Integral sliding mode control for active suspension systems of half-vehicle model

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Abstract
This paper proposes a design method of sliding mode controller with the robustness against actuator uncertainty for active suspension systems of half-vehicle model. The features of the proposed sliding mode controller are not to require any force sensors to constitute local force feedback loop and to avoid chattering, which will be often a problem in sliding mode control. Based on the concept of the second order sliding mode control, the switching control input is redesigned by the describing function method in order to occur limit cycles of the switching function. Occurring the limit cycles instead of perfect sliding mode can lead continuous control inputs to suppress deterioration in high frequency band. The describing function method shows the existing condition of the limit cycles for the design parameters of the redesigned switching input. From numerical simulations, it can be checked that the proposed sliding mode controller can occur almost desired limit cycles of the switching function. Also, it can be seen that the proposed sliding mode controller shows high robustness against actuator uncertainty while it can suppress chattering in high frequency band.

Key words: Second order sliding mode control, Active suspension systems, Describing function method, Limit cycle, Chattering

1. Introduction

For these three decades, vehicle active suspension systems have been studied, and various control theories have been applied to control strategies of active suspension systems (For example, Oya, et al., 2007, Du and Zhangdu, 2008, Suzuki and Takahashi, 2010a, 2010b, Wasiwitono and Saeki, 2011). One of those control theories is the sliding mode control (For example, Nishimura and Takahashi, 2007, Li, et al., 2013), which is extremely systematized in the nonlinear robust control theory. One of features of the sliding mode control is to show invariant performance against uncertainty of satisfying the matching condition theoretically, though the control law based on discontinuous switching relay input is relatively simple structure. Many researches on application of the sliding mode control have been advanced energetically (For example, Huang, et al., 2007, Chang and Ting, 2009, Toyama, et al., 2011). The sliding mode control seems to be suitable for control of the active suspension systems, which need high robustness against actuator uncertainty of active suspension systems. In practice, however, the discontinuous switching relay input through actuators might cause deterioration of suspension performance, such as ride comfort and driving stability, especially in high frequency band. Although the saturation function instead of the discontinuous switching relay input is well known as a way to settle chattering, finding adequate value of the design parameter to define the saturation region could take great effort. On the other hand, the second order sliding mode control has been investigated in order to lead continuous control input (For example, Bartolini, G., et al., 2008). With such continuous control input, the second order sliding mode control can highly be expected to make practical control systems, which can have sufficient robustness against modeling error and uncertainty and can compress chattering problem.

This paper proposes a design method of sliding mode controller to lead continuous control inputs with the describing function method for active suspension systems of half-vehicle model. In order to avoid chattering, the
proposed method redesigns the conventional nonlinear switching control input with the twisting algorithm of the second order sliding mode control (Levant, A., 1993). Some design parameters of the redesigned nonlinear control input are determined by the describing function method, so that a desired limit cycle instead of perfect sliding mode can be occurred in the vicinity of switching surfaces. As a result, deterioration of the control effect due to the chattering in the high frequency band, such as increase in acceleration of the center of mass or in angular acceleration of pitch motion related to ride comfort, can be suppressed while the high robustness against actuator uncertainty is secured. The basic concept of the proposed sliding mode control method has already been proposed for a quarter car model (Toyama and Ikeda, 2009). This paper tries the concept to extend for the half vehicle model. Finally, simulation results show basic effectiveness of the proposed controller.

