Non-contact identification of rotating blade vibration

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Abstract

This paper presents a non-contact measurement and diagnostic method for the parametric identification of vibrations of rotating engine blades, based on blade tip-timing (BTT) measured by optical sensors. Because of the inherent under-sampling nature of BTT measurements, effective algorithms are needed to extract key vibration parameters such as frequency and amplitude from the measurement. In this paper, an Enhanced Estimation of Signal Parameters via Rotational Invariance Technique (E-ESPRIT) is proposed. The main advantage of this technique is its ability to analyze both single and multi-mode blade vibrations spreading across a wide dynamic range, while accommodating the effect of varying rotational speeds and sensor installation errors. Analysis and numerical simulation have shown that the method can effectively improve the accuracy and robustness of vibration frequency and amplitude estimation compared to traditional ESPRIT.

Key words: Blade vibration, Non-contact measurement, Blade tip timing, Under-sampled signal, ESPRIT

1. Introduction

Accurate assessment of vibration parameters such as frequency, amplitude, and phase of rotating blades are key to ensuring early detection of possible blade defects and determining the remaining service life of a turbine engine. 60% of aero-engine failures can be attributed to vibration, of which 70% of these failures are blade failures [Amoo, 2013]. Traditionally, blade vibration is identified by attaching strain gages on to the blade(s) of interest and measuring the strain experienced by the blade when deflected due to vibrations. However, such a contact-based measurement technique requires high-quality telemetry for remote data retrieval, is time-consuming to design and install on to the blade(s) of the test engine, and is therefore costly and inefficient to implement. Additionally, strain gages can interfere with the aerodynamic and mechanical properties of the blade assembly, leading to uncertainty and error in the measurement.

An alternative is a non-contact stress measurement system (NSMS) [Roth, 1980] [Schlagwein and Schaber, 2006] that utilizes optical, magnetic, or capacitance sensors installed around the engine casing to measure the time-of-arrival (TOA) of each blade by analyzing the blade tip-timing (BTT) signals [Heath and Imregun, 1997]. Based on TOA data, the blade tip deflection signal (TDF) can be reconstructed and its corresponding vibration parameters can then be estimated using proper spectral analysis techniques [Zielinski and Ziller, 2000] [Carring, et al., 2001]. The NSMS technique is less costly to implement, and enables a longer test lifespan than the traditional, strain gage-based methods.

In spite of its advantages, four major challenges need to be overcome to successfully employ the NSMS technique for non-contact blade stress measurement: 1) under-sampling of data, 2) varying blade rotational speeds, 3) sensor installation errors, and 4) synchronous measurement vibration estimation [Dimitriadis, et al., 2002]. Under-sampling is an inherent issue in BTT data processing, due to the fact that NSMS takes only a limited number of data samples (one from each sensor) from each blade per revolution. Because of this limitation, the discrete number of data on blade tip deflections cannot be processed by traditional spectral analysis techniques such as FFT [Zoltowski and Mathews, 1994]. This leads to further uncertainty since the same amount of blade tip deflections can be obtained at differing or combined modes of vibrations, each of which are associated with a different level of blade tip stress [Prochazka and Vanek, 2011]. Varying rotational speeds lead to a varying sampling rate, causing significant challenges to blade vibration signal processing and parameter estimation. Additionally, sensor position installation errors are unavoidable.
in practice, and cause the same problems as varying rotational speeds. NSMS parameter estimation is straightforward for the case of asynchronous vibration, where the blade vibration frequencies are not integer multiples of the rotational speed. Conversely, when the engine is operating at an integer multiple of an engine order, the system would nominally measure the same displacement point of the blades every time they pass the sensors, preventing a signal to be rendered.

To address these challenges, this paper presents an Enhanced Estimation of Signal Parameters via Rotational Invariance Technique, or E-ESPRIT. ESPRIT is a sub-space estimation technique, first introduced by R. Roy to solve the direction of arrival (DOA) problem in the radar signal processing field [Roy and Kailath, 1989]. When applied to NSMS, the technique solves the under-sampling problem by employing multiple groups of sensors with multiple sensors within each group. The sensors are arranged with predetermined angular separations as a way to increase the number of sampled points in each revolution, while satisfying the Nyquist criterion. The strategy of using multiple groups of sensors would offer sampling points measured at different vibration displacements for some synchronous vibration frequencies, thus reducing the number of frequencies that cannot be estimated. Furthermore, enhanced ESPRIT introduces an interpolation method to ensure that data points are sampled at constant intervals as required by the ESPRIT algorithm, reducing the uncertainty caused by sensor installation errors and varying rotational speed.

