Self-identification method of arrangement and effective pressure areas for a vibration-isolation table supported with a redundant number of pneumatic actuators

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Abstract

For the purpose of developing a vibration-isolation table with flexibility and/or scalability (which means “with adaptability to desired load specification”), this study investigates a self-identification method with regard to positions or arrangement and effective pressure areas of pneumatic actuators which support a vibration-isolation table. The proposed identification method is a fundamental technique necessary to construct a scalable vibration-isolation table in which the actuators supporting the table can be added or removed flexibly according to the load demand, easily, and in a short period of time. First, this study constructs a mathematical model for a vibration-isolation table supported by a redundant number of actuators and a type 1 digital servo controller to control the position and inclination angles of the table. Next, we derive the identification method for obtaining the position of the added pneumatic actuator and the effective pressure areas of the pneumatic actuators. We then demonstrate the proposed identification method by numerical simulations using our detailed mathematical model. Finally, this study shows an example of the application of the proposed identification method coupled with a force redistribution technique by addition of an actuator into the vibration-isolation table system during continuous operation. The validation and success of our method was confirmed by the displacement and pressure responses of the vibration-isolation table.

Key words : Vibration-isolation table, Pneumatic actuator, Redundant actuator, Scalability, Identification

1. Introduction

Vibration-isolation tables have been widely used to support semiconductor exposure apparatuses among other applications, and are used in the prevention of transmitted vibrations from mechanical devices mounted on the apparatuses themselves as well as through the ground. For example, the vibration-isolation is necessary not only to protect the apparatuses from the damage by strong vibration due to earthquakes, but also to prevent the semiconductor apparatuses from any vibrations through the exposure-processes, because the vibrations directly deteriorate the processing resolution and throughput in photo-lithography used in the semiconductor exposure apparatuses. Much research has been devoted to active vibration control for vibration-isolation tables and is further being investigated to achieve higher vibration-isolation capability by many researchers including Erin, et al. (1998), Kim, et al. (2001), Zuo, et al. (2005), Hashimoto, et al. (2007), Hoshino, et al. (2007, 2009, 2010), Shirani and Wakui (2010), Shin, et al. (2010), and Chen and Shih (2010), to name a few.

In recent years, product development cycles have become shorter in the manufacturing industries. Lowering energy consumption in manufacturing is required for reducing not only the manufacturing cost but also environmental load.

Therefore, a more flexible production process, which can be reconstructed easily according to fast changes in market demands, is required for the future of manufacturing. Since semiconductor production apparatuses have become bigger due to the high demand for large-scale devices such as liquid crystal displays (LCDs), large-scale vibration-isolation tables with better vibration-isolation performance are necessary to support and avoid vibrations from these apparatuses. Therefore, the aforementioned flexible production process must strongly adhere to semiconductor production apparatuses that ensure reduction in both manufacturing cost and energy consumption.

For the purpose of developing such a scalable vibration-isolation table with adaptability to the desired load specification, this study investigates a self-identification method of positions and effective pressure areas of a redundant number of pneumatic actuators which support the vibration-isolation table. The proposed identification method gives a fundamental technique necessary to construct a scalable vibration-isolation table which can increase or decrease the number of actuators supporting the table flexibly, easily, and in a short period of time. This study demonstrates the proposed method by numerical simulations using the detailed model of the vibration-isolation table supported with a redundant number of pneumatic actuators.

