Joule heat welding of thin wires to thin films

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Abstract
To effectively use micro or nano materials and create advanced materials systems from them, welding techniques for such small scale materials are vital. For this purpose, the tips of two thin wires were welded together with Joule heat and the conditions required to achieve it was discovered experimentally. In this paper, the Joule heat welding of thin wires to thin films deposited on substrates is described. To derive the conditions required to achieve this, a heat conduction model for a cylindrical element in the film located at the contact between the wire and the film is considered and the critical thickness of the film above which the welding conditions are independent of film thickness is derived analytically. The validity of the heat conduction model was experimentally verified by welding 0.8 μm diameter Pt wires onto thin Pt films on electrically-insulated substrates. The experiments are performed for various combinations of the length of the wire and the thickness of the films. The current required for welding the wires onto the films having the thickness over the critical thickness are investigated and the conditions required for the welding are found to be coincident with those for welding the tips of two wires with Joule heat.

Key words: Joule heat, Welding, Thin wire, Thin film, Heat conduction

1. Introduction

There is a huge demand for manufacturing technologies that can be used to create advanced micro or nano material systems. These systems are usually composed of many components that need to be assembled into a single system. Welding is one of the emerging technologies for such purposes (Tan, et al., 2004; Xu, et al., 2005; Kim, et al., 2005; Hu, et al., 2012; Lu, et al., 2010; Tohmyoh, et al., 2007; Hirayama, et al., 2001; Dong, et al., 2007; Jin, et al., 2007; Peng, et al., 2009; Zhao, et al., 2013). Of the available techniques for joining small scale materials together, that utilizing Joule heat can be used in various situations to join micro or nanowires (Tohmyoh, et al., 2007) or carbon nanotubes (Hirayama, et al., 2001; Dong, et al., 2007; Jin, et al., 2007), etc. Joule heat has also been used to solder nanowires (Peng, et al., 2009) and to structurally modify thin wires (Hummelgård, et al., 2010; Tohmyoh and Ishihara, 2013).

Because the Joule heat technique requires only a current to be supplied to the welding point, it has some potentially important advantages in constructing materials systems. For example, provided that a current can be supplied there, materials can be welded together at points hidden from view that may be inaccessible to other techniques. Although welding techniques utilizing electron (Tan, et al., 2004; Xu, et al., 2005) or laser beams (Kim, et al., 2005; Hu, et al., 2012) are reliable, such techniques can hardly be applied to such points because the beams are unable to reach the points.

It has been found that the tips of two thin wires can be successfully welded together with Joule heat in a self-completed manner provided that a specific amount of constant direct current is supplied to the wire system (Tohmyoh and Fukui, 2009). The welding conditions in this case can be described by a parameter which governs the melting phenomenon at the point of contact between the two wires (Tohmyoh, 2009). Welding the tips of two thin wires together is classified as one-dimensional welding from the point of view of current flow. Moreover, the tip of a thin wire has been successfully welded to the side of a second wire and this is classified as two-dimensional welding (Fukui and Tohmyoh, 2011). To widen the applications, this welding technique can be extended further. For example, the
welding of a thin wire onto a film is necessary for creating advanced materials systems. Such welding of a wire to a planar surface is classified as three-dimensional welding.

In this paper the Joule heat welding of a thin wire to a planar surface, i.e., a thin film, is performed. For thicker films, the current required to weld the wire onto the film becomes independent of the film thickness, and in this case, the conditions for welding depend only on the properties of the wire. The purpose of this study is to find the critical thickness of the film at which the welding conditions become independent of the film thickness. First, an analytical heat conduction model to calculate the critical thickness of the film was developed. Thin 0.8 µm diameter Pt wires were examined in this study and these wires were welded onto Pt films of various thicknesses. The value of the critical thickness calculated from the model was verified by the experimental results.

2. Joule heat welding of thin wires in vacuum

The process to weld the tips of two thin wires together and the relevant physics are described in this section. As shown in Fig. 1(a), the length of the wire to the left is $l_1$ and that to the right is $l_2$. Thus, the total length of the wire system $l$ is given by $l_1 + l_2$. After bringing the tips of the two wires into contact, a constant direct current, $I$, is supplied to the wire system.

