Evaluation of mixed-mode thermal stress intensity factor in glass plates at various temperatures (Effect of crack width on the sign of the SIF)

Masahiro SUETSUGU* and Kouichi SEKINO**
*Department of Mechanical Engineering, Suzuka National College of Technology
Shiroko-cho, Suzuka-shi, Mie 510-0294, Japan
E-mail:suetsugu@mech.suzuka-ct.ac.jp
**College of Science and Engineering, Kanto Gakuin University
1-50-1 Mutsuura-higashi, Kanazawa-ku, Yokohama, Kanagawa 236-8501, Japan

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Abstract

Mixed-mode thermal stress intensity factors (SIFs) are investigated at various temperatures using the method of caustics. First, theoretical caustic patterns are obtained for various types of optical systems in addition to the mode II SIF $K_{II}$ and mode I SIF $K_{I}$ ratios. Next, the values of the SIF are experimentally investigated at various temperatures for glass plates with an inclined artificial notch or a natural crack, and the effect of the crack on the sign of the SIF is considered. Then, it is determined whether or not crack extension occurs. It is shown that a negative value of the SIF $K_{I}$ occurs at the notch tip under high-temperature conditions. In contrast, the sign of the SIF $K_{I}$ at the natural crack tip under high-temperature conditions is positive. At low temperatures, the signs of the SIFs at the notch and the natural crack tip are positive. Crack propagation is observed when the sign of $K_{I}$ is positive. The direction of crack propagation initiating from the natural crack at high temperatures is in accordance with the theory of maximum circumferential tensile stress ($\sigma_\theta_{\text{max}}$). At low temperatures, the crack extends slightly, and thereafter, the direction of crack propagation abruptly changes because of the compressive stress in front of the notch and the natural crack tip.

Key words: Thermal stress, Stress intensity factor, Mixed-mode fracture, Caustics, Glass plate

1. Introduction

Recently, it has become very important to analyze the mechanical behavior of materials at various temperatures for the safe operation of machines and nuclear reactors under severe loading conditions. In particular, the fracture behavior in the vicinity of a crack tip at high and low temperatures must be studied in detail since materials are often subject to fracture by thermal stresses. In the past, attempts have been made to theoretically analyze the thermal stress intensity factor (SIF) (Matsumaga, et al., 1989; Goshima, et al., 1991). On the other hand, numerous papers have been published on experimental approaches to these problems using the photoelastic method (Zhang and Burger, 1985; Matsumoto, et al., 1990) and the caustic method (Wang and Hwang, 1994; Wang and Chen, 1993; Aoki, et al., 1994). However, only a few studies have focused on the effect of the crack width on the crack propagation behavior. Shimizu (2005) studied the influence of the crack width on the mode I thermal SIF for epoxy resin plates with a natural crack or an artificial notch using the method of caustics. It was shown that a positive or negative value of the mode I SIF $K_{I}$ can arise depending on the crack width. In this study, the effect of the crack width on the thermal SIF under a mixed-mode loading condition and the crack propagation characteristics are investigated using the caustic method (Shimizu and Suetsugu, 1987). Glass plates, which are widely used as a practical material, were adopted as test specimens. A glass plate also has an advantage over epoxy resin plates for studying the crack propagation
phenomenon. As shown in the abovementioned paper (Shimizu, 2005), a positive or negative value of the SIF can arise under thermal stress loading conditions. In such a case, the method of caustics is very useful because it is possible to easily ascertain whether the value of the SIF is positive or negative through the shape of the obtained caustic pattern. The caustic method is thus advantageous compared to the photoelastic technique. The fundamental theory of caustics for optically isotropic materials under the mixed-mode loading condition has been described by Theocaris et al. (1972) and Kalthoff (1993); however, an analytical method for determining the positive or negative value of the SIF was not described by these authors.