2. Nomenclature

x_i, y_i	: Positions along the x - and y -axes of the i -th actuator.
z_{ai}	: Total displacement of the i -th pneumatic actuator.
x_{si}, y_{si}	: Positions of the displacement sensor along the x - and y -axes fixed on the i -th actuator.
N	: Number of pneumatic actuators.
M	: Mass of the table.
J_x, J_y	: Moments of inertia around the x - and y -axes of the table.
z	: Displacement of the gravity center of table along the z -axis.
θ_x, θ_y	: Inclination angle around the x - and y -axes of the table.
A_{ai}	: Effective pressure area of the i -th pneumatic actuator.
p_i	: Pressure of the i -th pneumatic actuator.
p_s	: Atmospheric pressure.
$p_{\phi i}$	: Output pressure of the pneumatic regulator of the i -th actuator.
f_i	: Force generated by the i -th actuator.
m_i	: Mass of air in the i -th pneumatic actuator.
V_i	: Volume of air in the i -th pneumatic actuator.
q_{mi}	: Mass flow rate into the i -th actuator.
c_i	: Proportional constant related with $p_{\phi i}$, p_i and q_{mi} .
ϕ_i	: Rotational angle of the stepping motor of the i -th pneumatic regulator.
C_p, C_v	: Specific heat at constant pressure and specific heat at constant volume.
κ	: Specific heat ratio (C_p/C_v).
R	: Gas constant of air.
L	: Initial length of the pneumatic actuator.
T_0	: Temperature of air in the pneumatic actuator at the equilibrium state.
z_{a0i}	: Displacement of the i -th pneumatic actuator at the equilibrium state.
p_{0i}	: Pressure of air in the i -th pneumatic actuator at the equilibrium state.
V_{0i}	: Volume of air in the i -th pneumatic actuator at the equilibrium state.
z_{ai}	: Total displacement of the i -th pneumatic actuator at the equilibrium state.
μ_i	: Constant with respect to the response between output pressure $p_{\phi i}$ and ϕ_i of the i -th pneumatic regulator.
H_i	: Constant with respect to the relationship between output pressure $p_{\phi i}$ and ϕ_i of the i -th pneumatic regulator.
u_i	: Input pulse per unit time for the stepping motor of the i -th pneumatic regulator.
K_i	: Rotation angle per one pulse input to the stepping motor of the i -th pneumatic regulator.
F_e	: External force in the direction along the z -axis acting at the center of the table.
M_{xe}, M_{ye}	: External moments around the x - and y -axes acting at the table.
g	: The acceleration due to gravity.
C_j	: The j -th equilibrium condition.
N_c	: Number of equilibrium conditions.

Fig. 1 3DOF model of the vibration-isolation table supported with a redundant number of actuators.

Fig. 2 Pneumatic actuator controlled by a pneumatic regulator driven by a stepping motor.

- $\dot{(\circ)}$: Differentiation operator with respect to time for the symbol $(\circ)$.
- $\Delta(\circ)$: Backward difference operator for the symbol $(\circ)$.
- Δt : Sampling period of the controller.
- $\hat{(\circ)}$: Symbol which indicates identified values for the symbol $(\circ)$.
- k : Number of sample steps of the controller.

3. Mathematical model and controller design

3.1. Mathematical model for the vibration-isolation table

This study derives a mathematical model for a vibration-isolation table supported with a redundant number of actuators as shown in Fig. 1 (Hoshino et al., 2012, 2013, Maeda et al., 2013). The coordinate system O-xyz is defined as an orthogonal coordinate system. The z-axis passes through the center of gravity of the table in the equilibrium state, and z represents the vertical displacement of the table. The inclination angles of the table around the x- and y-axes passing through the center of gravity of the table are represented by θ_x and θ_y , respectively. This study considers the 3 degrees of freedom (DOF) of the table's motion. The table is supported by a redundant number of pneumatic actuators, which are driven by stepping motors as shown in Fig. 2. The equation of motion for the table is expressed as

$$M\ddot{X} = G_1 A_a (p - p_s) - Mg + F_e, \quad (1)$$

where

$$M = \text{diag}[M \ J_x \ J_y], \quad g = [g \ 0 \ 0]^T, \quad A_a = \text{diag}[A_{a1} \ \cdots \ A_{aN}],$$

$$G_1 = \begin{bmatrix} 1 & \cdots & 1 \\ y_1 & \cdots & y_N \\ -x_1 & \cdots & -x_N \end{bmatrix}, \quad X = \begin{bmatrix} z \\ \theta_x \\ \theta_y \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix}, \quad p_s = \begin{bmatrix} p_s \\ \vdots \\ p_s \end{bmatrix}_{N \times 1}, \quad F_e = \begin{bmatrix} F_e \\ M_{xe} \\ M_{ye} \end{bmatrix}_{N \times 1}.$$