If the current is above a critical value for welding, the wires are successfully welded together by Joule heat, see Fig. 1(b). After the welding is completed, the maximum temperature is at the midpoint of the wire system. The steady state temperature at this point, $T_C$, is given by (Tohmyoh, 2009)

$$T_C = \frac{1}{8K\sigma}U^2 + T_0,$$  
(1)

where

$$U = I \frac{l}{A} f,$$  
(2)

and where $K$ is the thermal conductivity of the wire, $\sigma$ the electrical conductivity, $T_0$ the ambient temperature, $A (= \pi d^2 / 4)$ the cross sectional area of the wire, and $d$ the diameter of the wire. The function $f$ represents the thermal boundary conditions around the wire. If the temperature at each end of the wire is $T_0$, and the wire is thermally insulated, $f$ is unity (Tohmyoh, 2009). For the current required for welding, the value of $T_C$ is below the melting point, $T_M$, of the wire.

Fig. 1 Schematic of the welding process. (a) The tips of two thin wires are brought into contact and a constant direct current is supplied to the wire system. (b) If the current is above the critical value for welding, the two wires weld together. In this case $T_C$ is below $T_M$. (c) If $T_C$ is above $T_M$, the wire is severed at the midpoint.
When the current is increased, $T_C$ reaches $T_M$ and the wire is cut at the midpoint of the system (Fig. 1(c)). The value of $f$ can be determined for a given set of environmental conditions from the current required to cut the wire, $I_Q$, using the following relationship.

$$I_Q = \frac{I_0}{f}, \quad (3)$$

where

$$I_0 = \sqrt{\left(T_M - T_0\right)8K\sigma A} / T. \quad (4)$$

In the special case where the wire is supported by the materials with sufficient heat capacity and the wire is thermally insulated, the temperature of the supporting materials is left unchanged by the current and the temperature of the wire uniformly increased under the current supply. In this case, the temperature gradient of the wire becomes very small. On the other hand, there is a huge temperature gradient at the interfaces between the wire and the supporting materials. We call such interfaces "cold points". And if the cold points are located at the ends of the wire, $f$ can be determined analytically from the following equation (Tohmyoh, 2013).

$$f = \frac{2\sqrt{2}}{\pi} \ln \left[ \frac{\kappa (\pi/2)^2}{B} \right], \quad (5)$$

where $\kappa = K / (\rho c)$, $\rho$ is the density, and $c$ the specific heat. Note that $f$ depends on the thermal properties ($\kappa$) and geometry ($l$) of the wire and is independent of the electrical properties ($\sigma$) in this case.

The values of $U$ for successful welding can be expressed by (Tohmyoh, 2009)

$$U_C < U < U_U, \quad (6)$$

where $U_C$ and $U_U$ are the lower and upper value of $U$ for welding. $U_U$ corresponds to the melting point of the wire and $U_C$ is found to be 70% of $U_U$ for an ultrathin 0.8 $\mu$m diameter Pt wire.

![Electro-thermal problem for a thin wire in contact with the film on a substrate. The other electrode for supplying current is separated by some distance from the point of contact between the wire and the film. Here the heat conduction problem in a cylindrical element in the film with diameter $d$ and height $D$ is considered.](image)

3. Theoretical modeling

Based on the above, $f$ can be derived analytically provided that the cold points are located at the ends of the wire, in which case the value of $U$ can be found without the need for experimentation. Where a thin wire is welded by Joule heat to a metal film deposited on a substrate, the cold point for the film can be assumed to be located at the point of contact. This means that there is a critical thickness of the film for this type of welding. In this section this critical thickness is derived analytically.

The electro-thermal problem for a thin wire in contact with a film on a substrate is considered (Fig. 2). The wire is brought into contact with the film and a direct current $I$ is supplied via an electrode in contact with the film located at
some distance from the point of contact between the thin wire and the film. Because the temperature gradient in the film in this situation is estimated to be very small (see Appendix), the temperature variation close to the point of contact between the wire and the film should be considered. As shown in Fig. 2, the electro-thermal problem for a cylindrical element in the film is considered. The direction of the \( x \) axis is perpendicular to the planar surface of the film and its origin is set to be at the interface between the film and the substrate. The heat conduction in the cylindrical element is considered to be governed by the following one dimensional differential equation (Carslaw and Jaeger, 1959).

\[
\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} - \nu (T - T_0) + \frac{q}{\rho c},
\]