In this paper, the mathematical description of BTT data measurement is first introduced in Section 2. The proposed enhanced ESPRIT method is developed in Section 3. Simulations to verify the methods are discussed in Sections 4 and 5. The concluding section summarizes the main results and future work of this project.

2. Tip Timing and TDF Signals

The general procedure for the tip timing technique is illustrated in Fig. 1, which has the objective of characterizing blade vibration. Sensors measure the raw TOA data, which are then processed to reconstruct the TDF signal via parameter estimation algorithms. The TOA data and TDF signal together are called the BTT, whose mathematical formulation is introduced below.

\[
\theta_1(t) = \phi + 2\pi \Omega t \quad (1)
\]

where \(\phi\) is the initial angular position of blade \(l\) at time \(t = 0\), and \(\Omega\) is the rotational frequency of the rotor that can be determined by a shaft revolution sensor or 1/rev sensor. Since TOA represents the time series when blade \(l\) is detected by probe \(k\), the angular position \(\theta_1(t)\) of each probe (initial angular position is \(\theta_k\) and \(n\) is the number of revolutions carried out by the rotor) must satisfy the following condition:

\[
\theta_1(t) = \phi + 2\pi \Omega t = \theta_k + 2\pi n \quad (2)
\]
In a real-world situation, blades inevitably experience structural vibrations. The actual angular position of blades at any time can be expressed as:

$$\varphi_l(t) = \varphi_0 + 2\pi \Omega t + \arcsin(x_l(t) / R) = \Theta_l + 2\pi n$$

(3)

where \(x_l(t)\) is the deflection of blade \(l\) due to vibration, and \(R\) is the bladed rotor radius. The term \(\arcsin(x_l(t) / R)\) represents the angular deflection of the blades. When blades vibrate at a sub-mil level relative to \(R\), \(\arcsin(x_l(t) / R)\) approximately equals \(x_l(t) / R\). Thus Eq. (3) can be re-written as:

$$\varphi_l(t) = \varphi_0 + 2\pi \Omega t + x_l(t) / R = \Theta_l + 2\pi n$$

(4)

It should be noted that Eq. (1) ~ Eq. (4) represent the angular conditions for the TOA measurement. Let \(\bar{t}_{n,k,l}\) (subscript \(n, k, l\) denotes the number of revolution, sensor, and blade) be the solutions to Eq. (2) and Eq. (4), respectively. Then \(\bar{t}_{n,k,l}\) corresponds to the theoretical arrival time in the absence of vibration, and \(t_{n,k,l}\) corresponds to the actual arrival time. It can be seen from Eq. (4) that the estimation of blade vibration is tightly related to rotational speed. Actually, based on the relationship between vibration frequency and rotor rotational frequency, blade vibration can be classified into asynchronous and synchronous vibration, which will be discussed in more detail later.

Taking the center of the rotor as the origin point and the blade tip as the observation target, the distance of blade arrival, determined by the arrival time set \(\{t_{n,k,l}\}\), can be expressed as:

$$D_l(t_{n,k,l}) = R \cdot \Omega \cdot t_{n,k,l}$$

(5)

The un-deflected or constant blade distance of arrival \(D_0\) can be expressed similarly as:

$$D_0(\bar{t}_{n,k,l}) = R \cdot \Omega \cdot \bar{t}_{n,k,l}$$

(6)

Using Eq. (5) and Eq. (6), the blade deflection can be reconstructed or estimated as:

$$\tilde{x}_l = D_l - D_0$$

(7)

The deflection takes place about a datum line, typically from the centerline axis to the positions of the installed sensors, assuming there are no structural imperfections, as the lower part in Fig. 2 shows. \(P_1\) and \(P_2\) denote two adjacent sensors, \(\{t_{1,1,3}, \ldots, t_{1,1,3}\}\) and \(\{t_{1,2,3}, \ldots, t_{1,2,3}\}\) denote the actual arrival time (represented by \(t_{n,k,l}\) in the above equations) of a single blade measured by two sensors, and 1/rev denotes one revolution measurement, providing the theoretical arrival time \(\bar{t}_{n,k,l}\). The objective of this measurement system is to reconstruct TDF data based on the information of TOA measurement, according to Eq. (5) ~ Eq. (7), while multiple TDF segments can be pieced together to restore the original blade vibration signal and estimate the vibration characteristics.