In the present study, theoretical caustic patterns for various mode II SIF $K_{II}$ and mode I SIF $K_I$ ratios and types of optical systems were first obtained. Then, the value of the thermal SIF and crack propagation were investigated at various temperatures of the glass plates with either an inclined artificial notch or a natural crack. An artificial notch has a gap between the notch surfaces; however, the natural crack surfaces are completely closed. Therefore, the stress states at the notch and at the crack tip induced by thermal loading are considerably different from each other. The value of the thermal SIF arising at the notch or the crack tip is evaluated including a plus or minus sign by the method of caustics, and it is determined whether or not crack extension occurs. In addition, the crack propagation behavior is briefly discussed by comparing the stress distributions obtained by a finite element method (FEM) analysis.

2. Basic principle of caustics

The fundamental theory of caustics for optically isotropic materials under mode I and mode II mixed-mode loading conditions has been discussed by Theocaris et al. (1972). In that paper, the basic principle of caustics is described for positive values of $K_I$ and $K_{II}$. In this study, it is necessary to use the analysis method for positive or negative values of $K_I$ and $K_{II}$. Therefore, the shape of the caustic pattern (which depends on the plus/minus sign of $K_I$ and $K_{II}$, the ratio of $K_I$ to $K_{II}$, and the optical system used in the experiment) is calculated in detail by referring to that study (Theocaris, et al., 1972), and the technique for determining the values of $K_I$ and $K_{II}$ is indicated below.

2.1 For a parallel or divergent light source

A collimated light beam is projected onto a cracked specimen, as shown in Fig. 1. Here, $z_0$ is the distance between the specimen and the screen. Cartesian coordinate systems $(x, y)$ and $(x', y')$ are used for the specimen and the screen, respectively. The situation for a divergent light beam is the same as that shown in Fig. 1. An incident light ray at the point $P(r, \phi)$ on the specimen impinges the screen at the point $Q'$. The light vector $W$ on the screen is given by Eq. (1) using the deviation vector of the light beam $w$. As shown in Fig. 1, $W$ depends on the difference in the optical path length $\Delta s$,
\[ W = \lambda \cdot r + w \]  

(1)

where \( \lambda \) is the magnification factor for the light, as shown in Fig. 2, and is given by Eq. (2).

\[ \lambda = \left( z_i + z_0 \right) / z_i \]  

(2)

In Eq. (2), \( z_i \) is the distance between the specimen and the focal point of the light. The shape of the caustic pattern on the screen is calculated using the formulas given in the aforementioned paper (Theocaris, et al., 1972).

\[
W_x' = \lambda r_0 \left\{ \cos \phi + \frac{2}{3 \sqrt{K^2_i + K^2_{II}}} \left( K_i \cos \frac{3\phi}{2} - K_{II} \sin \frac{3\phi}{2} \right) \right\}
\]

and

\[
W_y' = \lambda r_0 \left\{ \sin \phi + \frac{2}{3 \sqrt{K^2_i + K^2_{II}}} \left( K_i \sin \frac{3\phi}{2} + K_{II} \cos \frac{3\phi}{2} \right) \right\}
\]

(3)

In Eq. (3), \( W_x' \) and \( W_y' \) are the \( x \) and \( y \) components of the light vector \( W \) on the screen, respectively. Also, \( r_0 \) in Eq. (3) is the initial curve on the specimen forming the caustic pattern on the reference plane given by Eq. (4):

\[ r_0 = \left( \frac{3\zeta}{2\lambda} \right)^{\frac{2}{3}} \left( K^2_i + K^2_{II} \right)^{\frac{1}{3}} \]  

(4)

Here,

\[ \zeta = \frac{z_0 \cdot d \cdot c_0}{\sqrt{2\pi}} \]  

(5)

In Eq. (5), \( d \) is the specimen thickness, and \( c_0 \) is the optical constant for the caustics. Using Eq. (3), the shape of the caustic pattern for positive or negative values of \( K_i \) and \( K_{II} \) can be obtained.

\[ \lambda = (z_0 - z_i) / z_i \]  

(6)

In this case, the light vector \( W \) on the screen is given by Eq. (7):

2.2 For a convergent light source

When a convergent light source (as shown in Fig. 3) is used, the magnification factor \( \lambda \) is given by Eq. (6):

\[ \lambda = (z_0 - z_i) / z_i \]  