(7)

where \( q = I^2 / (\sigma A^2) \), \( T \) is the temperature, \( t \) time, \( A \) the cross sectional area of the cylindrical element which is equal to that of the wire, and \( \nu = H p / (\rho c A) \) where \( H \) is the surface conductance and \( p \) the perimeter of the cylindrical element. The steady state temperature can be determined by setting \( \partial T / \partial t = 0 \) in Eq. (7) and with the following boundary conditions: \( T = T_M \) at \( x = D \), \( dT / dx = 0 \) at \( x = 0 \) (heat flux at \( x = 0 \) equals zero). We get

\[
T = \frac{e^{bx} + e^{-bx}}{e^{bD} + e^{-bD}} g + (T_M - g),
\]

(8)

and

\[
g = T_M - T_0 - \frac{q}{\rho c \nu},
\]

(9)

where \( b = (\nu / \kappa)^{0.5} \). The temperature at \( x = 0 \) is

\[
T_D = \frac{2}{e^{bD} + e^{-bD}} g + (T_M - g).
\]

(10)

Fig. 3  (a) The relationship between \( T_0 \) and \( D \) for \( I = 4.5 \) mA. (b) The relationship between \( dT_0 / dD \) and \( D \). If \( C = -200 \) K/\( \mu \)m is chosen, the value of \( D_{cr} \) is estimated to be 2.3 \( \mu \)m.
Let us consider the case in which a thin Pt wire with \( d = 0.8 \text{\,\(\mu\)m} \) and \( l = 100 \text{\,\(\mu\)m} \) is welded onto a Pt film on a substrate. From Eqs. (3), (4) and (5), the value of \( I_0 \) is determined to be 4.5 mA for the case where the cold points are located at each end of the wire \((f = 3.45, I_0 = 15.5 \text{\,mA})\). The following values for Pt were used in this calculation: \( K = 72 \text{\,W/(m\,K)} \), \( \sigma = 9.45 \times 10^6 \text{\,S/m} \), \( T_M = 2042 \text{\,K} \), \( T_0 = 293 \text{\,K} \), \( A = \pi (0.8 \times 10^{-6})^2 / 4 \). \( \rho = 2.15 \times 10^4 \text{\,kg/m}^3 \), \( c = 134 \text{\,J/(kg\,K)} \). The value of \( B \) is assumed to be 0.01. It had been previously verified that \( f \) is not sensitive to the value of \( B \) (Tohmyoh, 2013).

The values of \( T_D \) for \( I = 4.5 \text{\,mA} \) are plotted as a function of \( D \) in Fig. 3(a). Here, \( H_p \) is included in the term \( v \). This is for the heat transfer from the surface of the cylinder into the surrounding ambient. However the cylinder in this case is surrounded by solid material, so, in our calculations, \( H_p \) is replaced by \( K \). This is an important assumption for this calculation. As can be seen in Fig. 3(a), the value of \( T_D \) saturates with increasing \( D \). Provided \( T_D \) is sufficiently low, the cold point can be assumed to be beneath the point of contact between the thin wire and the film. Let us define the value of \( D \) where \( T_D \) is sufficiently low as the critical thickness, \( D_{cr} \). To determine \( D_{cr} \), the following condition on the slope of the \( T_D-D \) relationship is imposed.

\[
\frac{dT_D}{dD} = -2gb\left(e^{bD} - e^{-bD}\right) \leq C .
\] (11)

Figure 3(b) shows the relationship between \( dT_D/dD \) and \( D \). If \( C = -200 \text{\,K/\(\mu\)m} \) is chosen, \( D_{cr} \) is estimated to be 2.3 \( \mu \text{m} \). That is, if \( D \) is greater than about 2 \( \mu \text{m} \), the welding conditions, i.e., the welding current, is governed only by the properties of the wire and is independent of \( D \). Although \( T_D \) is a little larger than \( T_0 \) for \( C = -200 \text{\,K/\(\mu\)m} \), the authors consider this value of \( C \) to be reasonable. In the present model, the base of the film was assumed to be thermally insulating. However, this assumption gives an overestimate for \( T_D \) because, in actuality, there is a small amount of heat conduction from the film to the substrate. Therefore, the authors used this value of \( C \) to determine \( D_{cr} \). In our previous study, where tips of thin Pt wires were welded onto the tips of other Pt wires having the relatively thicker diameters, the critical diameter of the thicker Pt wire, where the welding condition was independent of the properties of the thicker wire, existed (Fujimori and Tohmyoh, 2013).