2. Problem formulation for half-vehicle model

Figure 1 shows a half-vehicle model (Li, et al., 2013). In this figure, $z_{sf}(t)$ is the front body displacement, $z_{sr}(t)$ is the rear body displacement, $l_1$ is the distance between the front axle and the center of mass, $l_2$ is the distance between the rear axle and the center of mass, $\varphi(t)$ is the pitch angle, and $z_c(t)$ is the displacement of the center of mass. $m_c$ is the mass of the car body. $m_{sf}$ and $m_{sr}$ are the unsprung masses on the front and the rear wheels. $I_p$ is the pitch moment of inertia about the center of mass. $z_{sf}(t)$ and $z_{sr}(t)$ are the front and the rear unsprung mass displacements respectively. $z_c(t)$ and $z_r(t)$ are the front and the rear terrain height displacements. $k_{sf}$ and $k_{sr}$ are the front and the rear stiffness of the springs respectively, while $c_{sf}$ and $c_{sr}$ are the front and the rear damping coefficient of the dampers. $k_f$ and $k_r$ are the front and the rear tire stiffness, $u_f(t)$ and $u_r(t)$ are the front and the rear actuator force inputs respectively. The goal of this paper is to achieve a desired ride comfort and a desired driving stability against the actuator uncertainty $\Delta u(t)$, when the state variable vector $x(t)$ is assumed to be measurable and the actuator forces $u_f(t)$ and $u_r(t)$ are assumed to be unmeasurable.

When the pitch angle $\varphi(t)$ is small enough, the front body displacement and the rear body displacement can be described as follows.

$$z_{sf}(t) = z_{c}(t) - l_1 \varphi(t)$$

$$z_{sr}(t) = z_{c}(t) + l_2 \varphi(t)$$

Then, the motion equations of the half-vehicle model can be represented as follows.

$$m_{sf} \ddot{z}_{sf}(t) = -k_{sf} [z_{sf}(t) - z_{af}(t)] - c_{sf} [\dot{z}_{sf}(t) - \dot{z}_{af}(t)] - k_{sr} [z_{cr}(t) - z_{ar}(t)] - c_{sr} [\dot{z}_{cr}(t) - \dot{z}_{ar}(t)] + u_f(t) + u_r(t)$$

$$I_p \ddot{\varphi}(t) = l_1 k_{sf} [z_{sf}(t) - z_{af}(t)] + l_1 c_{sf} [\dot{z}_{sf}(t) - \dot{z}_{af}(t)] - l_1 k_{sr} [z_{cr}(t) - z_{ar}(t)] - l_2 c_{sr} [\dot{z}_{cr}(t) - \dot{z}_{ar}(t)]$$

$$m_{sr} \ddot{z}_{sr}(t) = k_{sr} [z_{sr}(t) - z_{af}(t)] + c_{sr} [\dot{z}_{sr}(t) - \dot{z}_{af}(t)] - k_{sf} [z_{sf}(t) - z_{af}(t)] - u_f(t)$$

With Eqs. (3) - (6), the dynamics of the front body and the rear body can be described as follows.

$$\ddot{z}_{sf}(t) = \dot{z}_{sf}(t) - l_1 \dot{\varphi}(t) + a_1 [u_f(t) - k_{sf} [z_{sf}(t) - z_{af}(t)] - c_{sf} [\dot{z}_{sf}(t) - \dot{z}_{af}(t)] + a_2 [u_r(t) - k_{sr} [z_{cr}(t) - z_{ar}(t)] - c_{sr} [\dot{z}_{cr}(t) - \dot{z}_{ar}(t)]]$$

$$\ddot{z}_{sr}(t) = \dot{z}_{sr}(t) + l_2 \dot{\varphi}(t) + a_2 [u_f(t) - k_{sf} [z_{sf}(t) - z_{af}(t)] - c_{sf} [\dot{z}_{sf}(t) - \dot{z}_{af}(t)] + a_3 [u_r(t) - k_{sr} [z_{cr}(t) - z_{ar}(t)] - c_{sr} [\dot{z}_{cr}(t) - \dot{z}_{ar}(t)]]$$

where

When the state variable vector $x(t)$, the control input vector $u(t)$ and the road vector $w(t)$ are defined as follows,

$$x(t) = \begin{bmatrix} z_{sf}(t) - z_{af}(t) \\ z_{af}(t) - z_{ur}(t) \\ z_{af}(t) - z_{tr}(t) \\ z_{ur}(t) - z_{tr}(t) \\ \dot{z}_{sf}(t) \\ \dot{z}_{af}(t) \\ \dot{z}_{ur}(t) \end{bmatrix},$$

