Damping control of sloshing during liquid container transfer by active vibration reducer with 6-DOF parallel linkage (In the case of straight path on horizontal plane)

Masafumi HAMAGUCHI* and Takao TANIGUCHI*
* Interdisciplinary Graduate School of Science and Engineering, Shimane University
1060 Nishikawatsu-cho, Matsue-shi, Shimane 690–8504, Japan
E-mail: hamaguchi@ecs.shimane-u.ac.jp

Received 16 May 2014

Abstract
This paper presents a method to damp sloshing in a liquid container during its transfer in a cart equipped with an active vibration reducer having a six-degree-of-freedom parallel linkage. Sloshing is generated during the liquid container’s transfer. To damp this sloshing, the active vibration reducer tilts and horizontally moves the container. The reducer is controlled by a linear quadratic regulator. The weighting matrix of the quadratic performance index is optimized with a genetic algorithm (GA). The amplitude of sloshing is considered for the fitness of the GA. In this study, the cart was driven along a straight path on a horizontal plane. The usefulness of the control system was verified through a simulation and experiment.

Key words: Sloshing, Damping control, Active vibration reducer, Parallel linkage, Liquid container transfer, Linear quadratic regulator, Genetic algorithm

1. Introduction

Currently, extensive research and development is underway for the damping control of carried objects, which has practical applications in factories and various places. Examples of practical applications for damping control include the transfer of molten metals in the steel and casting industries, transfer of raw materials and mixed solutions in chemical plants, and transfer of precise products using carts.

Studies have been extensively conducted on the suppression of sloshing generated during the transfer of a liquid container. The transfer acceleration of the container can be shaped to suppress residual vibration of the liquid in the container (Aboel-Hassan et al., 2009; Yano and Terashima, 2005). A mechanism for tilting the container has been added to improve the damping performance (Hamaguchi and Taniguchi, 2009; Terashima and Yano, 2001; Yano and Terashima, 2001). In this paper, mechanisms with actuators to suppress vibration are termed active vibration reducers. Active vibration reducers are used for containers that are transferred in carts on uneven roads and curved paths and should have high degrees of freedom (DOF) (e.g., six).

This paper proposes an active vibration reducer with a 6-DOF parallel linkage. Sloshing in a liquid container being transferred by a cart is damped by the active vibration reducer installed in the cart. A good damping effect is obtained with high-speed and high-precision positioning mechanisms, such as the parallel linkage mechanism. Many studies have been conducted on this mechanism (Chen and You, 2000; Choi et al., 2001; Hong and Yamamoto, 2009), but most focused on parallel link manipulators. Few active vibration reducers have been based on the parallel linkage mechanism. Although Preumont et al. (2007) presented an active vibration isolator based on the parallel linkage mechanism, they did not consider the damping control of carried objects.

As a first step towards damping control, a liquid container being transferred along a straight path on a horizontal plane was considered in this study. Although a 2-DOF active vibration reducer is sufficient for this
case, a 6-DOF reducer is needed for curved paths on slopes and uneven roads. These cases will be considered in the next step.

![Photograph of experimental equipment](image1)

**Fig. 1** Photograph of experimental equipment

![Measurement point of liquid level](image2)

**Fig. 2** Measurement point of liquid level

![Schematic layout of experimental equipment](image3)

**Fig. 3** Schematic layout of experimental equipment

![Schematic of cart used in study](image4)

**Fig. 4** Schematic of cart used in study

### 2. Experimental equipment

Figure 1 shows a photograph of the experimental equipment. A liquid container is mounted on the endplate of a 6-DOF active vibration reducer. The endplate is supported by six links, each of which is connected to a linear actuator. The position and attitude of the endplate are controlled by the displacement of the linear actuators. This displacement is measured with a rotary encoder. The active vibration reducer is mounted on a cart that is manually driven along a straight path in the Y direction on the horizontal plane. The acceleration of the cart is measured with an accelerometer. The clear acrylic liquid container is cylindrical in shape with an inner diameter of 0.10 m, height of 0.30 m, and thickness of 0.005 m. Water was chosen to be the target liquid because it is inexpensive and easy to handle. The container is filled with water to a height of 0.10 m. Two laser displacement sensors are used to observe the liquid level at two measurement points, P1 and P2, as shown in Fig. 2. Each measurement point is located at a distance of 0.0255 m from the center of the container. In this
study, only the sensor at P1 was used because the cart was driven along a straight path. A white watercolor is mixed with the water so that the water surface can be detected with laser sensors.