(6)

In this case, the light vector \( W \) on the screen is given by Eq. (7):

\[ W = -\lambda \cdot r + w \]  

(7)

Using Eq. (7), the formula for the shape of the caustic pattern on the screen can be derived in the same manner as in 2.1 and is given by:

\[ W'_s = -\lambda r_0 \left\{ \cos \phi - \frac{2}{3\sqrt{K_I^2 + K_{II}^2}} \left( K_I \cos \frac{3\phi}{2} - K_{II} \sin \frac{3\phi}{2} \right) \right\} \]

\[ W'_y = -\lambda r_0 \left\{ \sin \phi - \frac{2}{3\sqrt{K_I^2 + K_{II}^2}} \left( K_I \sin \frac{3\phi}{2} + K_{II} \cos \frac{3\phi}{2} \right) \right\} \]

(8)

Consequently, the shape of the caustic pattern for positive or negative values of \( K_I \) and \( K_{II} \) can be obtained.

2.3 Shape of the caustic pattern and determination of SIFs \( K_I \) and \( K_{II} \)

Examples of caustic patterns calculated using Eqs. (3) and (8) are shown in Fig. 4, where \( \mu \) is given by Eq. (9).

\[ \mu = \frac{K_{II}}{K_I} \]  

(9)

Fig. 3 Optical system for convergent light

![Optical system for convergent light](image)

Fig. 4 Examples of caustic patterns (\( \zeta = 1, \lambda = 1 \))
Figure 4 shows that the caustic patterns obtained by a parallel or divergent light source and those obtained by a convergent light source are symmetrical with respect to the origin when the signs of $K_I$ and $K_{II}$ are opposite. From Fig. 4, a convenient caustic pattern for measuring the diameters, as shown in Fig. 5, can be obtained by choosing a suitable optical system irrespective of the combination of $K_I$ and $K_{II}$. The values of $K_I$ and $K_{II}$ are determined from Eqs. (10) and (11) for a diameter $D$, as shown in Fig. 5.

$$K_I = \frac{1.671}{z_0 \cdot d \cdot c_0} \cdot \frac{1}{\lambda^2} \left( \frac{D}{\delta} \right)^{2.5} \cdot \frac{1}{\sqrt{1 + \mu^2}} \tag{10}$$

$$K_{II} = \mu \cdot K_I \tag{11}$$

The signs of $K_I$ and $K_{II}$ can be determined by comparing the optical system shown in Fig. 4 and the caustic pattern obtained in the experiment. The values of $\mu$ and $\delta_\lambda$ in Eq. (10) are determined through the shape of the caustic pattern shown in Fig. 5. Moreover, the relationship between $\frac{(D_{\text{max}} - D_{\text{min}})}{D_{\text{max}}}$ (which can be determined using $D_{\text{max}}$ and $D_{\text{min}}$ shown in Fig. 5) and $\mu$ is shown in Fig. 6. As mentioned above, once the value of $\mu$ is evaluated from Fig. 6, the value of $\delta_\lambda$ can be obtained through Fig. 7.

2.4 Experimental verification

As shown in Fig. 8, a compressive loading test was performed with a centrally cracked circular polymethylmethacrylate (PMMA) disk. A pair of diametric compressive loads $P$ were applied to the disk at loading angles of $\theta = 0^\circ$, $45^\circ$, and $90^\circ$, and the caustic patterns were recorded using a divergent light source and a convergent light source, as shown in Figs. 2(b) and 3, respectively. Figure 9 shows examples of the caustic patterns obtained for various experimental conditions. As compared with Fig. 4, we can see that the sign of $K_I$ is positive and $K_{II} = 0$ at $\theta = 0^\circ$, both signs of $K_I$ and $K_{II}$ are negative at $\theta = 45^\circ$, and the sign of $K_I$ is negative and $K_{II} = 0$ at $\theta = 90^\circ$. Figure 10 shows the variation in SIFs $K_I$ and $K_{II}$ with $P$ evaluated through the caustic patterns obtained with a divergent light source ($\theta = 45^\circ$ and $90^\circ$). The theoretical result reported by Dong et al. (2004) is also shown in Fig. 10 by dotted lines. As shown in Fig. 10, positive or negative values of $K_I$ and $K_{II}$ obtained by the caustic method are in good agreement with the theoretical result.

