Steering law considering biased loads for control moment gyroscopes

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Abstract
A new steering law of a pyramid configuration of four control moment gyroscopes (CMGs) is proposed to suppress radical motion of gimbals, reduce the gimbal angular displacement and level the biased ones of the four CMGs. Additionally analytical findings on the execution timing of the proposed method in relation to multiple attitude maneuvers are newly presented. Because major failure mode of the CMG is the defective lubrication of the spin bearings, resulting from being put on the excessive radial loads by radical motion of gimbals, the proposed method focuses the relation between the gimbal angular displacement of each CMG and the initial condition of gimbal angles. A suitable set of initial gimbal angles is selected using the defined evaluation function in an off-line preliminary calculation. The evaluation function considers the manipulability of the upcoming maneuver related to the radical motion of gimbals and the biased gimbal angular displacements of four CMGs accumulated during the operational time. Simultaneously, the criteria for judgment and analytical findings on the execution timing of the proposed method in relation to multiple attitude maneuvers are newly presented, which consider the extra load by null motion and the accumulated gaps of gimbal angles between the initial condition and the terminal ones. The dynamics of a typical pyramid configuration of four single-gimbal CMGs was modeled and a numerical simulation in case the CMGs do not pass through the singularity was carried out. Numerical simulation confirmed that the proposed method not only keeps the manipulability greater but also levels the gimbal angular displacements of the CMGs without degrading attitude control.

Key words: Control Moment Gyros, Attitude control, Singularity, Steering law, Spacecraft

1. Introduction

Control Moment Gyros (CMGs) have been used for not only large space structures such as the International Space Station (ISS) and Mir, but also for Earth-observing satellites that must be able to rapidly acquire multiple targets in emergency monitoring and space science missions. CMGs are specifically used in these applications because they generate much more torque than reaction wheels. The pyramid configuration of four single-gimbal CMGs (SG-CMGs) is generally used for three-axis attitude control of a spacecraft from the viewpoint of hardware redundancy, as shown in Fig. 1. However, CMGs have failed in several cases, resulting in no or strictly limited attitude control of the space structure or satellite and difficulties in continuing the mission. Mir, which was equipped with SG-CMGs, could no longer operate properly after the failure of one of the SG-CMGs, preventing the station from properly using its solar panels. It was reported that two of the four double-gimbal CMGs used for the ISS failed and shut down, and the ISS subsequently operated with only the remaining two (Gurri, et al., 2010). Once the CMGs have failed and the spacecraft has lost its attitude control ability, the opportunity loss is critical and it would be impractical to repair or replace the CMG. Such a situation should be avoided.

One of the major failure modes of the CMG is the defective lubrication of the spin bearings, which are mounted on both sides of the spin axis and support the momentum wheel. It is estimated that the radical motion of gimbals put the spin bearings on the excessive radial loads, resulting in occurring ball skidding and then the defective lubrication (Burt
and Loffi, 2003). In other words, repetitive radical motion of gimbals would introduce shock to the spin bearings, resulting in the failure of the CMG. Since this kind of motion would occur when the CMGs are near singularity, a steering law should be considered. Additionally, there exist some cases that the loads are biased toward the specific CMGs in the multiple maneuvers. If the spacecraft repeats the same type of maneuver in which the CMGs do not pass the singularity, the gimbal angular displacement is biased toward specific CMGs (CMG 1 and CMG 3 in this simulation) when employing the previous method. Therefore, it is desired for the CMGs to avoid radical motion of gimbals, reduce the gimbal angular displacements, and level the biased gimbal angular displacements of four CMGs by the steering law.

On the other hand, most previous steering laws aimed to avoid singularity and have not considered such a viewpoint. The authors focused the dependence of gimbal angular velocities and gimbal angular displacements on the set of initial gimbal angles (Vadali, et al., 1990), and proposed the singularity avoidance and escape algorithm utilizing preferred initial gimbal angles. A procedure of the authors’ previous method, which is executed at non-observation phase of the spacecraft, is presented in Fig. 2. In this algorithm, the suitable set of initial gimbal angles are calculated by the defined evaluation function considering singularity and internal disturbance, using off-line simulation prior to an actual attitude maneuver. Then its suitable initial gimbal angles are realized by null motion, which the pyramid configuration of four CMGs can change its gimbals without generating any torque to the spacecraft (Nanamori and Takahashi, 2008). However, this method does not consider the radical motion of the gimbals resulting in the failure of the CMGs, and the additive amount of the gimbal angular displacement by null motion because it supposes to apply the null motion and change the set of initial gimbal angles before maneuvering.