Figure 3 shows the schematic layout of the experimental equipment. The signals from the rotary encoder of the reducer, accelerometer of the cart, and laser displacement sensor are sent to a digital signal processor board (DSP). The control signals calculated in the DSP are transmitted to the controller of the actuator through a V/F converter. Because this actuator is driven by pulse signals, the control signals of the voltage are converted to pulse signals through the converter.

Figure 4 shows the schematic of the cart used in the study. The axle direction, traveling direction, and vertical direction are represented by the X-axis, Y-axis, and Z-axis, respectively. The coordinate system O-XYZ is fixed on the ground. The wheelbase is 0.68 m; the cart is 0.68 m wide, 1.19 m long, and 0.70 m tall. Figure 4 also indicates the positions of the six linear actuators.

Figure 5 shows the schematic of the reducer. The linear actuators operate in the vertical direction. The links are connected to the actuators and the endplate via ball joints. The three pairs of actuators are placed on the baseplate at 120° intervals. The length of each link is 0.30 m, the radius of the endplate is 0.075 m, and the radius of the baseplate is 0.20 m. The movable distance of the actuator is 0.20 m. The coordinate system o-xyz is fixed on the cart.

3. System equation

A pendulum-type sloshing model in the Y-direction, as shown in Fig. 6, approximates (1, 1)-mode sloshing (Hamaguchi and Taniguchi, 2009). Although the sloshing model in the X-direction is similar to that in the Y-direction, the former model was neglected here because sloshing in the X-direction is not excited by traversal along straight paths in the Y-direction. The model in the Y-direction was used to design the damping control system. The linearized model in the Y-direction is expressed as follows:

\[ \ddot{\theta}_x = -\frac{g}{l_p} \theta_x - \frac{c}{m} \dot{\theta}_x + \frac{c}{l_p} \dot{\phi}_x - \frac{l_0}{l_p} \dot{\phi}_y + \frac{1}{l_p} Y_O, \]  

where \( \theta_x \) is the pendulum angle around the X-axis, \( \phi_x \) is the container angle around the X-axis, \( g \) is the gravitational acceleration, \( l_p \) is the equivalent pendulum length, \( l_0 \) is the distance between the container rotation center \( O_r \) and liquid surface center \( o_p \), \( m \) is the liquid mass, \( Y_O \) is the position of \( O_r \) on the Y-axis, \( y_c \) is the
position of the cart on the Y-axis, $y_O$ is the position of $O_r$ on the Y-axis, and $c$ is the equivalent coefficient of viscosity for sloshing. The liquid level displacement $h_y$ at the measurement point $P_1$ is given by

$$h_y = L(\theta_x - \phi_x),$$

where $L$ is the distance between the measurement point and the center of the container.

The system equation in the Y-direction is given by

$$\dot{x} = Ax + Bu + b_d a_y, \quad y = Cx,$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{g}{l_p} & -\frac{c}{m} & 0 & \frac{c}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{l_p}{l_p} - \frac{l_0}{l_p} \\ 0 \\ 1 \end{bmatrix}, \quad b_d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} L & 0 & -L & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} \theta_x \\ \dot{\theta}_x \\ \phi_x \\ \dot{\phi}_x \\ y_O \\ \dot{y}_O \end{bmatrix}, \quad y = \begin{bmatrix} h_y \\ \phi_x \\ y_O \end{bmatrix}, \quad u = \begin{bmatrix} u_y \\ u_{\phi_x} \end{bmatrix}. \tag{4}$$

$a_y = \ddot{y}_c$ is the disturbance of the cart acceleration, $u_y = \ddot{y}_O$ is the control input of acceleration, and $u_{\phi_x} = \ddot{\phi}_x$ is the control input of the angular acceleration. The actual values of the parameters in the system equation are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$ [Ns/m]</td>
<td>1.67</td>
</tr>
<tr>
<td>$l_p$ [m]</td>
<td>0.0261</td>
</tr>
<tr>
<td>$m$ [kg]</td>
<td>0.785</td>
</tr>
<tr>
<td>$l_0$ [m]</td>
<td>0.0500</td>
</tr>
<tr>
<td>$L$ [m]</td>
<td>0.0255</td>
</tr>
</tbody>
</table>

4. Control system

The container is tilted and horizontally moved by the active vibration reducer to damp the sloshing generated by the acceleration of the cart. The reducer is controlled by a linear quadratic regulator (LQR). The weighting matrix of the quadratic performance index is optimized with a genetic algorithm (GA).