Fig. 2: Measurement schematic of BTT

The above derivation explains how TOA and TDF data are obtained, given the existence of blade vibration. In
general, blade vibration can be classified into two groups: *synchronous* vibration and *asynchronous* vibration. Synchronous vibrations have frequencies that are an integer multiple of the rotor’s rotational frequency as shown in the Campbell diagram [Lalanne and Ferraris, 1998]. In other words, if a blade vibrates at a synchronous frequency when the rotor is rotating with a constant speed, sensors will measure the same vibration positions and deliver the same TOA data, causing insufficient information for estimation. The frequencies of asynchronous vibrations are a non-integer multiple of the rotational frequency, and occur within the rest of the regions between the lines of the EO’s, along the natural frequency lines. If a blade vibrates at an asynchronous frequency, different locations in the vibration signal are sampled each revolution, leading to sufficient TOA data and information for vibration estimation. In general, synchronous vibration is much more difficult to estimate than asynchronous vibration, under the situation of constant rotational speed. However, the rotor often spins at varying speeds. When a rotor rotates around the synchronous points (intersections between natural frequencies and engine orders), blades undergo different amplitudes and phases at different rotational speeds. This phenomenon is also called resonance vibration, which provides different TOA measurement in different revolutions, making it possible to estimate synchronous vibration.

3. Enhanced ESPRIT Algorithm

The analysis of tip timing data aims at estimating the frequencies and amplitudes of blade vibrations as a non-invasive alternative to using strain gages for blade vibration analysis. A major challenge to solving the problem is under-sampling, due to that only one data point per probe per blade is produced each revolution (i.e. sampling rate is identical to the rotational rate, less than most vibration frequencies). Furthermore, the rotational speed of a rotor may fluctuate, causing the measured BTT data to be non-uniform such that it cannot be precisely estimated by traditional methods such as Fourier Transform. ESPRIT, as one of the sub-space techniques [Salhi, et al., 2007], provides an alternative solution to this problem. Compared with other sub-space algorithms, ESPRIT features reduced computation and storage cost [Roy and Kailath, 1989].

The essence of ESPRIT lies in the rotational property between staggered subspaces that are invoked to produce frequency estimation [Ottersten, et al., 1991]. In the case of a discrete-time signal, this property relies on observations of the signal over two identical intervals staggered in time. In other words, ESPRIT requires that the sampling is performed uniformly. Thus, when applying ESPRIT to NSMS, two sensors are needed to create required two staggered data set. However, the practical working condition of the rotor invokes several application limitations to conventional ESPRIT:

- Varying rotational speed
- Sensor installation error

Varying rotational speed can cause the varying sampling rate, violating the requirement by conventional ESPRIT. Error is inevitable in practical sensor installation, causing the time intervals between two adjacent sensors among different groups to be inconsistent. To address these limitations, algorithmic enhancement of ESPRIT, together with corresponding sensor placement strategy, have been developed. After introducing the sensor placement strategy for Enhanced ESPRIT (E²ESPRIT) in Section 3.1, Section 3.2 will introduce E²ESPRIT algorithm mathematically.

3.1 Sensor Placement Strategy

*Multiple Groups with Multiple Sensors (versus varying speed and installation error)*

Conventional sensor placement scheme for ESPRIT when applied to NSMS uses one pair of sensors arranged next to each other, which delivers a single data-pair each revolution, causing the estimation vulnerable to varying rotational speed. To maintain the sampling rate, multiple sensors in each group are employed for the enhanced ESPRIT (E²ESPRIT), as illustrated the first sensor group $P_{11} \sim P_{13}$ in Fig. 3, providing three TOA points each revolution. When sensor installation error or varying rotational speed are present, interpolation techniques can be applied to these three TOA points measured by each sensor group to retrieve one data-pair, which satisfy the required constant sampling rate. Another way to reduce the effect of varying rotational speed is multiple sensor groups, as illustrated in Fig. 3 with three sensor groups. Multiple sensor groups can provide multiple retrieved TOA data-pairs, reducing the time duration (i.e. number of revolutions) for estimation each time, when certain number of data length is required for estimation.
Angle Selection between Adjacent Sensors (versus under-sampling)