$$u(t) = \begin{bmatrix} u_f(t) \\ u_s(t) \end{bmatrix},$$

$$w(t) = \begin{bmatrix} \ddot{z}_{sf}(t) \\ \ddot{z}_{af}(t) \end{bmatrix},$$

the state space model of the half-vehicle model is given by

$$\dot{x}(t) = Ax(t) + Bu(t) + \Delta u(t) + Cw(t),$$

where the actuator uncertainty $\Delta u(t)$ is defined as

$$\Delta u(t) = \begin{bmatrix} \Delta u_f(t) \\ \Delta u_s(t) \end{bmatrix}.$$  

Each matrix of the state space model shown in Eq. (13) are described as follows.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -a_1k_{sf} & -a_2k_{sr} & 0 & 0 & -a_1c_{sf} & -a_2c_{sr} & a_1c_{sf} & a_2c_{sr} \\ -a_2k_{sf} & -a_2k_{sr} & 0 & 0 & -a_2c_{sf} & -a_2c_{sr} & a_2c_{sf} & a_2c_{sr} \\ k_{sf} & 0 & -k_{sf} & 0 & c_{sf} & 0 & -c_{sf} & 0 \\ 0 & -k_{sf} & k_{sf} & 0 & 0 & c_{sf} & 0 & -c_{sf} \\ 0 & m_{af} & m_{af} & 0 & 0 & 0 & m_{af} & 0 \\ 0 & m_{af} & m_{af} & 0 & 0 & 0 & m_{af} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & a_1 & a_2 & -\frac{1}{m_{af}} & 0 \\ 0 & 0 & 0 & a_2 & a_3 & 0 & -\frac{1}{m_{af}} \end{bmatrix}^T.$$
3. Integral sliding mode control

The actuator uncertainty $\Delta u(t)$ could affect ride comfort and driving stability, since the uncertainty might cause resonance of the sprung mass or the unsprung mass system. In order to secure the robustness against the uncertainty, this paper employs the integral sliding mode theory. Different from the conventional sliding mode control design approach, the order of the motion equation in integral sliding mode is equal to the order of the original system, rather than reduced by the dimension of the control input (Utkin, V., Guldner, J., and Shi, J., 1999). Because the actuator uncertainty fulfills the following matching condition obviously

$$\Delta u(t) \in \text{span}(B),$$

it is expected that the plant in the sliding mode will be invariant with respect to the uncertainty $\Delta u(t)$.

Figure 2 shows the block diagram of the integral sliding mode control system for the active suspension systems of the half-vehicle model. The control input vector $u(t)$ is defined as follows.

$$u(t) = u_0(t) + u_i(t)$$

where $F$ is the feedback gain vector. On the other hand, $u_i(t)$ is a nonlinear input vector to reject the influence of the actuator uncertainty $\Delta u(t)$.

$$u_i(t) = -\gamma \cdot \text{sign}(\sigma(t)),$$

where the relay gain matrix $\gamma$ is time-invariant. The switching function $\sigma(t)$ is generally defined as follows

$$\sigma(t) = Cx(t) - \int_0^t C\{Ax(\tau) + Bu_0(\tau) + Gw(\tau)\}d\tau.$$

Calculating the switching function $\sigma(t)$ based on Eq. (22) needs to measure the front and the rear terrain height velocities $\dot{z}_f(t)$ and $\dot{z}_r(t)$ of the vector $w(t)$ as shown in Eq. (12). If the coefficient vector $C$ is given by

$$C = [0_{2 \times 6} \quad I_{2 \times 2}],$$

$CG=0_{2 \times 2}$ can avoid difficulty of measuring those velocities. Then the switching function and the relay gain matrix are described as follows

$$\sigma(t) = Cx(t) - \int_0^t C\{Ax(\tau) + Bu_0(\tau)\}d\tau = [\sigma_f(t) \quad \sigma_r(t)]^T,$$

$$\gamma = \text{diag}(\gamma_f, \gamma_r).$$

With the equivalent control method (Utkin, V., 1992), the equivalent control input $u_{eq}(t)$ of the nonlinear input vector Eq. (21) in the sliding mode is given by

$$u_0(t) = -Fx(t),$$
That is, the equivalent control input \( u_{eq}(t) \) can behave to remove the influence of the uncertainty \( \Delta u(t) \). As a result, the plant enforced in the sliding mode has invariant property against the uncertainty \( \Delta u(t) \). In order to clarify the existing condition of the sliding mode on the relay gain matrix \( \gamma \), the following positive definite function is considered as a candidate of the Lyapunov function.