4.1. Damping control system

Figure 7 shows the damping control system. The cart acceleration $a_y$ is considered to be a system disturbance. The Kalman filter estimates the state $x$ and removes observation noise from the output $y$. The LQR gain is $F$, and the Kalman filter gain is $K_{lim}$.

The control system is discretized to enable computer implementation. The equation for a discrete system is

$$x(k+1) = A_dx(k) + B_d u(k), \quad y(k) = Cx(k), \tag{5}$$

where

$$A_d = \exp(A\Delta T), \quad B_d = \int_0^{\Delta T} \exp(A\tau) d\tau B, \tag{6}$$
where

\begin{equation}
\frac{\Delta T}{\Delta t} = \frac{1}{k}
\end{equation}

\(k\) is the sampling number, and \(\Delta T\) is the sampling period. Here, \(\Delta T = 1.0\) ms.

The gain of the Kalman filter is calculated by solving the discrete Riccati equation, where the dispersions of observation noise on \(h_y\), \(\phi_x\), and \(y_O\) are set to \(8.065 \times 10^{-5} \text{m}^2\), \(1.000 \times 10^{-10} \text{rad}^2\), and \(1.000 \times 10^{-10} \text{m}^2\), respectively, and the system noise is assumed to be zero.

The LQR gain \(F\) is obtained by minimizing the discrete quadratic performance index in Eq. (7), i.e., by solving the discrete Riccati equation:

\begin{equation}
J = \sum_{k=0}^{\infty} \left(x^T(k)Qx(k) + u^T(k)Rx(k)\right),
\end{equation}

where \(Q \geq 0\) and \(R > 0\) are weighting matrices for the state and input, respectively.

4.2. Target position and attitude of endplate

The target position and attitude of the endplate are calculated from the control input \(u\). Figure 8 shows the endplate with \(\phi_x\) and \(\phi_y\). The point \(P_r\) represents the center of the endplate, \(L_e\) is the rotation radius of \(O_x, P_r\), and \(\phi_e\) is the angle between the z-axis and \(O_y, P_r\). The coordinates \((x_e, y_e, z_e)\) of \(P_r\) in the o-xyz coordinate system are

\begin{equation}
x_e = -L_e \cos \phi_x \tan \phi_y, \quad y_e = -L_e \cos \phi_x \tan \phi_y, \quad z_e = (1 - \cos \phi_x)L_e,
\end{equation}

where

\begin{equation}
\phi_x = \sqrt{\phi_x^2 + \phi_y^2}, \quad \phi_y = \int \phi_x \, dt^2, \quad \phi_z = \int \phi_y \, dt^2.
\end{equation}

The target position of the endplate, written in terms of coordinates \(x_{er}, y_{er},\) and \(z_{er}\), is

\begin{equation}
x_{er} = -L_e \cos \phi_x \tan \phi_y + x_u, \quad y_{er} = -L_e \cos \phi_x \tan \phi_y + y_u, \quad z_{er} = (1 - \cos \phi_x)L_e,
\end{equation}

where

\begin{equation}
x_u = \int u_x \, dt^2, \quad y_u = \int u_y \, dt^2.
\end{equation}

The target attitude of the endplate, written in terms of coordinates \(\phi_{ex}\) and \(\phi_{ey}\), is

\begin{equation}
\phi_{ex} = \phi_e, \quad \phi_{er} = \phi_y.
\end{equation}

The radius \(L_e\) is given as

\begin{equation}
L_e = l_0 + h_s + t_b,
\end{equation}

where \(l_0\) is the static liquid level and \(t_b\) is the thickness of the bottom of the container. In this study, \(L_e = 0.155\) m was taken based on \(l_0 = 0.050\) m, \(h_s = 0.100\) m, and \(t_b = 0.005\) m.

Fig. 8 Position of endplate
4.3. Inverse kinematics of active vibration reducer and actuator control

The target displacement of each linear actuator is calculated from the target position and attitude of the endplate by using the inverse kinematics of the reducer. Figure 9 shows the geometric model of the reducer. Equation (14) is established in Fig. 9.

\[ p + R s_i - b_i = v_i a_i + l_i z_i, \quad i = 1, 2, 3, \ldots, 6, \]

where \( p \) is the position vector of the center of gravity of the endplate, \( R \) is the rotation matrix that represents the endplate attitude, \( s \) is the vector representing the location of the link connection on the endplate, \( b \) is the vector representing the location of the actuator on the baseplate, \( v \) is the displacement of the actuator, \( a \) is the unit vector representing the movement direction of the actuator, \( l \) is the link length, \( z \) is the unit vector representing the link direction, and the suffix \( i \) is the actuator number. Using the vector \( d \),

\[ d_i = p + R s_i - b_i, \]

