) \rightarrow (Y, B)$ be a k -fold G -covering, $(X, A) = (X_1, A_1) \amalg (X_2, A_2)$ and $p_i = p|_{(X_i, A_i)}$ are k_i -fold G -coverings, $i=1, 2$. The*

diagram

$$\begin{array}{ccc} Y/B & \xrightarrow{t(p)} & Q_G(X/A) = Q_G(X_1/A_1 \vee X_2/A_2) \\ & \searrow t(p_1) & \downarrow c \\ & & Q_G(X_1/A_1) \end{array}$$

is commutative in $\mathcal{T}_{G*}$, where c is induced by the collapsing map $X_1/A_1 \vee X_2/A_2 \rightarrow X_1/A_1$.

ii) Let $p_1: (X, A) \rightarrow (Y, B)$ be a k_1 -fold G -covering and $p_2: (Y, B) \rightarrow (W, D)$ a k_2 -fold G -covering. The diagram

$$\begin{array}{ccc} W/D & \xrightarrow{t(p_2)} Q_G(Y/B) \xrightarrow{Q_G(t(p_1))} & Q_G Q_G(X/A) \\ & \searrow t(p_2 \circ p_1) & \downarrow \eta \\ & & Q_G(X/A) \end{array}$$

is commutative in $\mathcal{T}_{G*}$, where η is the canonical infinite loop G -map.

§ 4. The equivariant Becker-Gottlieb transfer and Atiyah transfer

Let $\pi: X \rightarrow Y$ be a fiber bundle with G -action in the sense of [14] with compact fiber over a finite G -complex Y . The equivariant Becker-Gottlieb transfer $\tau(\pi): \Sigma^V(Y_+) \rightarrow \Sigma^V(X_+)$ for π is defined by Nishida [12]. Especially, k -fold G -covering $p: X \rightarrow Y$ over a finite G -complex Y has such an equivariant Becker-Gottlieb transfer $\tau(p)$. We show that $\tau(p)$ is an adjoint of $t(p)$. Let $\tilde{p}: Z \rightarrow Y$ be a principal (G, Γ) -bundle associated with p . Define $Z \times_r [k] \subset Z \times_r C^k$ by $[z, i] \rightarrow [z, 0, \dots, 0, 1, 0, \dots, 0]$ with 1 in the i -th place. $Z \times_r C^k$ can be embedded in $Y \times V$ as a subbundle for a representation V of G because $Z \times_r C^k$ is a complex G -vector bundle over a finite G -complex Y . Let α be the composition map $X \approx Z \times_r [k] \subset Z \times_r C^k \subset Y \times V \xrightarrow{pr_2} V$. Then there is an embedding $i: X \times DV \rightarrow Y \times DV$ defined by $i(x, v) = (p(x), \alpha(x) + \varepsilon v)$ for small $\varepsilon > 0$. $\tau(p)$ is defined by

$$\tau(p)(v \wedge y) = \begin{cases} v' \wedge x, & \text{if } y = p(x) \text{ and } v = \alpha(x) + \varepsilon v', \\ *, & \text{otherwise.} \end{cases}$$

V may be replaced by $n\rho$ for large n . Let

$$c_{z(i)}: (\dot{D}\rho)^n \rightarrow (\dot{D}\rho)^n, \quad z \in Z,$$

be the map defined by $c_{z(i)}(v) = \alpha(z(i)) + \varepsilon v$, where we identify $(\dot{D}\rho)^n$ with $\dot{D}(n\rho)$. The map $d: Z \rightarrow \mathcal{C}_{\infty+}(k; G)$ defined by $d(z) = (c_{z(1)}, \dots, c_{z(k)})$ is a $G \times \Gamma$ -map and we use it to define $t(p)$. Let $y \in Y$ correspond to $[z]$ by $Y \approx_G Z/\Gamma$. Then

$t(p)(y): \Sigma^{m\rho} \rightarrow \Sigma^{n\rho} \wedge X_+$ is defined by

$$t(p)(y)(v) = \begin{cases} c_{z(i)}^{-1}(v) \wedge z(i), & \text{if } v \in \text{Im } c_{z(i)}, \\ *, & \text{otherwise.} \end{cases}$$

Therefore the adjoint of $t(p)$ is equal to $\tau(p)$.

Next we observe that the Atiyah transfer for K_G -theory is the induced homomorphism in K_G -theory induced from the adjoint of the equivariant transfer. Let $\tilde{h}_G^*$ be a reduced G -cohomology theory represented by an Ω - G -spectrum $E = \{E_n\}$, $E_n \simeq_G \Sigma^\rho E_{n+1}$, (Araki-Murayama [1], Theorem 3.4). Let $p: X \rightarrow Y$ be as above. Then $t(p): Y_+ \rightarrow \Omega^{m\rho} \Sigma^{m\rho}(X_+)$ for some m . Define $\text{tr}[\alpha] \in [Y_+, E_n]^G$ for $[\alpha] \in [X_+, E_n]^G$ as the composition

$$Y_+ \xrightarrow{T/\Gamma} \mathcal{C}_m(k; G)_+ \wedge_{\Gamma} (X_+)^k \xrightarrow{1_+ \wedge \Gamma \alpha^k} \mathcal{C}_m(k; G)_+ \wedge_{\Gamma} (E_n)^k \xrightarrow{\theta_{m,k}} E_n.$$

By the same proof as in [6], Proposition 1.4, we get

LEMMA 4.1. *The following diagram is commutative.*

$$\begin{array}{ccc} [X_+, E_n]^G & \xrightarrow{\text{tr}} & [Y_+, E_n]^G \\ \downarrow \cong & & \downarrow \cong \\ [\Sigma^{m\rho}(X_+), E_{n+m}]^G & \xrightarrow{(\text{adj } t(p))^*} & [\Sigma^{m\rho}(Y_+), E_{n+m}]^G \end{array}$$

Let $\pi: E \rightarrow X$ be a complex G -vector bundle and p_*E be the direct image bundle (Atiyah [2], p. 26). The fiber $(p_*E)_y$ is defined as the direct sum $\bigoplus_x E_x$ where $x \in p^{-1}(y)$. G acts on p_*E obviously and this induces the Atiyah transfer $p_*: K_G(X) \rightarrow K_G(Y)$. Let BU_G be the 0-th term of Ω - G -spectrum representing the reduced K_G -theory. To show $(\text{adj } t(p))^* = p_*$, it is sufficient to prove that p_* is represented by $\text{tr}[\alpha]$ when π is represented by $\alpha: X \rightarrow BU_G$. $1 \times \pi^k: \mathcal{C}_\infty(k; G) \times E^k \rightarrow \mathcal{C}_\infty(k; G) \times X^k$ is a $G \times \Gamma$ -vector bundle and the base space is Γ -free. Therefore $1 \times \pi^k$ is a G -vector bundle. An isomorphism $\phi: (T/\Gamma)^*(1 \times \pi^k) \rightarrow p_*E$ is defined by $\phi([z], [d(z), e_1, \dots, e_k]) = (e_1, \dots, e_k) \in (p_*E)_y$, $y = [z]$, and hence $(\text{adj } t(p))^* = p_*$.

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