Here, the mass of the table is represented by M , and the moments of inertia about the x- and y-axes are represented by J_x and J_y , respectively. The effective pressure area and the pressure of air of the i -th actuator are represented by A_{ai} and p_i , respectively. The acceleration due to gravity and the atmospheric pressure are represented by g and p_s , respectively. The matrix G_1 is constructed from the positions of the actuators. In addition, the external force acting on the table is represented by F_e , and the external moments about the x- and y-axes acting on the table are represented by M_{xe} and M_{ye} . The number of actuators is represented by N and the relationship between the pressure p_i and the total displacement z_{ai} of the i -th pneumatic actuator is expressed as follows (Erin, et al., 1998):

$$\dot{p}_i = -\kappa \frac{p_{0i} A_{ai}}{V_{0i}} \dot{z}_{ai} - \kappa \frac{RT_{0i} c_i}{V_{0i}} p_i + \kappa \frac{RT_{0i} c_i}{V_{0i}} p_{\phi i} \quad (2)$$

where $\kappa = C_p/C_v$ represents specific heat ratio determined by the specific heat at constant volume, C_v , and the specific heat at constant pressure, C_p . The gas constant is represented by R , and c_i represents the proportionality constant related with the pressure p_i , the output pressure of pneumatic regulator $p_{\phi i}$, and the mass flow rate q_{mi} of the i -th actuator. There

are short time delays in the responses from the rotations of the stepping motors ϕ_i to the output pressures of the pneumatic regulators $p_{\phi i}$. The relation between ϕ_i and $p_{\phi i}$ is expressed as a first order delay system,

$$\dot{p}_{\phi i} = \mu_i(H_i\phi_i - p_{\phi i}). \quad (3)$$

The rotation angles of the stepping motors that control the output pressures of the regulators are expressed as

$$\dot{\phi}_i = K_u u_i \quad (4)$$

where K_u represents the proportionality constants. Also, z_{ai} in Eq. (2) satisfies

$$[\dot{z}_{a1} \ \cdots \ \dot{z}_{ai} \ \cdots \ \dot{z}_{aN}]^T = \mathbf{G}_1^T \dot{\mathbf{X}} \quad (5)$$

from the geometric relationship of the position of the actuators. From Eqs. (1)–(5), the state equations for the vibration-isolation table are expressed as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{\phi} + \mathbf{D}\mathbf{p}_s + \mathbf{E}_g\mathbf{g} + \mathbf{E}_e\mathbf{F}_e, \quad (6)$$

$$\dot{\boldsymbol{\phi}} = \mathbf{K}_u\mathbf{u}, \quad (7)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}^{-1}\mathbf{G}_1\mathbf{A}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_1\mathbf{G}_1^T & -\mathbf{W}_2 & \mathbf{W}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\boldsymbol{\mu} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\mu}\mathbf{H} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{G}_1\mathbf{A}_a \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{E}_g = \begin{bmatrix} \mathbf{0} \\ -\mathbf{I}_{3 \times 3} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{E}_e = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$\mathbf{x} = [\mathbf{X}^T \ \dot{\mathbf{X}}^T \ | \ \mathbf{p}^T \ \mathbf{p}_\phi^T]^T, \quad \mathbf{u} = [u_1 \ \cdots \ u_N]^T, \quad \boldsymbol{\phi} = [\phi_1 \ \cdots \ \phi_N]^T, \quad \mathbf{p}_\phi = [p_{\phi 1} \ \cdots \ p_{\phi N}]^T,$$

$$\boldsymbol{\mu} = \text{diag}[\mu_1 \ \cdots \ \mu_N], \quad \mathbf{H} = \text{diag}[H_1 \ \cdots \ H_N], \quad \mathbf{K}_u = \text{diag}[K_u \ \cdots \ K_u]_{N \times N},$$

$$\mathbf{W}_1 = \text{diag} \left[-\frac{\kappa p_{01} A_{a1}}{V_{01}} \ \cdots \ -\frac{\kappa p_{0N} A_{aN}}{V_{0N}} \right], \quad \mathbf{W}_2 = \text{diag} \left[\frac{\kappa R T_0 c_1}{V_{01}} \ \cdots \ \frac{\kappa R T_0 c_N}{V_{0N}} \right].$$