4. Experiments

In the previous section, the value of \( D_{cr} \) was estimated to be 2.3 \( \mu \text{m} \). Here the validity of this estimate is verified experimentally. The wires used in the experiments were ultrathin Pt wires with \( d = 0.8 \text{\,\(\mu\)m} \), as shown in the scanning electron microscope (SEM) image in Fig. 4(a). The wires were initially covered with Ag with a diameter of 75 \( \mu \text{m} \). This kind of wire is known as Wollaston wire (Sacharoff and Westervelt, 1984). The wires were dipped into HNO₃ solution to expose the Pt wire. It was confirmed from previous experiments that the experimental value of \( f \) for a thin Pt wire with its end covered with Ag was in good agreement with that obtained using Eq. (5). The cold point is located at the end of the Pt wire (Tohmyoh, 2013).

A cross sectional view of the Pt film sample is shown in Fig. 4(b). Pt films were deposited on oxidized Si substrates by sputtering. A 300 nm thick SiO₂ layer had been grown by oxidizing the Si substrates at 1223 K for 60 minutes. This was followed by deposition of a TiN adhesion layer. The thicknesses of the Pt films, \( D \), were 0.8 and 2 \( \mu \text{m} \).

Figure 4(c) shows the experimental setup. A 75 \( \mu \text{m} \) diameter Ag probe was used as the electrode to supply the current. The thin Pt wire was attached to a 3-axis manipulator. The Ag probe was fixed on a z-stage, and the substrate with the thin Pt film was fixed on an x-y stage. Using the manipulator and the stages, the Pt wire and Ag probe were brought into contact with the Pt film at a separation of 125 \( \mu \text{m} \). This separation, 125 \( \mu \text{m} \), was far greater than the values of \( D \), meaning that the experimental results were not influenced by the Ag probe. A constant direct current was passed through the film between the Ag probe and the Pt wire for 10 seconds. Following this it was confirmed whether welding had been achieved or not. If the weld was incomplete, the current was increased by 0.1mA, and the trial repeated until the wire was severed.
5. Results and discussion

Figure 5(a) shows an example of the relationship between the voltage across the welding circuit and the time after the current was switched on. Immediately after switching on the current, the voltage decreased within a few seconds, and it then remained at a constant value. This behavior was also found for the self-completed welding of the tips of two Pt wires (Tohmyoh and Fukui, 2009). From this similarity, the authors concluded that the Pt wire was welded onto the Pt film in a self-completed manner a few seconds after the current had been switched on.

Figure 5(b) shows a SEM micrograph of the welded Pt wire just after it has been severed at its midpoint by Joule heat. The tip of the Pt wire hanging from the Ag probe was manipulated such that the tip of the wire attached to the film could be moved (Fig. 5(c)). Although the Pt wire welded onto the film was deflected (Fig. 5(d)), the wire remained welded to the film, and returned to its original position after removing the force (Fig. 5(e)). The experimental results confirm that the Pt wire was rigidly welded onto the Pt film by Joule heat.

Table 1 summarizes the results of the welding experiments. The symbol S indicates that welding was successful. The symbols M and N show unsuccessful welding. The symbol M indicates that the wire system was severed due to excessive Joule heat. The symbol N indicates that welding was not achieved due to insufficient Joule heat.

From the data for M in Table 1, the values of \( f \) for \( D = 0.8 \) and \( 2 \) \( \mu \)m were determined, and these are plotted as functions of \( d / l \) in Fig. 6. The value of \( f \) determined from Eq. (5) which is independent of \( D \) is also shown in Fig. 6. As can be seen in Fig. 6, the values of \( f \) for \( D = 2 \) \( \mu \)m are in good agreement with those determined from Eq. (5) for larger values of \( d / l \). This indicates that the value of \( f \) for \( D = 2 \) \( \mu \)m is independent of \( D \), as predicted in section 3, where \( D_c \) was calculated to be around 2 \( \mu \)m. On the other hand, the values of \( f \) for \( D = 0.8 \) \( \mu \)m are clearly higher than those determined from Eq. (5). This indicates that the temperature increase in the Pt film around the point of contact with the Pt wire was not negligible in this case. Therefore, the function \( f \) for \( D = 0.8 \) \( \mu \)m was determined by fitting the following polynomial to the experimental data:
Fig. 5 An example of the welding experiment. (a) Voltage history. (b) to (e) show photographs taken during the procedure to confirm whether the wire was rigidly welded onto the film or not.