Therefore, this study proposes a new method of selecting the suitable set of initial gimbal angles to suppress radical motion of gimbals and gimbal angular displacement and level those biased loads. The advantage of the proposed method is to suppress the motion of gimbals that leads to the failure of the CMGs, by adding the null motion to the conventional steering law before starting to maneuver. The effect of leveling the gimbal angular displacements of the CMGs is shown in Fig. 3, which is based on the numerical analysis. Additionally, the criteria for judgment and analytical findings on the execution timing of the proposed method in relation to multiple attitude maneuvers are newly presented in this study. The proposed method is executed between every predefined number of maneuvers considering the extra load by null motion and the accumulated gaps of gimbal angles between the initial condition and the terminal ones. Besides, considering the limited resource of on board computer, off-line attitude maneuver simulation for selecting the suitable set of initial gimbal angles is supposed to be run at ground facility and its calculation result will be transmitted from the earth station as a command. To verify the feasibility of the proposed steering law, the dynamics of a pyramid configuration of four CMGs is modeled and numerical simulations are carried out. In this paper, the effect of keeping the manipulability greater and the leveling effect for the load are evaluated in case that the CMGs do not pass through the singularity. Additionally, the execution timing of the proposed method is also analyzed assuming the spacecraft repeats the same pattern of the maneuver repeatedly.

![Diagram](image)

Fig. 1 Typical pyramid configuration of four single-gimbal CMGs (SG-CMG).
2. Concept and overall design

2.1 Concept

The proposed method calculates a suitable set of initial gimbal angles at the start of the maneuver by employing a defined evaluation function, and then implements the set of initial gimbal angles through null motion. The proposed method utilizes the dependence of gimbal angular velocities and gimbal angular displacements on the set of initial gimbal angles. The proposed method selects a suitable set of initial gimbal angles to achieve a better leveling of the gimbal angular displacements from several sets of gimbal angles. In addition, the manipulability is evaluated in terms of radical motion of gimbals.

The proposed method consists of three steps beginning at the non-observation phase before maneuver. Step 1 is the off-line attitude maneuver simulation and evaluation for each set of nominated initial gimbal angles to select the suitable set of initial gimbal angles. Step 2 is to change the gimbal angles from the present state to the suitable state derived in step 1. Step 3 is the attitude maneuver adopting conventional steering law.
Step 1: At non-observation phase before starting the maneuver, several sets of initial gimbal angles implemented through null motion between every predefined number of maneuvers from the present set of gimbal angles are prepared as the nomination of the suitable set. Time interval of generating null motion is design parameter. In this study, it is supposed to be set as 5 seconds. In case that the CMGs start to steer from each set of initial gimbal angles, the gimbal angle displacement in each case is calculated and evaluated. Gimbal angle displacements are quantities stored for null motion, and subsequent maneuvers. The average and variance of each gimbal angular displacement are then calculated to evaluate the bias of the load. Simultaneously, the manipulability is evaluated to judge the distance from the singularity and the radical motion of gimbals while maneuvering. The evaluation values are introduced by substituting the average and variance of each gimbal angular displacement and the manipulability into the evaluation function. The set for which the evaluation value is the smallest is selected as the suitable set of initial gimbal angles.

Step 2: After selecting the suitable set of initial gimbal angles, the CMGs execute null motion and the set of initial gimbal angles is changed without generating any control torque to the spacecraft. At this point, the spacecraft does not execute the other mission while changing the gimbal angles by null motion.

Step 3: The CMGs start to steer its gimbals from the suitable state and the spacecraft starts to maneuver. The proposed method employs the Singular Direction Avoidance (SDA) steering law expressed by Eq. (11) as conventional steering law to avoid the unexpected singularity.

2.2 Execution timing of the proposed method during operation

Considering the extra load by null motion and the accumulated gaps of gimbal angles between the initial condition and the terminal ones, the number of maneuvers before applying the proposed method $n_r$, $r_{\min} \leq n_r \leq r_{\max}$ is set. $r_{\min}$ is determined under the condition that the average and variance of the gimbal angular displacements of the four CMGs are better than those for the previously used method. $r_{\max}$ is determined according to the effect duration of the proposed method after applying the null motion. In case that the CMGs do not pass through singularity during maneuver after changing the set of initial gimbal angles, there is the tendency that most steering laws will return the gimbal angles to the same values at the end of the maneuver (Kanzawa, et al., 2013). Therefore the CMGs can steer its gimbals as the same pattern as the just before steering. However, the substantial gaps between the initial condition and the terminal ones are accumulated after repeating the maneuvers because of the external disturbances, etc., resulting in falling into the state that the CMGs cannot do the same steering as just after applying the null motion. Before reaching this state, the proposed method should apply null motion and modify the set of initial gimbal angles appropriately. The effect duration of the proposed method depends on the profile of the attitude maneuver, it should be determined analytically. The practical number of $r_{\max}$ and the criteria for judgment is proposed in chapters 5 and 6.