3.2. Controller for position and inclination angle of the vibration-isolation table

As an example, this study utilizes a type 1 digital optimal servo controller to control and stabilize the position and inclination angle of the vibration-isolation table (Hoshino et al., 2012, 2013, Maeda et al., 2013). To construct the type 1 digital servo controller effectively using the integral characteristics of the stepping motor given by Eq. (7), the discrete-time state equation of state equation (6) is reconstructed as

$$\mathbf{x}(k+1) = \mathbf{A}_d\mathbf{x}(k) + \mathbf{B}_d\boldsymbol{\phi}(k) + \mathbf{D}_d\mathbf{p}_s + \mathbf{E}_{gd}\mathbf{g} + \mathbf{E}_{ed}\mathbf{F}_e, \quad (8)$$

where

$$\mathbf{y}(k) = \mathbf{C}_d\mathbf{x}(k) = [z(k) \ \theta_x(k) \ \theta_y(k)]^T, \quad (9)$$

and $\mathbf{A}_d = e^{\mathbf{A}\Delta t}$, $\mathbf{B}_d = \left(\int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \right) \mathbf{B}$, $\mathbf{C}_d = \mathbf{C}$, $\mathbf{D}_d = \left(\int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \right) \mathbf{D}$, $\mathbf{E}_{gd} = \left(\int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \right) \mathbf{E}_g$, $\mathbf{E}_{ed} = \left(\int_0^{\Delta t} e^{\mathbf{A}\tau} d\tau \right) \mathbf{E}_e$. The backward difference of the state vector, $\Delta\mathbf{x}(k+1) = \mathbf{x}(k+1) - \mathbf{x}(k)$, is expressed as

$$\Delta\mathbf{x}(k+1) = \mathbf{A}_d\Delta\mathbf{x}(k) + \mathbf{B}_d\Delta\boldsymbol{\phi}(k). \quad (10)$$

The error signal is defined as

$$\mathbf{e}(k) = \mathbf{R}_r(k) - \mathbf{y}(k), \quad (11)$$

where $\mathbf{R}_r(k) = [z_r(k) \ \theta_{xr}(k) \ \theta_{yr}(k)]^T$ represents the reference signal for vertical displacement and inclination angle of the table. From Eqs. (10) and (11), the extended state equation is obtained as

$$\begin{bmatrix} \mathbf{e}(k+1) \\ \Delta\mathbf{x}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{C}_d\mathbf{A}_d \\ \mathbf{0} & \mathbf{A}_d \end{bmatrix} \begin{bmatrix} \mathbf{e}(k) \\ \Delta\mathbf{x}(k) \end{bmatrix} + \begin{bmatrix} -\mathbf{C}_d\mathbf{B}_d \\ \mathbf{B}_d \end{bmatrix} \Delta\boldsymbol{\phi}(k) + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta\mathbf{R}_r(k+1). \quad (12)$$

When the reference signal $\mathbf{R}_r(k)$ does not change, $\Delta\mathbf{R}_r(k+1) = \mathbf{0}$ is satisfied. Therefore,

$$\mathbf{X}_0(k+1) = \boldsymbol{\Phi}\mathbf{X}_0(k) + \boldsymbol{\Gamma}\Delta\boldsymbol{\phi}(k) \quad (13)$$

is obtained, where $X_0 = [e^T(k) \Delta x^T(k)]^T$. The type 1 optimal servo controller can be realized by the following input

$$\Delta \phi(k) = [K_e \ K_x]X_0(k) = KX_0(k), \quad (14)$$

where K_e and K_x are optimal feedback gain, which minimizes the evaluation function

$$J = \frac{1}{2} \sum_{k=1}^{\infty} [X_0^T(k)QX_0(k) + \Delta \phi^T(k)R\Delta \phi(k)], \quad (15)$$

determined from optimal regulator theory. Here, Q and R are positive-definite weighting matrices.

4. Identification of actuator position and effective pressure areas

4.1. Identification of actuator position

This study develops an identification method of the inserted position of an additional actuator. The additional actuator, which is not included initially in the closed-loop servo-system, is inserted at the position indicated in Fig. 1. Assuming that the external force F_e is generated by the added actuator inserted at position (x_{N+1}, y_{N+1}) , F_e is expressed as

$$F_e = G_{\text{add}}(x_{N+1}, y_{N+1})f_{N+1}, \quad (16)$$

where

$$G_{\text{add}}(x_{N+1}, y_{N+1}) = [1 \ y_{N+1} \ -x_{N+1}]^T, \quad f_{N+1} = A_{N+1}(p_{N+1} - p_s).$$