Selection of the angle $\theta$ that separates the sensor depends on the rotational speed and maximum vibration frequency. Based on the Nyquist criterion, the angular interval between two adjacent sensors should be smaller than half of the blade rotational angle in one vibration period, which means that:

$$\theta < (180^{\circ} \times \Omega / (60 \times f_{\text{max}}))$$

where $f_{\text{max}}$ is the maximum vibration frequency and $\Omega$ is the rotational speed in RPM. When the vibration frequency approaches the Nyquist frequency, parameter estimation accuracy decreases.

3.2 Enhanced ESPRIT Algorithm

The ESPRIT algorithm assumes that the blade vibration signal is reconstructed with $P$ frequency components, in the form of a complex exponential:

$$s(n) = \sum_{l=1}^{P} a_l e^{j2\pi f_l n} + w(n)$$

where $a_l$, $f_l$, $\varphi_l$ are amplitude, frequency, and phase of the blade vibration, respectively. The symbol $w(n)$ denotes external perturbation caused by mechanical noise or shaft vibration. Let $b_l = a_l e^{j\varphi_l}$, then Eq. (9) can be rewritten as:

$$s(n) = \sum_{l=1}^{P} b_l e^{j2\pi f_l n} + w(n)$$

It is seen that $b_l$ contains information on both the amplitude ($a_l$) and phase ($\varphi_l$) of the blade vibration. Thus the problem of estimating vibration is transformed to estimate $f_l$ and $b_l$. Based on $L$ ($L = 3$ in this paper) sampling points measured by each sensor group, two retrieved data points with the same time interval can be obtained through interpolation, which will be introduced in next subsection. The time delay between two retrieved data points provided by each group is essentially caused by the phase-shift at these two sampling points. Let $x(1)$ and $y(1)$ be the two data points obtained from the first sensor group, $x(2)$ and $y(2)$ be the two data points obtained from the second sensor group, and so on.
and so on. Then \( x(1) = s(1), y(1) = s(2), x(2) = s(Q) \), \( y(2) = s(Q+1) \), and so on, where \( Q \) is defined as \( \theta_{x}/\theta \).

To estimate the vibration characteristics, a shifted observability matrix \( \Phi \) containing the frequency information will be established next step. Reconstructed TDF data are regrouped into two matrices \( X \) and \( Y \). Elements in \( X \) come from sensor \( P_1 \), while elements in \( Y \) come from sensor \( P_2 \). The row vectors in matrix \( X \) are obtained by the repeated interceptions of the data set \( \{ x(1) = s(1), x(2) = s(Q), \ldots \} \) by a sliding \( m \)-length window, as shown in Fig. 4. The size of \( X \) is \( (n-m+1) \) by \( m \). Through this operation, the corresponding elements in the neighboring row vectors are staggered with one unit time delay, which is later employed to derive the matrix \( \Phi \). Furthermore, the matrix \( X \) can be written as:

\[
X = \begin{bmatrix}
1 & 1 & \ldots & 1 \\
e^{j2\pi f_1} & e^{j2\pi f_1} & \ldots & e^{j2\pi f_m} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j(n-m)2\pi f_1} & e^{j(n-m)2\pi f_1} & \ldots & e^{j(n-m)2\pi f_m}
\end{bmatrix}
\]

\[
\begin{bmatrix}
b_{1}e^{j2\pi f_1} & b_{1}e^{j2\pi f_1}Q & \ldots & b_{1}e^{j2\pi f_1}(n-1)Q \\
b_{2}e^{j2\pi f_1} & b_{2}e^{j2\pi f_1}Q & \ldots & b_{2}e^{j2\pi f_1}(n-1)Q \\
\vdots & \vdots & \ddots & \vdots \\
b_{p}e^{j2\pi f_1} & b_{p}e^{j2\pi f_1}Q & \ldots & b_{p}e^{j2\pi f_1}(n-1)Q
\end{bmatrix}
= A \ast S
\]  

(11)

In Eq. (11), \( A \) is fully described using the vibration frequencies. Once the frequencies are obtained, amplitudes can be estimated through calculation of the matrix \( S \). Similarly, the matrix \( Y \) can be written as:

\[
Y = A \ast \Phi \ast S,
\]

where \( \Phi = \begin{bmatrix} e^{j2\pi f_1} & e^{j2\pi f_1} & \ldots & e^{j2\pi f_m} \end{bmatrix} \)