\[
V = \frac{1}{2} \sigma(t)^T \sigma(t)
\]

In order to become asymptotically stable, the time derivative of the Lyapunov function Eq. (27) should be negative definite as follows.

\[
\dot{V} = \sigma(t)^T \dot{\sigma}(t)
= \sigma(t)^T \left[ C \dot{x}(t) - C [A \dot{x}(t) + B \dot{u}_0(t)] \right]
= \sigma(t)^T \left[ C \left[ A \dot{x}(t) + B \left[ u_0(t) + u_1(t) + \Delta u(t) \right] + G \omega(t) \right] - C [A \dot{x}(t) + B \dot{u}_0(t)] \right]
= \sigma(t)^T \left[ C B \left[ u_1(t) + \Delta u(t) \right] \right]
= \left[ \gamma_f(t) \quad \gamma_r(t) \right] \left[ -\frac{1}{m_{uf}} \left\{ \gamma_f \left( \sigma_f(t) \right) + \Delta \gamma_f(t) \right\} \
-\frac{1}{m_{ur}} \left\{ \gamma_r \left( \sigma_r(t) \right) + \Delta \gamma_r(t) \right\} \right]
= -\frac{\sigma_f(t)}{m_{uf}} \left\{ \gamma_f \left( \sigma_f(t) \right) + \Delta \gamma_f(t) \right\} - \frac{\sigma_r(t)}{m_{ur}} \left\{ \gamma_r \left( \sigma_r(t) \right) + \Delta \gamma_r(t) \right\} < 0
\]

Then, the diagonal elements of the relay gain matrix \( \gamma \) should satisfy the following inequalities.

\[
\gamma_f > |\Delta \gamma_f(t)|
\]

\[
\gamma_r > |\Delta \gamma_r(t)|
\]
4. Redesign of nonlinear input

In order to introduce continuous inputs against chattering problem, the nonlinear input vector \( u_i(t) \) shown in Eq. (21) is redesigned based on the twisting algorithm of the second order sliding mode control (Levant, A., 1993) as follows.

\[
\mathbf{u}_i(t) = \begin{bmatrix} u_{i_1}(t) \\ u_{i_2}(t) \end{bmatrix},
\]

\[
U_i(s) = \text{diag}\left(\frac{\omega_j^2}{s^2 + 2 \zeta_j \omega_j s + \omega_j^2}, \frac{\omega_r^2}{s^2 + 2 \zeta_r \omega_r s + \omega_r^2}\right) U_2(s),
\]

\[
\mathbf{u}_2(t) = \begin{bmatrix} u_{2,1}(t) \\ u_{2,2}(t) \end{bmatrix} = \begin{bmatrix} -\gamma_{j1}\text{sign}(\sigma_j(t)) - \gamma_{j2}\text{sign}(\dot{\sigma}_j(t)) \\ -\gamma_{r1}\text{sign}(\sigma_r(t)) - \gamma_{r2}\text{sign}(\dot{\sigma}_r(t)) \end{bmatrix},
\]

where \( s \) denotes the Laplace operator, \( U_{i_1}(s), U_{i_2}(s), U_{2_1}(s) \) and \( U_{2_2}(s) \) are the Laplace transform of \( u_{i_1}(t), u_{i_2}(t), u_{2_1}(t) \) and \( u_{2_2}(t) \) respectively. \( \omega_j, \omega_r, \zeta_j, \zeta_r, \gamma_{j1}, \gamma_{j2}, \gamma_{r1}, \gamma_{r2} \) are the design parameters.