When displacement X_0 is kept in state $X_0 = \mathbf{0}$ by using the output pressures of the actuators p excepting the $(N+1)$ -th actuator and applying the servo controller introduced in previous section, Eq. (1) is rewritten as

$$G_1 A_a(p - p_s) - Mg + G_{\text{add}}(x_{N+1}, y_{N+1})f_{N+1} = \mathbf{0} \quad (17)$$

for the equilibrium state. The identification method of the inserted position of the additional actuator is expressed as the problem in solving the matrix $G_{\text{add}}(x_{N+1}, y_{N+1})$ of Eq. (17) by applying the force f_{N+1} , which is generated by the inserted actuator as

$$G_{\text{add}}(\hat{x}_{N+1}, \hat{y}_{N+1}) = \begin{bmatrix} 1 \\ \hat{y}_{N+1} \\ -\hat{x}_{N+1} \end{bmatrix} = \frac{Mg - G_1 A_a(p - p_s)}{f_{N+1}} = \frac{Mg - G_1 A_a(p - p_s)}{A_{a(N+1)}(p_{N+1} - p_s)}, \quad (18)$$

where the hat ($\hat{\cdot}$) symbol indicates the identified or estimated value. However, when the effective pressure areas, A_{ai} ($i = 1, \dots, N+1$), are difficult to measure directly, estimation is necessary to identify the position of the inserted actuator.

4.2. Identification of effective pressure areas

From the first row of the vectors in Eq. (17), the equilibrium condition of the force along the z -axis is given as

$$(p - p_s)^T [A_{a1} \ \cdots \ A_{aN}]^T + (p_{N+1} - p_s)A_{a(N+1)} = ([p_1 \ \cdots \ p_N | p_{N+1}] - (p_{s(N \times 1)})^T) A_a^* = Mg. \quad (19)$$

Given more than $(N+1)$ different equilibrium conditions achieved by changing the pressure, p_{N+1} , generated by the inserted actuator, we can construct a matrix equation

$$\begin{bmatrix} [p_1 \ \cdots \ p_N | p_{N+1}]_{C_1} - (p_{s(N \times 1)})^T \\ \vdots \\ [p_1 \ \cdots \ p_N | p_{N+1}]_{C_{N_c}} - (p_{s(N \times 1)})^T \end{bmatrix} A_a^* = \begin{bmatrix} Mg \\ \vdots \\ Mg \end{bmatrix}, \quad (20)$$

where C_j and N_c ($N_c \geq N+1$) represents the j -th equilibrium condition and the number of equilibrium conditions, respectively. Rewriting Eq. (20) as

$$\Pi A_a^* = \Psi, \quad (21)$$

the vector A_a^* can be solved by applying the least squares method or by using the pseudo inverse matrix $\Pi^\dagger$ as

$$A_a^* = \Pi^\dagger \Psi. \quad (22)$$

Finally, we can estimate the effective pressure areas of the pneumatic actuators by Eq. (22) and the position of the inserted actuator by Eq. (18).

Fig. 3 Identification procedure of the effective pressure areas of pneumatic actuators.

4.3. Identification procedure

The identification procedure using Eq. (22) is shown in Fig. 3. Equilibrium conditions C_j ($j = 1, \dots, N_c$) in Eq. (22), necessary for identification of effective pressure areas, are realized by using the additional actuator output. Adding an actuator at the approximately appropriate position as shown in Fig. 1 and applying force f_{N+1} to the closed loop system (type 1 digital servo system given by Eqs. (10) and (14)), the closed loop system converges to an equilibrium state C_j . In the equilibrium condition C_j , both $e(k) \Rightarrow 0$ and Eq. (19) are satisfied under different output pressures, p_{N+1} . Constructing the matrices H and Ψ , the effective pressure areas are identified as A_a^* by Eq. (22). Further, by applying Eq. (18), the position of the added actuator (x_{N+1}, y_{N+1}) is estimated.

5. Results of Simulation

Parameters of the vibration-isolation table are shown in Table 1. In this study, the number of actuators, N , are set to eight initially, and arrangement of the eight actuators at initial conditions is shown in Table 2. Setting the weighting

Table 1 Parameters of the mathematical model for vibration-isolation table.

M	5.30×10^2	kg	g	9.80	m/s ²
J_x	29.6	kg·m ²	R	287	J/(kg·K)
J_y	30.1	kg·m ²	κ	1.40	—
T_0	293	K	p_{0i}	0.184	MPa
z_{0i}	2.00×10^{-2}	m	K_{ti}	5.03×10^{-5}	rad/pulse
p_s	0.1013	MPa	H_i	2.20×10^4	Pa/rad
L_i	1.00×10^{-2}	m	γ_i	55.0	1/s
A_{ai}	7.85×10^{-3} (Nominal value)	m ²	c_i	6.30×10^{-7}	m·s

Table 2 Positions of pneumatic actuators at the initial conditions.