Equation (14) is modified as follows:

\[ d_i = v_i a_i + l_i z_i. \]

Subsequently, Eq. (16) is arranged as follows:

\[ (d_i - v_i a_i)^T (d_i - v_i a_i) = (l_i z_i)^T (l_i z_i), \quad v_i^2 - 2 d_i^T a_i v_i + d_i^T d_i - l_i^2 = 0. \]

The displacement \( v \) is calculated as follows by using the quadratic formula:

\[ v_i = d_i^T a_i \pm \sqrt{(d_i^T a_i)^2 - d_i^T d_i + l_i^2}, \]

where the \( \pm \) sign is in the reducer. The rotation matrix \( R \) is

\[ R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & \cos \gamma & -\sin \gamma \\ -\sin \beta & \cos \gamma & 0 \end{bmatrix} = \begin{bmatrix} \cos \phi_r & 0 & \sin \phi_r \\ 0 & \cos \phi_r & -\sin \phi_r \\ -\sin \phi_r & \cos \phi_r & 0 \end{bmatrix}, \]

where \( \alpha, \beta, \) and \( \gamma \) are the endplate angles around the x-axis, y-axis, and z-axis, respectively. The angle \( \gamma \) is zero because it is not used in damping control. The target displacement of the linear actuator \( v_r \) is

\[ v_{ri} = d_r^T a_r - \sqrt{(d_r^T a_r)^2 - d_r^T d_r + l_r^2}, \]

where

\[ d_r = p_r + R s_i - b_i, \quad p_r = \begin{bmatrix} x_{er} \\ y_{er} \\ z_{er} + z_0 \end{bmatrix}, \quad R_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_r & -\sin \phi_r \\ 0 & \sin \phi_r & \cos \phi_r \end{bmatrix} \begin{bmatrix} \cos \phi_r & 0 & \sin \phi_r \\ 0 & \cos \phi_r & -\sin \phi_r \\ -\sin \phi_r & \cos \phi_r & 0 \end{bmatrix}; \]

here, \( z_0 \) is the initial height of the endplate, and \( z_0 = 0.37 \) m and \( \phi_r = 0 \) in this study.

The actuator is controlled such that the displacement \( v \) equals the target value \( v_r \). The transfer function \( M(x) \) from the target input \( v_r \) to the displacement \( v \) is a linear first-order system:

\[ M(x) = \frac{V(x)}{V_r(x)} = \frac{1}{1 + Ts} \]

where \( T \) is the time constant of the actuator.

Fig. 9 Geometric model of active vibration reducer
where $T$ is a time constant; its values are presented in Table 2. The actuator is controlled by applying the input $v_{in}$:

$$V_{in}(s) = V_r(s) + \Delta V(s) = V_r(s) + (V_r(s) - M(s)V_r(s)) = 2V_r(s) - M(s)V_r(s), \quad (23)$$

where $\Delta V$ is the displacement error, which is given as $\Delta V = v_r - v$.

### Table 2  Values of time constant $T$

<table>
<thead>
<tr>
<th>Actuator No.</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ [s]</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.016</td>
<td>0.017</td>
<td>0.017</td>
</tr>
</tbody>
</table>

#### 4.4. Direct kinematics of active vibration reducer

The position and attitude of the endplate are calculated by using the direct kinematics of the reducer. Although this position and attitude can be calculated from the output of accelerometers and gyro sensors installed on the endplate, drift errors occur because of the integration of the sensor’s output with noise once or twice. The displacement of the linear actuator is measured by using a rotary encoder. No noise is present in the output of the rotary encoder. The direct kinematics of the active vibration reducer is approximated by using minute kinematics (Masuda et al., 1999). The minute position and attitude vector $\delta e$ of the endplate are defined as follows:

$$\delta e = \left[ \frac{\delta p}{\delta \Omega} \right], \quad \delta p = \left[ \delta x_e \quad \delta y_e \quad \delta z_e \right], \quad \delta \Omega = \left[ \frac{\delta \alpha}{\delta \beta} \right]. \quad (24)$$

Further, the minute displacement vector $\delta v$ of the actuator is defined as follows:

$$\delta v = \left[ \delta v_1 \quad \delta v_2 \quad \delta v_3 \quad \delta v_4 \quad \delta v_5 \quad \delta v_6 \right]^T. \quad (25)$$