Then, the closed-loop system can be transformed into a nonlinear feedback system shown in Fig. 3 (a). In this analysis model, the linear part \( G \) is given by

\[
G(s) = \text{diag}(G_f(s), G_r(s)) = \text{diag}\left(\frac{\omega_j^2}{m_{u_f}s^2 + 2 \zeta_j \omega_j s + \omega_j^2}, \frac{\omega_r^2}{m_{u_r}s^2 + 2 \zeta_r \omega_r s + \omega_r^2}\right),
\]

and \( N(a) \) denotes the describing function of the nonlinear part as follows

\[
N(a) = \begin{bmatrix} N_f(a) \\ N_r(a) \end{bmatrix} = \begin{bmatrix} \frac{4}{\pi a}(\gamma_{j1} + \gamma_{j2}) \\ \frac{4}{\pi a}(\gamma_{r1} + \gamma_{r2}) \end{bmatrix},
\]

where the variable \( a \) denotes the amplitude of the limit cycle. With the describing function method for the analysis model, the existence condition for a limit cycle of the switching function Eq. (24) can independently be described for the front and the rear suspension as follows.

\[
G_f(j\omega) = -\frac{1}{N_f(a)},
\]

\[
G_r(j\omega) = -\frac{1}{N_r(a)},
\]

where \( \omega = j\omega \). The describing function method states that if Eqs. (36) and (37) have a solution \((a, \omega)\), then there might be a periodic solution of the system with frequency \( \omega \) and amplitude \( a \). Figure 3 (b) shows an interpretation of the describing function method in the complex plane. The redesigned input shown in Eqs. (32) and (33) are chosen, since the existing condition of the limit cycle is that the locus of \( G_f(j\omega) \) and the locus of \( -1/N_r(a) \) can cross in \((a, \omega)\) as shown in Fig. 3 (b).

The existence condition Eqs. (36) and (37) can be concretely described as follows.

\[
\frac{2(a m_{u_f} \pi \zeta_j \omega_j^2 - 2 \gamma_{j1} \omega_j^2)}{a \pi \omega_j} - \frac{4 \gamma_{j2} \omega_j^2 + a m_{u_f} \pi \omega_j (\omega_j^2 - \omega^2)}{a \pi \omega_j} j = 0
\]

(38)

\[
\frac{2(a m_{u_r} \pi \zeta_r \omega_r^2 - 2 \gamma_{r1} \omega_r^2)}{a \pi \omega_r} - \frac{4 \gamma_{r2} \omega_r^2 + a m_{u_r} \pi \omega_r (\omega_r^2 - \omega^2)}{a \pi \omega_r} j = 0
\]

(39)

From Eqs. (38) and (39) when \( \zeta_j \) and \( \zeta_r \) are set to a constant \( \zeta \) in the first step, the design parameters of the redesigned nonlinear input Eqs. (31) - (33) should be determined as follows.
\[ \omega > \omega_j, \omega_i \]  
\[ \gamma_{f1} = \frac{a_{mf} \pi \omega_j^2}{2 \omega_j} \]  
\[ \gamma_{f2} = \frac{a_{mf} \pi (\omega_j^2 - \omega_i^2)}{4 \omega_j^2} \]  
\[ \gamma_{r1} = \frac{a_{mr} \pi \omega_i^2}{2 \omega_i} \]  
\[ \gamma_{r2} = \frac{a_{mr} \pi (\omega_i^2 - \omega_j^2)}{4 \omega_i^2} \]  

5. Numerical simulation

Numerical simulations were carried out with the numerical analysis software MATLAB/Simulink. Table 1 shows the system parameter values for the half-vehicle model (Li, et al., 2013) shown in Fig.1. The movement speed of the half-vehicle was assumed to be 30 [km/h]. The velocity of the road disturbance was assumed to be a band-limited white Gaussian signal with mean zero. The sampling period for control was 1 [ms].