#1	x_1	-272	mm	#5	x_5	272	mm
	y_1	-272	mm		y_5	272	mm
#2	x_2	0	mm	#6	x_6	0	mm
	y_2	-272	mm		y_6	272	mm
#3	x_3	272	mm	#7	x_7	-272	mm
	y_3	-272	mm		y_7	272	mm
#4	x_4	272	mm	#8	x_8	-272	mm
	y_4	0	mm		y_8	0	mm

matrices $\mathbf{Q}$ and $\mathbf{R}$ in Eq. (15) to

$$\mathbf{Q} = \text{diag}[10^3 \ 10^2 \ 10^2 \mid 10^3 \ 10^2 \ 10^2 \mid 10^2 \ 10^1 \ 10^1 \mid \underbrace{10^{-10} \ \cdots \ 10^{-10}}_N \mid \underbrace{0 \ \cdots \ 0}_N], \mathbf{R} = \text{diag}[\underbrace{1 \ \cdots \ 1}_N], \quad (23)$$

the feedback gain of a type 1 digital servo system, $\mathbf{K}$, is determined. First, the vibration-isolation table is supported by the eight pneumatic actuators, and the position and inclination angles are set to and kept in $z = 0$ mm, $\theta_x = 0$ rad, and $\theta_y = 0$ rad by the servo controller. Next, we insert a pneumatic actuator at the position $(x_{N+1}, y_{N+1}) = (0.136$ mm, 0.136 mm). It is assumed that the added actuator position is only approximately known, however, the position (x_{N+1}, y_{N+1}) can be estimated by using the proposed method as shown in the following subsection.

5.1. Identification results

First, the effective pressure areas are identified by the procedure shown in Fig. 3. The initial output force of the added actuator, f_{N+1} , is increased by 2 N steps by a step function every 3 s from 0 N to 1600 N ($N_c = 800$). The steady-state pressures of all actuators are obtained to construct the matrix $\mathbf{II}$ in Eq. (21) in each step. The identified effective pressure areas are obtained by Eq. (22) and the identification results of the effective pressure areas are shown in Table 3. Here, actual values with uncertainty are assumed as unknown values and identified by the proposed method. The actual and identified values are shown in the upper and lower lines of Table 3, respectively. Here, relative identification errors of the effective pressure areas are calculated by

$$\frac{\hat{A}_{ai} - A_{ai}}{A_{ai}} \times 100 \text{ [\%]}. \quad (24)$$

Identification results which have less than approximately 8 % of the relative identification errors are obtained.

Next, using the estimated results of the effective pressure areas, the position of the added actuator is also estimated by Eq. (18). Identification result of the position of the added actuator is shown in Table 4. Actual values assumed as unknowns and the identified values are shown in the left and right columns of Table 4, respectively. The obtained results were achieved to a very high degree of accuracy. Here, the relative estimation errors calculated by

$$\frac{\hat{x}_9 - x_9}{x_9} \times 100 \text{ [\%]}, \quad \frac{\hat{y}_9 - y_9}{y_9} \times 100 \text{ [\%]} \quad (25)$$

are indicated in the parentheses in Table 4.

5.2. Application example

As an example of an application of the proposed identification method, this section demonstrates the method for increasing the number of actuators of the vibration-isolation table (supported by a redundant number of actuators) while working continuously, and shows the method for obtaining adaptability to the desired load specification. Note that it is difficult to construct the method for decreasing the number of actuators because the method has to realize removing the active and working actuator smoothly and changing the controller continuously, so the authors are currently investigating

Table 3 Identification results of the effective pressure areas.