Fig. 5 Caustic pattern under mixed-mode condition

Fig. 6 Relationship between $\frac{(D_{\text{max}} - D_{\text{min}})}{D_{\text{max}}}$ and $\mu$

Fig. 7 Numerical factor $\delta_\lambda$ for determining the SIF from mixed-mode crack-tip caustics
3. Test specimen

Rectangular glass plate specimens, as shown in Fig. 11, were prepared. A natural crack or an artificial notch was formed in the specimens with various inclined angles of $\theta = 30^\circ$, $45^\circ$, $70^\circ$, and $90^\circ$. The length $a$ was 12 mm for a natural crack and 6 mm for an artificial notch. These dimensions were decided based on a study (Shimizu, 2005) using an epoxy resin plate with a natural crack or an artificial notch with $\theta = 90^\circ$. In that paper, it was shown that a large value for the SIF can be obtained under these conditions. A natural crack is produced in the brittle glass specimen by the following technique. First, a small crack is formed in the long dimension of the rectangular glass plate by cooling the edge with liquid nitrogen. Thereafter, this small crack was extended to the intended location by heating that point with a soldering iron. Finally, a specimen with the dimensions shown in Fig. 11 was cut from the glass plate using a diamond saw, and thereafter, the cut ends were made smooth with emery paper. An artificial notch was produced using a diamond wheel. The notch width was 0.3 mm, and the notch tip was semicircular in shape with a radius of $\rho = 0.15$ mm.

It is known that glass is an optically anisotropic material; therefore, the shape of the caustic pattern should be a double curve. However, we did not see double caustic curves in the pictures obtained in this study because the optical constant of caustics $c_0$ is very small. Consequently, it is considered that the caustic patterns obtained in this study include a single caustic curve, and the value of $c_0$ is found to be $-0.047 \times 10^{-10}$ m$^2$/N using Eq. (10) with a value of $\delta_i = 3.17$ by carrying out a three-point bending test with the Mode I loading condition at room temperature. The SIFs were evaluated with this value of $c_0$ over the entire range of test temperatures in this study.
4. Experimental apparatus and test procedure

The experimental setup used for the caustics method in the study of the thermal SIF is shown in Fig. 12. In addition to the parallel laser light source shown in Fig. 12, a divergent or convergent light source was also employed, depending on the sign of the SIF. First, the heating or cooling device was adjusted to the test temperature \( T \), and then, the bottom surface of the specimen was placed on the device. The variation in the caustic pattern formed by the thermal stress over time \( t \) was recorded. The starting time \( (t = 0) \) was the moment when the specimen was placed on the heating or cooling device, and the initial temperature of the specimen was room temperature. The cooling device used for the experiments at low temperatures is shown in Fig. 13. This device uses liquid nitrogen and was made in our laboratory.

5. Experimental results and discussion

5.1 Thermal SIF of notch

5.1.1 At high temperatures

Figure 14 shows examples of caustic patterns obtained for a glass plate with a notch at \( T = 673 \) K, the highest temperature that the heating device can achieve. The crack extension phenomenon was not observed at this temperature. These patterns were obtained using a convergent light source, and therefore, the sign of \( K_I \) is negative. As shown in Fig. 14(b), the caustic patterns formed by an inclined notch are asymmetric because of mixed-mode loading. Moreover, by comparing the caustic patterns at \( t = 5 \) s and \( t = 180 \) s in Fig. 14(b), it is evident that the directions of deviation of these two caustic patterns are opposite. This means that the sign of \( K_{II} \) varies from positive to negative with time \( t \). Figure 15 shows the variation in \( K_I \) with time \( t \). This figure shows that the absolute value of \( K_I \) increased with time until \( t = 30 \) s, and thereafter, it decreased. Figure 16 shows the variation in \( K_{II} \) with time \( t \). This figure also shows that generally, the absolute value of \( K_{II} \) increased with time until \( t = 30 \) s, and thereafter, it decreased. However, \( K_{II} \) was first positive, and then became negative. Figure 17 shows the relationship between \((K_I)_{\text{max}}\) and \((K_{II})_{\text{max}}\) and the notch angle \( \theta \) at \( T = 673 \) K. It is seen that the value of \( K_I \) reached a maximum at \( \theta = 90^\circ \), and the value of \( K_{II} \) reached a maximum at \( \theta = 30^\circ - 45^\circ \) within the range of tested angles.