(12)

It is seen that all elements in matrix \( Y \) are one unit time delayed from the elements in matrix \( X \). Based on the time delay between \( X \) and \( Y \), the rotation matrix \( \Phi \) is derived, which is fully described by the vibration frequencies \( \{ f_1, f_2, \ldots, f_p \} \). Therefore the blade vibration frequencies can be estimated by solving \( \Phi \) based on the measurements from two neighboring sensors \( P_1 \) and \( P_2 \). Constructing a Hankel matrix \( Z \) by making use of the phase delay information between \( X \) and \( Y \), and performing auto-correlation of the Hankel matrix, the effect of noise, expressed as [Manolakis, et al., 2000], can be reduced:

\[
R_n = E\left[ Z(n) \right] Z^H(n) = \frac{1}{n-m+1} \begin{bmatrix} XX^H & XY^H \\ YX^H & YY^H \end{bmatrix}
\]

(13)

In Eq. (13), \( XX^H \) and \( YY^H \) represent auto-correlation of the \( X \) and \( Y \) matrices, respectively, and \( XY^H \) is the cross correlation of \( X \) and \( Y \). Because of this construction, the eigenvectors of \( R_n \) contain the phase delay information, and \( \Phi \) can be estimated by a singular value decomposition (SVD) calculation. As a result, Eq. (13) can be written as:

\[
R_n = \frac{1}{n-m+1} \begin{bmatrix} XX^H & XY^H \\ YX^H & YY^H \end{bmatrix} = \Sigma U = \Sigma \begin{bmatrix} U_1 & U_2 \\ U_3 & U_4 \end{bmatrix} = \Sigma \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}
\]

(14)

In Eq. (14), \( \Sigma \) and \( U \) are the diagonal singular values and corresponding eigenvectors of \( R_n \), respectively. \( U \) is separated into two parts, as the inverse construction of \( Z \), followed by dividing the space into the signal and noise subspaces while the size of the signal subspace is determined by the first \( K \) singular values. After sorting the singular values in a descending way, \( K \) can be selected according to typical information rules such as the Akaike information criterion (AIC) and minimum description length (MDL) [Cichocki and Amari, 2002]. Furthermore, \( \Phi \) can be calculated through:

\[
\Phi = U_{11}U_{22}^T = \text{diag} \left\{ 2\pi f_1, 2\pi f_2, \ldots, 2\pi f_p \right\}
\]

(15)

Once frequencies are determined, amplitudes can be calculated by:

\[
\begin{bmatrix}
x(1) \\
x(2) \\
\vdots \\
x(n-m+1)
\end{bmatrix} = \begin{bmatrix}
e^{j2\pi f_1} & e^{j2\pi f_1} & \ldots & e^{j2\pi f_m} \\
e^{j0+2\pi f_1} & e^{j0+2\pi f_1} & \ldots & e^{j0+2\pi f_m} \\
\vdots & \vdots & \ddots & \vdots \\
e^{j(n-m)0+2\pi f_1} & e^{j(n-m)0+2\pi f_1} & \ldots & e^{j(n-m)0+2\pi f_m}
\end{bmatrix}\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_p
\end{bmatrix}
\]

(16)

The absolute amplitudes of \( \{ b_1, b_2, \ldots, b_p \} \) are the amplitudes of each frequency component.
Above derives the estimation of vibration parameters starting from measured data. Different from conventional ESPRIT, E²SPRIT applies retrieved data from interpolation to construct the Hankel matrix, to satisfy the requirement of constant sampling rate. In the next subsection, interpolation method applied in this paper will be introduced.

### 3.3 Interpolation Method

Interpolation is investigated to improve the estimation accuracy under varying rotational speed and sensor installation errors. The objective is to retrieve data points using \( L \) sampling points provided by the sensor groups during one revolution. Two criteria were taken into account for the selection of an interpolation method that can be effectively applied to NSMS: 1) passing sampling points; 2) piecewise, to avoid Runge’s phenomenon [Li and Heap, 2014]. The Lagrange interpolation method is investigated in this presented study due to its computation efficiency, in order to maintain the on-line real time performance of the whole program manipulation.