First, numerical simulations were carried out to investigate the accuracy of the limit cycles led by the proposed sliding mode controller. The accuracy of the limit cycle should be verified, since the describing function method is known as an approximation method of finding a periodic solution. Also the accuracy of the limit cycle for the design parameters should be verified, since there are various combinations of those parameter values under the existing conditions Eqs. (40) – (44). In the numerical simulations of this paper, the feedback gain vector \( F \) of the nominal control inputs shown in Eq. (20) was determined based on the sky-hook damper system theory, which generally feedbacks the body absolute velocity, as follows.

\[
F = \begin{bmatrix}
0 & 0 & 0 & -4000 & 0 & -4000 & 0 & -4000 & 0
\end{bmatrix}
\]

The (1, 5) and (2, 6) elements of the matrix Eq. (45) means the feedback of the front and the rear body absolute velocities in order to get good ride comfort. In addition to the theory, the feedback gain vector \( F \) employs the feedback of the front and the rear unsprung mass absolute velocities as shown in the (1, 7) an (2, 8) elements of Eq. (45). As a result, the driving stability can be expected to improve, because resonance of the unsprung masses is suppressed.
The design parameter $\zeta$ is assumed to have three kinds of values, such as 1, 0.75 and 0.5, and the design parameters $\omega_f$ and $\omega_r$ are assumed to have three kinds of same values, such as 0.9, 0.7 and 0.5. The desired amplitude $a$ of the limit cycles is 0.01 and the desired frequency $\omega$ of the limit cycles is 100 [rad/s]. Based on Eqs. (41) - (44), the remaining design parameters are calculated to each combination of the parameter $\zeta$ and the parameters $\omega_f$, $\omega_r$. The actuator uncertainty $\Delta u(t)$ is assumed to be

$$\Delta u(t) = -b \cdot u(t),$$

(46)

where the perturbation gain $b$ is a constant with three kind of values such as 0, 0.1 and 0.2. The value 0 means the nominal case, since the uncertainty $\Delta u(t)$ is zero. The value 0.1 and 0.2 mean that the actuator forces decrease 10% and 20% respectively.

Table 2 shows the accuracy of the limit cycles for the design parameter $\zeta$. From this table, it can be seen that the errors have a tendency to become small as the parameter $\zeta$ becomes large. Also, it can be seen that high robustness against the perturbation Eq. (46) is shown irrespective of the parameter $\zeta$. Table 3 shows the accuracy of the limit cycles to the design parameters $\omega_f$, $\omega_r$. From this table, it can be seen that the errors have a tendency to become small as the parameters $\omega_f$, $\omega_r$ become large. Also it can be seen that high robustness against the perturbation Eq. (46) is shown, though the errors are relatively large. From these tables, it can be seen that the desired limit cycle occurs with sufficient accuracy and high robustness especially when the parameter $\zeta$ is chosen to be 1 and the parameters $\omega_f$, $\omega_r$ are chosen to be 0.9.

Figure 4 shows the time response of the switching function for the actuator perturbation gain $b=0.2$. For the conventional sliding mode controller with pure relay inputs shown in Eq. (21), the plant state can be enforced into the sliding mode by the fast switching of the control input $u_1(t)$ shown in Fig. 5. As aforementioned, this fast switching input could deteriorate the control efforts especially in high frequency band. On the other hand, although the slight influence caused by the actuator uncertainty can be seen, the proposed sliding mode controller can generate the almost desired limit cycle as shown in Fig. 4. And the proposed sliding mode controller can realize continuous control inputs as shown in Fig. 5.

Figure 6 shows PSD for acceleration of center of the mass, and Fig. 7 shows PSD for angular acceleration of the pitch motion. Compared with the nominal case of the linear state feedback control, which employs only the control input shown in Eqs. (20) and (45), those figures show robustness against actuator uncertainty with the linear state and with the proposed sliding mode controller. From those figures, it can be seen that the proposed sliding mode controller can settle chattering for both acceleration of center of the mass and angular acceleration of the pitch motion in high frequency band of more than about 11 [Hz], which is the resonant frequency of the unsprung mass systems. Although chattering of the pure relay inputs Eq. (21) in the conventional sliding mode controller can obviously cause remarkable increase of those PSD values in the high frequency band, the proposed sliding mode controller can achieve the performance equal to the nominal case.