As mentioned above, a relatively large negative value of \( K_I \) was induced at the notch tip with a width of 0.3 mm by the compressive stress generated at the bottom area of the specimen adjacent to the heating surface, and crack extension did not occur.
5.1.2 At low temperatures

(1) At $T = 233$ K

The caustic patterns for a glass plate with a notch at $T = 233$ K are shown in Fig. 18. The crack extension phenomenon was not observed at this temperature. These patterns were obtained using a collimated light source, and therefore, the sign of $K_I$ is positive. Figures 19 and 20 show variations in $K_I$ and $K_{II}$ with time $t$, respectively. Figure 21 shows the relationship between $(K_{I})_{\text{max}}$ and $(K_{II})_{\text{max}}$ versus the notch angle $\theta$ at $T = 233$ K, and it reveals that the value of $K_I$ reached a maximum at $\theta = 90^\circ$, and the value of $K_{II}$ reached a maximum at $\theta = 30^\circ$–$45^\circ$ within the range of tested angles. This tendency is similar to that at high temperatures.
At $T = 123$ K

An example of the caustic patterns at $T = 123$ K is shown in Fig. 22. At this temperature, the crack extension phenomenon was observed. In both cases (a) and (b) in the figure, the third frame shows the critical state of the stationary notch, and the fourth frame shows the running crack. The variation in the $K_{\text{eff}}$ value with time is shown in Fig. 23. Here, $K_{\text{eff}}$ is the effective SIF and is given by Eq. (12):

$$K_{\text{eff}} = \sqrt{K_I^2 + K_{II}^2}$$  \hspace{1cm} (12)

$K_{\text{eff}}$ is a significant and convenient parameter related to the initiation of crack extension for glass materials under a mixed-mode loading condition (Ikeda and Igaki, 1988). In Fig. 23, the values of $K_{\text{eff}}$ at the initiation of the crack extension are denoted by arrows, and the fracture toughness value at 123 K is 1.33 MPa·m$^{1/2}$ on average.

As stated above, a positive value of $K_I$ arises at the notch tip with a width of 0.3 mm due to the tensile stress generated at the bottom area of the specimen adjacent to the cooled surface, and crack extension is initiated when the value of $K_{\text{eff}}$ reaches the fracture toughness of the notch.

5.2 Thermal SIF of natural crack

5.2.1 At high temperatures

At $T = 373$ K

Figure 24 shows an example of the caustic patterns for a glass plate with a natural crack at $T = 373$ K. At this temperature, crack propagation did not occur. These patterns were obtained using a divergent light source, and therefore, the sign of $K_I$ is positive. Figure 25 shows the variations in $K_I$ and $-K_{II}$ with time $t$ obtained from the caustic patterns for glass plates with a natural crack at 373 K. Figure 26 shows the relationship between $(K_I)_{\text{max}}$ and $-(K_{II})_{\text{max}}$ and the crack angle $\theta$. As shown in Figs. 25 and 26, the value of $K_I$ is small compared with the value of $K_{II}$. 