Equation (26) is obtained by the total differentiation of both sides of Eq. (14):

$$\delta p + \delta Rs_i = a_i \delta v_i + l_i \delta z_i, \quad i = 1, 2, 3, \ldots, 6. \quad (26)$$

The second term on the left-hand side in Eq. (26) is arranged as follows:

$$\delta Rs_i = R_{\delta \Omega} \delta \Omega \times R_{s_i} = \left[ \begin{array}{ccc} 0 & -\delta \gamma & \delta \beta \\ \delta \gamma & 0 & -\delta \alpha \\ -\delta \beta & \delta \alpha & 0 \end{array} \right] \delta \Omega = \delta \Omega \times R_{s_i}. \quad (27)$$

Calculation of the inner product of the vector $z_i$ with both sides of Eq. (26) yields

$$z_i \cdot \delta p + z_i \cdot (\delta \Omega \times R_{s_i}) = z_i \cdot a_i \delta v_i + l_i z_i \cdot \delta z_i, \quad z_i \cdot \delta p + (R_{s_i} \times z_i) \cdot \delta \Omega = z_i \cdot a_i \delta v_i. \quad (28)$$

Equation (28) is rewritten by using Jacobian matrices $J_1$ and $J_2$ as follows:

$$J_1 \delta e = J_2 \delta v, \quad (29)$$

where

$$J_1 = \left[ \begin{array}{c} z_1^T \cdot (R_{s_1} \times z_1)^T \\ \vdots \\ z_6^T \cdot (R_{s_6} \times z_6)^T \end{array} \right], \quad J_2 = \left[ \begin{array}{c} z_1^T a_1 \quad \ldots \quad 0 \\ \vdots \\ 0 \quad \ldots \quad z_6^T a_6 \end{array} \right]. \quad (30)$$

Then, $\delta e$ is calculated as follows:

$$\delta e = J_1^{-1} J_2 \delta v. \quad (31)$$

Therefore, the position and attitude vector $e$ of the endplate is calculated from the following recurrence equation:

$$e(k) = e(k - 1) + \delta e(k) = e(k - 1) + J_1^{-1} (k - 1) J_2 (k - 1) \delta v(k), \quad k \geq 1. \quad (32)$$

$e$ is used to calculate $\phi$ and $\gamma_o$ of $y$ in Eq. (4) and to evaluate the constraint conditions in Eq. (36).
4.5. Optimization of weighting matrix in LQR

The weighting matrices $Q$ and $R$ in Eq. (7) are set as follows:

$$
Q = \text{diag}(q_1, q_2, q_3, q_4, q_5, q_6), \quad R = \text{diag}(1, 1).
$$

These variables, i.e., $q_1, q_2, q_3, \ldots, q_6$, are set using a binary-coded simple GA. One variable has binary codes of 11 bits, so one gene has 66 bits. The genotype is shown in Fig. 10. The binary codes are transformed to real values by the following equation:

$$
q_j = \frac{10(n_j + 1)}{256} \times 10^e_j, \quad j = 1, 2, 3, \ldots, 6,
$$

where $n$ and $e$ denote the most significant 8 bits and least significant 3 bits, respectively, of each variable. The flowchart of the GA is shown in Fig. 11. Roulette wheel selection, elitism selection, single-point crossover, and mutation by bit inversion are used in genetic operations. The following GA parameters were used in this study: the number of populations was 30, the crossover probability was 60%, the mutation probability was 10%, and the maximum number of generations was 500. The fitness $J$ of the GA is defined as follows:

$$
J = \frac{1}{J_1 + P}, \quad J_1 = \int_0^{t_f} |h_1| \, dt / h_{stf},
$$

where $t_f$ is the end time of the simulation and $P$ is a penalty term. The value of $h_1$ was evaluated in the damping control simulation, where the cart acceleration $a_y$ shown in Fig. 13 was applied and $t_f = 15$ s. $P = 0$ when the following constraint conditions are all satisfied:

$$
|y_e| \leq 0.1 \text{ m}, \quad |\alpha| \leq 20 \degree, \quad |\dot{v}_{1-6}| \leq 0.2 \text{ m/s}.
$$

These conditions are introduced from the constraints on the hardware of the reducer. $P = 10$ is imposed on $J$ when any one of the constraint conditions is not satisfied. A smaller value of $h_1$ means a greater value of $J$. The optimum weighting matrix $Q^*$ is obtained by maximizing the value of $J$. Searching with GA is executed 10 times, where the initial population is changed at random. The searching process is shown in Fig. 12, where
the largest values of $J$ for each generation are plotted. Figure 12 shows that all runs had good convergent characteristics, and run #5 had the largest value of $J$. The weighting matrix $Q$ with the largest value of $J$ in the 10 searching runs of GA is called the optimum weighting matrix $Q^\ast$. This matrix and the LQR gain $F^\ast$ are presented in Table 3.