No.	Effective pressure areas [m ²]	Relative error [%]	No.	Effective pressure areas [m ²]	Relative error [%]
#1	A_{a1}	7.46×10^{-3}	#5	A_{a5}	8.01×10^{-3}
	$\hat{A}_{a1}$	7.86×10^{-3}		$\hat{A}_{a5}$	8.08×10^{-3}
#2	A_{a2}	8.25×10^{-3}	#6	A_{a6}	8.64×10^{-3}
	$\hat{A}_{a2}$	8.47×10^{-3}		$\hat{A}_{a6}$	8.60×10^{-3}
#3	A_{a3}	7.62×10^{-3}	#7	A_{a7}	7.78×10^{-3}
	$\hat{A}_{a3}$	7.77×10^{-3}		$\hat{A}_{a7}$	7.15×10^{-3}
#4	A_{a4}	7.23×10^{-3}	#8	A_{a8}	8.17×10^{-3}
	$\hat{A}_{a4}$	6.84×10^{-3}		$\hat{A}_{a8}$	8.21×10^{-3}
			#9 (Added)	A_{a9}	7.85×10^{-3}
				$\hat{A}_{a9}$	7.75×10^{-3}

Table 4 Identification results of the position of the added actuator.

#9 (added)	x_9	136	mm	#9 (Estimated)	$\hat{x}_9$	136.0059	mm (0.0043 %)
	y_9	136	mm		$\hat{y}_9$	136.0048	mm (0.0035 %)

Fig. 4 Procedure for increasing the number of actuators which support a vibration-isolation table.

the method for decreasing the number of actuators smoothly. The procedure is outlined in Fig. 4. First, applying the identification method of the effective pressure areas given by Eq. (22), the effective pressure areas A_a are estimated and the position of the $(N + 1)$ -th added actuator is estimated by Eq. (18). The servo controller K for the vibration-isolation table supported by $(N + 1)$ actuators is prepared. Applying force redistribution (Hoshino et al., 2012, Maeda et al., 2013) represented by a redistribution matrix $W(k)$ as

$$A_a K_1 \phi'(k) = W(k) G_1 A_a K_1 \phi(k), \quad (26)$$

the $(N + 1)$ -th actuator is introduced into the servo controller (which means the $(N + 1)$ -th added actuator is activated as one of the actuators of the servo controller). Here, the forces generated by each actuator to yield $F^* = [F_x^*, M_x^*, M_y^*]^T$ are obtained by the least-squares method (Hoshino et al., 2012, Maeda et al., 2013). For adequate distribution of output force on the i -th actuator, a cost function is defined as

$$S = \frac{1}{2} \alpha_F (F - F^*)^2 + \frac{1}{2} \alpha_x (M_x - M_x^*)^2 + \frac{1}{2} \alpha_y (M_y - M_y^*)^2 + \frac{1}{2} \alpha_\Delta \sum_{i=1}^{N+1} \left(f_i - \frac{F}{N} \right)^2 + \frac{1}{2} \sum_{i=1}^{N+1} \gamma_i f_i^2, \quad (27)$$

where α_F , α_x , α_y , α_Δ , and γ_i represent non-negative weighting coefficients. Additional details are given in the references (Hoshino et al., 2007, 2010, 2012, Maeda et al., 2013). The fifth term is the sum of the squares of the forces generated by the actuators and is introduced to consider the conditions where the force distribution on the i -th actuator is stopped and, consequently, the force is appropriately or optimally redistributed onto the other actuators. The force vector $f = A_a(p - p_s)$ that minimizes the cost function (27) can be expressed as

$$f = W F^* \quad (28)$$

where W represents the transformation matrix obtained by the least-squares method that minimizes the cost function (27). Setting γ_i to a large value, the pressure distributed on the i -th actuator is stopped (Hoshino et al., 2012, Maeda et al.,

2013). Setting $\gamma_{N+1}(k)$ to 1000, which is sufficiently large value to stop the distribution (Hoshino et al., 2012, Maeda et al., 2013), $W(k)$ is derived to stop the distribution on the $(N + 1)$ -th actuator. Then the controller is switched from the controller for the vibration-isolation table supported by N actuators to one which uses $(N + 1)$ actuators. In this study, the type 1 digital servo controller is determined by substituting $(N + 1)$ into N in Eq. (23). Decreasing $\gamma_{N+1}(k)$ step-by-step to 0 and updating $W(k)$ at the each sample step of the controller, the $(N + 1)$ -th actuator is activated gradually and the distributed force on the $(N + 1)$ -th actuator increases gradually as well. Although W has been derived by assuming the weighting coefficient γ_{N+1} with a constant value stated in the reference (Hoshino et al., 2012, Maeda et al., 2013), this paper extends the method for the case when γ_{N+1} is changed step-by-step at each sample step of the controller.