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**Fig. 20** $K_{II}$ versus $t$ for a notch ($T = 233$ K)

**Fig. 21** $(K_I)_{\text{max}}$ and $-(K_{II})_{\text{max}}$ versus $\theta$ ($T = 233$ K)

**Fig. 22** Caustic patterns for a notch at $T = 123$ K (collimated light, $z_0 = 3000$ mm)

**Fig. 23** $K_{\text{eff}}$ versus $t$ for a notch at $T = 123$ K

**Fig. 24** An example of the caustic patterns at $T = 373$ K

**Fig. 25** Variations in $K_I$ and $-K_{II}$ with time $t$

**Fig. 26** Relationship between $(K_I)_{\text{max}}$ and $-(K_{II})_{\text{max}}$ and the crack angle $\theta$
(2) At $T = 473$ K

Crack propagation occurred at $T = 473$ K. Figure 27 shows an example of the caustic patterns at the initiation of the crack extension at that temperature. Since the caustic patterns shown in Fig. 27 were obtained using a divergent light source, the sign of $K_I$ is positive. In both cases (a) and (b) in Fig. 27, the third frame shows the critical state of the stationary crack, and the fourth frame shows the running crack. As seen in Fig. 27(b), the state of the inclined crack tip is mixed-mode loading before the crack extension; however, it experiences pure mode I loading after the crack extension. Figure 28 shows the variation in the $K_{eff}$ value with time. In the figure, the values of $K_{eff}$ at the initiation of the crack extension are denoted by arrows, and the fracture toughness value at $T = 473$ K is 0.69 MPa·m$^{1/2}$ on average. For $\theta = 30^\circ$, the velocity of the running crack is relatively high, and the crack tip reaches the left side of the specimen in ten seconds. For $\theta = 70^\circ$ and $90^\circ$, the crack slowly propagated and stopped at the center of the specimen. On the other hand, for $45^\circ$, the extended crack stopped close to the left side of the specimen.

In contrast to the notch with a width of 0.3 mm described in Section 5.1.1, a positive $K_I$ value arises at the natural crack tip due to the tensile stress generated in the vicinity of the crack tip, and crack extension is initiated when the $K_{eff}$ value reaches the fracture toughness. Based on the result of the FEM analysis that will be shown in Section 5.3, the appearance of the tensile stress at the crack tip is considered to be caused by the contact between the crack surfaces at the bottom portion of the specimen.
5.2.2 At low temperatures

Caustic patterns for a glass plate with a natural crack at \( T = 123 \text{ K} \) are shown in Fig. 29. At \( T = 123 \text{ K} \), the initiation of a crack extension occurred. These patterns are obtained using a collimated light source, and therefore, the sign of \( K_I \) is positive. In the same way as the notch described in Section 5.1.2, a positive value of \( K_I \) arises at the natural crack tip due to the tensile stress generated at the bottom area of the specimen adjacent to the cooled surface, and crack extension is initiated when the value of \( K_{eff} \) reaches the fracture toughness of the notch. In both cases (a) and (b) in Fig. 29, the second frame shows the critical state of the stationary crack, and the third frame shows the running crack. The fracture toughness value of \( K_{eff} \) at \( T = 123 \text{ K} \) is 0.9 MPa·m\(^{1/2}\) on average. The value of 0.9 MPa·m\(^{1/2}\) is higher than the fracture toughness of 0.69 MPa·m\(^{1/2}\) at \( T = 473 \text{ K} \) mentioned in Section 5.2.1. Shimizu and Ji (2005) studied the influence of low temperature on the fracture toughness of several nonmetal materials, including ceramics, glass, and some polymers. They reported that the values of \( K_I \) for these materials generally increased with decreasing temperature, and in particular, the value of \( K_I \) at \( T = 93 \text{ K} \) for glass is 1.5 to 2.0 times as large as that at room temperature. The direction of crack propagation under various conditions will be discussed next in Section 5.3.