### Table 3 Optimum weighting matrix and LQR gain

<table>
<thead>
<tr>
<th>$Q^\ast$</th>
<th>$F^\ast$</th>
</tr>
</thead>
<tbody>
<tr>
<td>diag($3.05 \times 10^2, 4.73 \times 10^3, 2.07 \times 10^3, 8.16 \times 10^3, 9.61 \times 10^6, 8.48 \times 10^7$)</td>
<td>$\begin{bmatrix} 161.3 &amp; 16.5 &amp; 142.0 &amp; 3.1 &amp; 122.8 &amp; 370.5 \ -2862.0 &amp; -21.8 &amp; 3603.0 &amp; 327.4 &amp; 284.9 &amp; 748.2 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

5. Simulation and experimental results

A simulation and experiment were performed to validate the damping control system with $F^\ast$. Figure 13 shows the result of the simulation, where the cart was driven along a straight path with an acceleration/deceleration of 2.5 m/s$^2$ and velocity of 1.25 m/s. Sloshing was generated by the cart acceleration $a_y$. The displacement of the liquid level $h_y$ was damped by controlling the position $y_e$ and angle $\alpha$ of the endplate. The endplate was moved and tilted backward in acceleration and forward in deceleration to damp sloshing. As shown in Fig. 13, the constraint conditions on $y_e$ and $\alpha$ were satisfied.

In the damping control experiment, higher-order mode sloshing was generated, and the endplate vibrated; that is, a spillover phenomenon occurred (Hamaguchi et al., 1995). To avoid this phenomenon, the Kalman filter gain was recalculated from a dispersion value of $8.065 \times 10^{-4}$ m$^2$ of the observation noise on $h_y$, which was larger than the measured value. This means that higher-order modes were regarded as noise on $h_y$. The experimental result is shown in Fig. 14. In the experiment, the cart was driven manually as the maximum acceleration was almost the same as that in the simulation. In the damping control experiment, the displacement of the liquid level $h_y$ was also damped. The value of $h_y$ was equal to zero when the angle of the liquid surface $\theta_e$ was equal...
to the angle of the container $\phi$, or that of the endplate $\alpha$. This means that a greater maximum amplitude of $h_y$ without control correlates to a greater value of $\alpha$ with control. As shown in Fig. 14, the maximum amplitude of $h_y$ without control was smaller than that in Fig. 13, so the maximum amplitude of $\alpha$ with control in the experiment was smaller than that in the simulation. The value of $y_e$ in Fig. 14 shows a greater change than that in Fig. 13 because the acceleration/deceleration time of the cart in the experiment was longer than that in the simulation. Although the constraint condition for $y_e$ was not satisfied in the experiment, a margin was allowed for safety under this condition.

Figure 15 shows a scene of the liquid container with damping control. The container was confirmed to move horizontally and tilt with the active vibration reducer to damp sloshing. Figure 16 shows a scene of the liquid container without damping control. The damping effect is demonstrated via a comparison of Fig. 15(b) with Fig. 16.

Terashima et al. (1996) showed the robustness of a similar controller. The robustness of the control system is strong when the damping ratio or damping coefficient is changed and poor when the natural frequency or static liquid level of the plant is changed. When a good control performance is needed for any static liquid level, an adaptive controller or gain scheduling method should be adopted for the control system.

6. Conclusion

An active vibration reducer with 6-DOF parallel linkage is proposed to damp sloshing in a liquid container being transferred by a cart. A damping control system equipped with the proposed active vibration reducer was designed by means of an LQR. The weighting matrix of the quadratic performance index was optimized using a GA. The usefulness of the damping control system was demonstrated through a simulation and experiment.

Although straight paths on a horizontal plane were considered in this study, the proposed active vibration reducer can also be applied to curved paths on slopes and uneven roads. These cases will be considered in future work.

References


Hong, J. and Yamamoto, M., A calculation method of the reaction force and moment for a Delta-type parallel link robot fixed with a frame, Robotica, Vol. 27, No. 4 (2009), pp. 579-587.