Table 1 Results of welding experiments.

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<th>Sample</th>
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<th>Symbol</th>
<th>l [μm]</th>
<th>I [mA]</th>
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Experiments were conducted in a SEM. S indicates successful welding and N incomplete welding. Joints that were severed after welding due to high Joule heat are classified as M. The diameter of all the wires was 0.8 μm, and the separation between the two electrodes for supplying the current was kept constant at 125 μm.
6. Conclusions

The Joule heat welding of thin 0.8 µm diameter Pt wires onto thin 0.8 and 2 µm thick Pt films was achieved. A heat conduction model was developed to determine the critical thickness of the film for which the current for welding the wire onto the film became independent of the film thickness. From the theoretical model, the critical thickness of the Pt film was predicted to be around 2 µm, and this estimation was verified by experiment.

Acknowledgment

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Appendix

Let us consider the electro-thermal problem for a film on a substrate (Fig. A1). A direct current, \( I \), is passed between two electrodes in contact with the film. The separation between the electrodes is \( L \). The electrical conductivity of the film is \( \sigma_1 \) and that of the substrate is \( \sigma_2 \). The thickness of the film is \( D \). The origin of the \( x \) coordinate is set to be at the midpoint between the electrodes. The temperature at \( x \) can be expressed in terms of the electric potential, \( \phi_x \), by (Saka, et al., 2009)

\[
T_x = T_0 - \frac{\sigma_1}{2\lambda_1} (\phi_x - \phi_0)^2,
\]

where \( \lambda_1 \) is the thermal conductivity of the film, and \( \phi_0 \) and \( T_0 \) are the electric potential and temperature at a reference point. The maximum temperature of the film, \( T_{\text{max}} \), is at \( x = 0 \), so if \( x = 0 \) is chosen as the reference point, \( \phi_0 \) becomes zero. Therefore, the temperature gradient between \( x = 0 \) and \( x = x \) is given by

\[
T_{\text{max}} - T_x = \frac{\sigma_1}{2\lambda_1} \phi_x^2,
\]

Here \( \phi_x \) can be expressed by (Takeo, 2006)

\[
\phi_x = \frac{\rho_1(-I)}{2\pi(0.5L - x)} \left[ 1 + 2 \sum_{n=1}^\infty k^n \left( 1 + 4n^2 \left( \frac{D}{0.5L - x} \right)^2 \right)^{-0.5} \right] + \frac{\rho I}{2\pi(0.5L + x)} \left[ 1 + 2 \sum_{n=1}^\infty k^n \left( 1 + 4n^2 \left( \frac{D}{0.5L + x} \right)^2 \right)^{-0.5} \right].
\]

where \( k = (\sigma_1 - \sigma_2) / (\sigma_1 + \sigma_2) \). If a non-conducting material is used as the substrate (\( \sigma_2 = 0 \)), \( k \) becomes unity. Moreover, \( \phi_x \) can be approximated by considering just the first term in the summation, i.e. \( n = 1 \):

\[
\phi_x \approx \frac{\rho_1(-I)}{2\pi(0.5L - x)} \left[ 1 + 2 \left( 1 + 4 \left( \frac{D}{0.5L - x} \right)^2 \right)^{-0.5} \right] + \frac{\rho I}{2\pi(0.5L + x)} \left[ 1 + 2 \left( 1 + 4 \left( \frac{D}{0.5L + x} \right)^2 \right)^{-0.5} \right].
\]

From Eqs. (A2) and (A4), the temperature gradient in the film can be estimated. Because \( \phi_x \) diverges at \( x = L/2 \), \( x \) is chosen to be 0.99 \((L/2)\). For the case of \( I = 4.5 \text{ mA} \), \( D = 0.8 \mu\text{m} \), \( L = 125 \mu\text{m} \), \( T_{\text{max}} - T_x \) was determined to be 0.001 K, showing that the temperature in the film is almost uniform under the supply of such a current.

Fig. A1 Electro-thermal problem for a thin film on a substrate. Here the current is supplied to the film via two electrodes separated by a distance \( L \).

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