From Eq. (26), force $f'(k)$ obtained by $W(k)$ at the sample step k and $(k - 1)$ are expressed as

$$f'(k) = A_a(p'(k) - p_s) = A_a K_1(\phi'(k) - \phi_0) = W(k) G_1 A_a K_1 \left\{ \phi(0) + \sum_{i=1}^k \Delta\phi(i) - \phi_0 \right\}, \quad (29)$$

and

$$f'(k-1) = A_a(p'(k-1) - p_s) = A_a K_1(\phi'(k-1) - \phi_0) = W(k-1) G_1 A_a K_1 \left\{ \phi(0) + \sum_{i=1}^{k-1} \Delta\phi(i) - \phi_0 \right\}, \quad (30)$$

where $p_s = K_1 \phi_0$. Subtracting Eq. (30) from (29), the backward difference of the redistributed force is derived as

$$\Delta f'(k) = A_a K_1 \Delta\phi'(k) = \{W(k) - W(k-1)\} G_1 A_a K_1 \left\{ \phi(0) + \sum_{i=1}^{k-1} \Delta\phi(i) - \phi_0 \right\} + W(k) G_1 A_a K_1 \Delta\phi(k). \quad (31)$$

Substituting

$$\phi(k) = \phi(0) + \sum_{i=1}^k \Delta\phi(i)$$

into Eq. (31), the dynamically redistributed control input $\Delta\phi'(k)$ obtained from $\Delta\phi(k)$ is derived as

$$\Delta\phi'(k) = K_1^{-1} A_a^{-1} [W(k) G_1 A_a K_1 \Delta\phi(k) + \{W(k) - W(k-1)\} G_1 A_a K_1 \{\phi(k-1) - \phi_0\}]. \quad (32)$$

Note that the aforementioned procedure, as shown in Fig. 4, can be applied recursively to accommodate any increase in the number of actuators.

Simulation results are shown in Fig. 5. In this simulation, the vibration-isolation table is supported by eight pneumatic actuators with position z and inclination angles (θ_x, θ_y) controlled by the type 1 digital servo system designed for the model

Fig. 5 Time histories of the pressures of actuators and the displacement z and the inclination angles (θ_x, θ_y) of table when increasing the number of actuators from 8 to 9 by using the proposed procedure. In this simulation, the vibration-isolation table is supported by eight actuators and position z and inclination angles (θ_x, θ_y) are controlled by the type 1 digital servo system designed for the model with eight actuators, and then actuator #9 is inserted. By first setting γ_9 to 1000 to stop distribution of the force on the actuator #9, the controller is switched to the model with nine actuators. Then, decreasing γ_9 gradually, the distribution of the force on the actuator #9 is increasing after 8 s.

Fig. 6 Enlarged view of the actuator pressure #9. This figure shows enlarged view of the region around 8 s indicated by the square in the figure of actuator pressure #9 of Fig. 5 (a) and shows that smooth change in distribution rate is achieved.

of table supported by eight actuators. Then, actuator #9 was placed at position $(x_9, y_9) = (136 \text{ mm}, 136 \text{ mm})$. By first setting γ_9 to 1000 to stop distribution of the force on the actuator #9, which then redistributes the force optimally to the other actuators by the proposed redistribution method in Eq. (32), the controller is switched to the model which contains nine actuators. Then, decreasing γ_9 gradually, the distribution of the force on the actuator #9 is increasing from around 8 s, while the actuator forces generated by all actuators (including the actuator #9) are maintaining appropriate force distribution by the redistribution method Eq. (32). Figure 6 shows enlarged view of the region indicated by the square in the figure of actuator pressure #9 of Fig. 5 (a). We can see smooth change in distribution rate from Fig. 6. Finally, the displacement and inclination angles are controlled by using all nine actuators. Through this simulation, smooth responses in the displacement and inclination angles were observed despite switching both the controller and the number of actuators (Fig. 5 (b)).

6. Conclusions

This study investigated a self-identification method of positions and effective pressure areas of pneumatic actuators which support a vibration-isolation table containing a redundant number of actuators. The proposed identification method can be represented simply by a self-identification algorithm. This study evaluated the accuracy of the estimation values and reliability of the estimated parameters for constructing the controller such as linear quadratic regulators (LQR) to configure the control system automatically. By using the proposed method, we demonstrated the simple addition of an actuator into the vibration-isolation system according to the load change. In addition, by using this process recursively, the method can be extended to accommodate the desired number of actuators into the active control system of the vibration-isolation table.

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