Table 4 shows robustness of the proposed sliding mode controller for the suspension strokes and the road holding forces against the actuator perturbation gain $b=0.2$. From this table as compared with the linear state feedback control, it can be seen that the proposed sliding mode controller can show high robustness while chattering is suppressed.
Table 1  System parameter values for the half-vehicle model (Li, et al., 2013)

<table>
<thead>
<tr>
<th>Actuator perturbation (b)</th>
<th>Design parameter</th>
<th>Front</th>
<th>Rear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m (_2)</td>
<td>m (_{2f})</td>
<td>k (_{2f})</td>
</tr>
<tr>
<td>0 (Nominal)</td>
<td>600 kg</td>
<td>40 kg</td>
<td>18000 N/m</td>
</tr>
<tr>
<td></td>
<td>1222 kgm(^2)</td>
<td>45 kg</td>
<td>22000 N/m</td>
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Table 2  Accuracy of limit cycle to design parameter \(\zeta\)
(Each cell shows average for \(\omega_f, \omega_r = 0.9, 0.7, 0.5\))

<table>
<thead>
<tr>
<th>Actuator perturbation (b)</th>
<th>Design parameter</th>
<th>Error of amp. [%]</th>
<th>Error of freq. [%]</th>
<th>Error of amp. [%]</th>
<th>Error of freq. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Nominal)</td>
<td>(\zeta)</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>36</td>
<td>9</td>
<td>36</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>69</td>
<td>34</td>
<td>83</td>
<td>34</td>
</tr>
<tr>
<td>0.1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>35</td>
<td>9</td>
<td>35</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
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<td>67</td>
<td>33</td>
</tr>
<tr>
<td>0.2</td>
<td>1</td>
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<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
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<tr>
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<td>0.5</td>
<td>69</td>
<td>30</td>
<td>67</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 3  Accuracy of limit cycle to design parameters \(\omega_f, \omega_r\)
(Each cell shows average for \(\zeta = 1, 0.75, 0.5\))

<table>
<thead>
<tr>
<th>Actuator perturbation (b)</th>
<th>Design parameters (\omega_f, \omega_r)</th>
<th>Error of amp. [%]</th>
<th>Error of freq. [%]</th>
<th>Error of amp. [%]</th>
<th>Error of freq. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (Nominal)</td>
<td>(\zeta)</td>
<td>0.9</td>
<td>29</td>
<td>4</td>
<td>42</td>
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<tr>
<td></td>
<td>0.7</td>
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<td></td>
<td>0.5</td>
<td>42</td>
<td>25</td>
<td>41</td>
<td>25</td>
</tr>
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Fig. 4  Time response of switching function

(a) Front suspension  
(b) Rear suspension

Fig. 5  Time response of control input \(u(t)\)

(a) Front suspension  
(b) Rear suspension

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(a) Linear state feedback control

(b) Proposed sliding mode control

Fig. 6 PSD of acceleration for center of mass against actuator uncertainty

(a) Linear state feedback control

(b) Proposed sliding mode control

Fig. 7 PSD of angular acceleration for pitch motion against actuator uncertainty
6. Conclusion

This paper proposed a design method of sliding mode controller to avoid chattering for active suspension systems of half-vehicle model. In the proposed sliding mode controller, the switching control input was redesigned by the describing function method in order to occur limit cycles of the switching function. Occurring the limit cycles instead of perfect sliding mode can lead continuous control inputs to suppress deterioration in high frequency band. The describing function method showed the existing condition of the limit cycles for the design parameters of the redesigned switching input. From numerical simulations, it could be checked that the proposed sliding mode controller could occur almost desired limit cycles of the switching function as determined the design parameter values adequately. Also, it could be seen that the proposed sliding mode controller could show high robustness against actuator uncertainty while it could suppress chattering in high frequency band without any force sensors.

Because this paper first indicates the basic validity of the proposed sliding mode controller, the design parameters of the proposed controller are tuned by trial and error in only one driving condition. As future work, the design method of the design parameters that makes the active suspension system robust against the uncertainty of the actuator in any road conditions should be clarify. As an idea to lead the design method, the existing condition of the limit cycle could be investigated for other analysis model with the actuator uncertainty, since this paper employs the analysis model without the actuator uncertainty in order to make the analysis easy.

References


Suzuki, T., and Takahashi, M., Active Suspension Control Considering Lateral Vehicle Dynamics due to Road Input,