Because three sampling points are measured by each group and applied to describe the vibration function, quadratic interpolation or parabolic interpolation is investigated. Based on the basic function, the interpolation function can be expressed as:

\[
L_2(t) = \frac{(t-t_3)(t-t_1)}{(x_3-x_1)(x_3-x_2)} x_1 + \frac{(t-t_1)(t-t_2)}{(x_1-x_2)(x_1-x_3)} x_2 + \frac{(t-t_2)(t-t_3)}{(x_2-x_3)(x_2-x_1)} x_3
\]

(17)

In above equation, \( t_1, t_2 \) and \( t_3 \) are measured TOA data, \( t \) denotes the time that needs to be interpolated, \( x_1, x_2 \) and \( x_3 \) are reconstructed TDF data points, and \( L_2(t) \) denotes the interpolated value at time \( t \). Due to sensor installation error or varying rotational speed, time intervals between \( x_1, x_2 \) and \( x_3 \) are not the same, violating the requirement of constant sampling rate. With interpolation as described in Eq. (17), given two predefined time spots with constant interval, two TDF data points can be retrieved, which are subsequently substituted into the ESPRIT to estimate vibration parameters.

### 4. Numerical Simulations

Performing tip timing measurement consists of recording blades’ passing time in front of the optical sensors, which is affected by vibration mode (i.e. frequency, amplitude and phase). Thus, to evaluate the effectiveness of the Enhanced ESPRIT method, both asynchronous and synchronous vibrations were considered when setting up a numerical model for simulation. The model uses frequency sweeping to simulate a wide range of potential vibration frequencies when the blade is going through asynchronous vibrations. For synchronous vibration, the simulation investigates the scenario when a resonance frequency is crossed. As discussed above, blade vibration is tightly related to rotor rotational speed. Also, the vibration estimation may be affected by sensor installation error. Hence, rotational speed and sensor installation error are investigated in the following simulations. For asynchronous simulation, the amplitude, phase and frequency in each revolution are assumed to be constant, and sensor installation errors are taken into consideration. For synchronous simulation, the effect of varying rotational speed, amplitude and frequency are taken into consideration.

In Table 1, parameters used for the simulation are listed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Asynchronous Simulation</th>
<th>Synchronous Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor groups (( N ))</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Sensors in each group (( L ))</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Angular interval (( \theta ))</td>
<td>2°</td>
<td>1°</td>
</tr>
<tr>
<td>Rotational speed (( \Omega ))</td>
<td>12,000 RPM (constant)</td>
<td>3,582-4,179 RPM (E.O. 20, mode 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6,304-6,444 RPM (E.O. 85, mode 6)</td>
</tr>
<tr>
<td>Sensor Installation Errors</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Synchronous vibrations occur when the blade vibration frequency is an integer multiple of the rotor rotational frequency. The ratio of the vibration frequency to the rotational frequency is called the engine order (E.O.). The ascending lines on the Campbell diagram represent increases in vibration frequency as a function of rotational speed, according to various engine orders. The horizontal mode lines are drawn to highlight the resonance frequencies.
corresponding to the vibration modes. The points where the engine order lines intersect the mode lines indicate rotational speeds where resonance will occur. To demonstrate the effectiveness of the developed computational method, vibrations with low vibration mode and low engine order, as well as high vibration mode and high engine order, have been applied for analysis.

4.1 Asynchronous Vibration Simulations

4.1.1 One Sensor with Installation Error

This simulation investigates the case when only one sensor $P_{12}$, has an installation error in the range of $-0.5^\circ$ to $0.5^\circ$. Simulation was performed with increments of $0.05^\circ$, considering the angular interval between two adjacent sensors is $2^\circ$.

Fig. 5 and Fig. 6 show the frequency and amplitude estimation results for asynchronous vibration. The left figures of both Fig. 5 and Fig. 6 are 3-D plots with vibration frequency as the $x$-axis, and sensor installation error as the $y$-axis. Figures on the right are mean estimation errors for each vibration frequency. The calculation indicates that the asynchronous frequency estimation error is less than 0.5%. With the increase of vibration frequency, estimation error increases, since they are closer to the Nyquist frequency (18,000 Hz) limit.