![Caustic patterns for a natural crack at T = 123 K](image)

5.3 Behavior of crack extension

Because of the existence of compressive stress near the notch tip, crack extension from the notch does not occur under high-temperature conditions. Figure 30 shows the state of the extension from a natural crack at \( T = 473 \text{ K} \), and Fig. 31 gives the relationship between \( \alpha \) and the inclination angle \( \theta \). Here, \( \alpha \) is the direction of the crack just at the beginning of crack extension and is evaluated by observing the photograph of the enlarged crack tip. On the other hand, \( \alpha' \) in Fig. 31 is the direction of \((\sigma_0)_{max}\) calculated theoretically using Eq. (13) for a given value of \( \mu \).

\[
\mu = \frac{-\sin\left(\frac{\alpha}{2}\right) - \sin\left(\frac{3\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right) + 3\cos\left(\frac{3\alpha}{2}\right)}
\]

(13)

Figure 31 shows that the direction of crack propagation for a glass plate is in accordance with the theory of maximum circumferential tensile stress \((\sigma_0)_{max}\) (Yuki, 1993).

Figure 32 shows the state of crack extension from a notch and a natural crack at \( T = 123 \text{ K} \). As seen in the figure, the crack extension is initiated at the crack tip, and thereafter, the direction of crack propagation abruptly changes. This tendency is considerably different from that at high temperatures.

To clarify the reason for this phenomenon, FEM analysis was performed for glass plates with a natural crack of \( \theta = 90^\circ \) at \( T = 463 \text{ K} \) and 123 K (see Fig. 33). The length of the crack was 12 mm. Figure 33 shows the distribution of \( \sigma_1 \) along the crack line at \( t = 40 \text{ s} \). Here, \( \sigma_1 \) is the normal stress perpendicular to the notch or crack line. At an
elevated temperature ($T = 463$ K), the crack surfaces at the bottom portion of the specimen (denoted by * in Fig. 33) came into contact with each other due to the compressive stress. Therefore, the elements in that area were put together. It is recognized from Fig. 33 that compressive stress arises in front of the crack tip at a low temperature of $T = 123$ K, thereby causing the propagation of the crack to change direction. FEM analysis for a glass plate with a notch of $\theta = 90^\circ$ at $T = 123$ K was also performed. As a result, it was confirmed that compressive stress exists in front of the notch tip just as in the natural crack.

**Fig. 30** Crack extension for various inclined cracks at $T = 473$ K

**Fig. 32** Crack extension for various inclined cracks at $T = 123$ K

**Fig. 33** Distribution of $\sigma_\alpha$ along the crack line at $t = 40$ s ($\theta = 90^\circ$)

6. **Conclusions**

A mixed-mode thermal SIF for notched and cracked glass plates was determined at high and low temperatures using the method of caustics. The effect of the crack width on the sign of the SIF was considered, and the crack extension phenomenon was studied based on this consideration. The following results were obtained from this study:

1. The theoretical caustic patterns were shown for various signs of $K_I$ and $K_{II}$, different ratios of $K_{II}$ to $K_I$, and different optical system types.

2. In the heated plate case, by comparing the experimental and theoretical caustic patterns, it was determined that the sign of $K_I$ for a notch was negative, whereas that for a natural crack was positive. In this case, thermal compressive stress must be caused in the bottom area of the specimen; however, a positive value of $K_I$ arises at the natural crack tip due to the contact between the crack surfaces. The negative value of $K_I$ arising at the notch tip was relatively large; however, crack extension did not occur.

3. The values of $K_I$ and $K_{II}$ for an inclined crack or notch reached their maximum at $\theta = 90^\circ$ and $\theta = 30^\circ–45^\circ$, respectively.
(4) In the case of a cooled plate, thermal tensile stress arises in the specimen, and the sign of $K_I$ for both the notch and the crack was positive.

(5) Crack propagation was observed for the crack at high temperatures and for both the crack and the notch at low temperatures. The values of $K_{eff}$ at crack extension are shown below.

- Natural crack at $T = 473\ K$: 0.69 MPa·m$^{1/2}$
- Natural crack at $T = 123\ K$: 0.9 MPa·m$^{1/2}$
- Notch with a width of 0.3 mm at $T = 123\ K$: 1.33 MPa·m$^{1/2}$

(6) The direction of crack propagation was in accordance with the theory of maximum circumferential tensile stress ($\sigma_\theta_{max}$) at high temperatures. The direction of crack propagation abruptly changed just after slight crack extension at low temperatures. This phenomenon was caused by the existence of compressive stress.

**References**