For amplitude estimation, the maximum error associated with the $E^{2}$SPRIT algorithm is less than 5% when vibration frequencies are in the range of 500 Hz - 7,000 Hz (the Nyquist frequency is 18,000 Hz in this simulation). When vibration frequencies are over 7,000 Hz, asynchronous amplitude estimation error increases exponentially, due to the following reasons: 1) error propagation from frequency estimation, because the amplitude estimation is based on the result of frequency estimation; 2) error amplified by the exponential calculation of the frequency matrix in Eq. (16). It should be noted that the estimation results shown in Fig. 5, Fig. 6, as well as in Fig. 8 do not contain synchronous frequencies and frequencies in the vicinity ($\pm 20$ Hz) of the synchronous region.
4.1.2 Error from Sensor Installation and Varying Rotational Speed

Simulations were also performed to investigate the effects of sensor installation errors and various rotational speeds. First, all 9 sensors are assumed to be installed with random errors, with the maximum being 0.2°. Second, the rotational speed is assumed to increase stepwise, starting from 12,000 RPM at an increment of ΔΩ (ΔΩ = 2 RPM) for each revolution, as illustrated in Fig. 7. Additionally, vibration frequencies and amplitudes are assumed to vary with changing rotational speed in a linear way. Calculations show that for asynchronous vibration, frequency estimation error from the enhanced ESPRIT algorithm is less than 0.5% (when vibration frequencies are below 7,000 Hz), which is significantly lower (from 0.82% to 0.15% for 3,900 ~ 4,000 Hz, and from 1.01% to 0.67% for 6,900 ~ 7,000 Hz, as shown in Table 2) than the estimation error from the conventional ESPRIT, as shown in Fig. 8. As the frequency increases to near the Nyquist frequency (18,000 Hz), estimation errors by the two methods are comparable, due the physical limitation of the sampling theory. Furthermore, the amplitude estimation error rapidly increases when frequency estimation error is larger than 1%, due to the exponentially propagating nature of the error from the frequency estimation. Table 2 compares the estimation accuracy of enhanced ESPRIT with conventional ESPRIT in three frequency ranges, up to 10,000 Hz. It is seen that the enhanced ESPRIT significantly improves the estimation accuracy. A possible way to improve the Enhanced ESPRIT performance is to decrease the distance between two adjacent sensors to increase the Nyquist frequency limit to allow for higher estimated frequency.

Table 2: Comparison of estimation accuracy between enhanced and conventional ESPRIT algorithms

<table>
<thead>
<tr>
<th></th>
<th>Conventional ESPRIT</th>
<th>Enhanced ESPRIT</th>
<th>Conventional ESPRIT</th>
<th>Enhanced ESPRIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation 1</td>
<td>Simulation 2</td>
<td>Simulation 1</td>
<td>Simulation 2</td>
</tr>
<tr>
<td>Frequency 3,900~4,000 Hz</td>
<td>0.59%</td>
<td>0.82%</td>
<td>0.10%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Estimation 6,900~7,000 Hz</td>
<td>0.80%</td>
<td>1.01%</td>
<td>0.23%</td>
<td>0.67%</td>
</tr>
<tr>
<td>Error 9,900~10,000 Hz</td>
<td>0.87%</td>
<td>1.50%</td>
<td>0.49%</td>
<td>1.49%</td>
</tr>
<tr>
<td>Amplitude 3,900~4,000 Hz</td>
<td>76.69%</td>
<td>82.30%</td>
<td>1.05%</td>
<td>9.52%</td>
</tr>
<tr>
<td>Estimation 6,900~7,000 Hz</td>
<td>80.45%</td>
<td>93.66%</td>
<td>5.21%</td>
<td>88.49%</td>
</tr>
<tr>
<td>Error 9,900~10,000 Hz</td>
<td>83.26%</td>
<td>91.16%</td>
<td>55.49%</td>
<td>89.30%</td>
</tr>
</tbody>
</table>
4.1.3 Multi-Frequency Vibration

The performance of enhanced ESPRIT on multi-frequency vibrations (i.e., when vibration signal is composed of multiple frequency components) has also been investigated. For this purpose, natural frequencies of blade vibration were obtained from an ANSYS model. The first three natural frequencies are found to be 1294.2 Hz, 2946.1 Hz and 5581.2 Hz, respectively. For the simulation, sensor installation errors and varying rotational speeds are taken into consideration, as shown in Tab. 3. $\Delta \theta_{\text{max}}$ denotes the maximum installation errors, which are initially selected, and $\Delta \Omega$ is the variation in the rotational speed between revolutions.

It can be seen from Table 3 that estimation errors for both the two-frequency and three-frequency cases are less than 1%, and even less than 0.3% when there are only two frequency components in the vibration signal. Estimation accuracy decreases with the increasing number of frequency components under the same simulation conditions. This is due to the fact that the vibration signal becomes more complicated when there are more frequency components, leading to both higher interpolation error and higher error on distinguishing those frequency components.

Table 3: Estimation results of enhanced ESPRIT on multi-frequency component vibration

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Estimated frequencies (Hz)</th>
<th>Average frequency estimation error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \sigma_{\text{max}}$</td>
<td>$\Delta \Omega$</td>
</tr>
<tr>
<td>Two frequency</td>
<td>0.1°</td>
<td>0 RPM</td>
</tr>
<tr>
<td>components</td>
<td>0.1°</td>
<td>1 RPM</td>
</tr>
<tr>
<td></td>
<td>0.2°</td>
<td>0 RPM</td>
</tr>
<tr>
<td></td>
<td>0.2°</td>
<td>1 RPM</td>
</tr>
<tr>
<td>Three frequency</td>
<td>0.1°</td>
<td>0 RPM</td>
</tr>
<tr>
<td>components</td>
<td>0.1°</td>
<td>1 RPM</td>
</tr>
<tr>
<td></td>
<td>0.2°</td>
<td>0 RPM</td>
</tr>
<tr>
<td></td>
<td>0.2°</td>
<td>1 RPM</td>
</tr>
</tbody>
</table>

4.2. Synchronous Vibration Simulation

To evaluate the performance of the enhanced ESPRIT algorithm under actual engine conditions, a simulator for synchronous vibrations at resonance frequencies was developed. The TDF data corresponding to a mode 1 resonance vibration of a single blade at E.O. 20 measured by each sensor are drawn in Fig. 9. As can be seen, each sensor measures a different blade deflection due to the changing phase of vibration. The rotational speed is considered to be continuously increasing at a rate of $\Delta \Omega = 1$ RPM between two revolutions. The results for frequency estimation across the resonance speed are plotted in Fig. 10.
Table 4 gives the simulation results for amplitude and frequency estimation with the E$^2$SPRIT algorithm at engine orders 20 and 85. These engine orders were chosen to represent realistic engine conditions. Because the simulated blade rotational speed and vibration frequency is different when the blade passes each sensor, the problem of measuring the same point of a synchronous vibration is avoided, unlike in the constant speed case. The simulation result shown is the average of every possible sensor position to avoid bias from the phase of vibration, as the simulation is not continuous from rev to rev; the simulation starts at the top dead center of rotation. As the engine order and mode both increase, the subsequent higher vibration frequency leads to a larger estimation error. This is because the higher frequency is closer to the Nyquist limit, and higher engine orders are associated with exciting higher frequencies. In addition, the phase of vibration changes more rapidly as resonance is approached at engine order 85 than engine order 20, due to more damping at the higher engine order. As can be seen in Fig. 10, as the resonance speed is approached, the estimation error increases due to the effect of the changing phase of vibration. The dynamic phase of blade vibration affects the estimation accuracy because, as described in section 3.2, the ESPRIT algorithm relies on the phase delay of the measured samples in performing the estimation.

5. Conclusion

The Enhanced ESPRIT algorithm for blade tip timing analysis has shown to be effective as a non-contact stress measurement technique for rotating blade vibrations analysis. Compared to the conventional ESPRIT algorithm, the Enhanced ESPRIT groups multiple sensors to adapt to varying engine speeds by means of interpolation and employing multiple groups of sensors to shorten the sampling period, consequently reducing errors due to both under-sampling and varying sampling rates. To validate the efficiency of the algorithm, practical scenarios of sensor installation errors and various rotational speeds that cause a non-uniform BTT signal have been evaluated. Simulation results indicate that the Enhanced ESPRIT algorithm effectively improves the frequency estimation accuracy, which correspondingly can also improve the amplitude estimation accuracy. Furthermore, simulation of the engine speed passing through resonance was also evaluated, and the result shows that the maximum frequency estimation error is less than 0.5% and maximum amplitude estimation error is less than 5%. Future research will further investigate multi-mode blade vibration and vibrations of an entire multi-bladed rotor assembly.

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References