3. Spacecraft attitude motion and CMG dynamics model

To control the attitude of the spacecraft around three axes, it is necessary to operate more than three CMGs cooperatively. This paper deals with the typical pyramid configuration of four CMGs shown in Fig. 1. The coordinate systems are inertial reference frame \{N\}, spacecraft body frame \{B\}, and CMG frame \{C\}.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_B$</td>
<td>Spacecraft moment of inertia</td>
</tr>
<tr>
<td>$I_C$</td>
<td>CMG moment of inertia</td>
</tr>
<tr>
<td>$\omega_{B,C}$</td>
<td>Angular velocity of the {C} frame relative to the {B} frame</td>
</tr>
<tr>
<td>$\omega_{N/B}$</td>
<td>Angular velocity of the {B} frame relative to the {N} frame</td>
</tr>
<tr>
<td>$\theta_{N/B}$</td>
<td>Euler angle of spacecraft, $\theta_{N/B} = [\theta_x \ \theta_y \ \theta_z]^T$</td>
</tr>
<tr>
<td>$I_{C_y}$</td>
<td>CMG moment of inertia around the y axis (rotor axis) in the {C} frame</td>
</tr>
<tr>
<td>$\omega_{\text{avg}}$</td>
<td>Angular velocity of the four CMG rotors</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Skew angle of the pyramid configuration</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>Gimbal angle of the $i$-th CMG</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Directional cosine matrix of the $i$-th CMG</td>
</tr>
</tbody>
</table>
The rotational equation of motion of a rigid spacecraft equipped with CMGs is (Kawai, 2004):

\[
\sum_{i=1}^{4} C_i^T I_i \omega_{i/C} + \omega_{N/B} \times \sum_{i=1}^{4} C_i^T I_i \omega_{i/C} = -\tau_{\text{cmg}}
\]  

(1)

where \( \tau_{\text{cmg}} \) is output torque from the CMGs and \( i \) is the identification number of the CMG. This expression is based on the assumption that the moment of inertia of the CMG \( I_C \) is far smaller than that of the spacecraft \( I_B \). Equation (1) can be rewritten as:

\[
\begin{bmatrix}
\delta_1 \\
\delta_2 \\
\delta_3 \\
\delta_4
\end{bmatrix} + \omega_{N/B} \times 
\begin{bmatrix}
-s \delta_1 c \beta -c \delta_2 + s \delta_1 c \beta + c \delta_4 \\
c \delta_1 - s \delta_2 c \beta -c \delta_1 + s \delta_4 c \beta \\
s \delta_1 s \beta + s \delta_2 s \beta + s \delta_3 s \beta + s \delta_4 s \beta
\end{bmatrix}
I_{C}, \omega_{\text{cmg}} = -\tau_{\text{cmg}}
\]  

(2)

Matrix \( A \) is a 3 × 4 Jacobian matrix of the pyramid configuration of four-CMGs.

\[
A = J_{C}, \omega_{\text{cmg}}
\begin{bmatrix}
-c \delta_1 c \beta & s \delta_1 & c \delta_1 c \beta & -s \delta_1 \\
-s \delta_1 & -c \delta_1 c \beta & s \delta_1 & c \delta_1 c \beta \\
-c \delta_1 s \beta & c \delta_1 s \beta & c \delta_1 s \beta & c \delta_1 s \beta
\end{bmatrix}
\]  

(3)

where \( c \beta = \cos \beta, s \beta = \sin \beta \).

4. Steering laws

4.1 Pseudoinverse steering law

To determine the required torque, it is necessary to solve the inverse kinematics of Eq. (2). In this calculation, each CMG consists of a velocity servo system. The gimbal angular velocity \( \dot{\delta} \) realizes \( \tau_{\text{cmg}} \). This inverse kinematics solution is called a steering law. The simplest steering law is to use the pseudoinverse matrix of \( A \) according to:

\[
\dot{\delta} = A^+ \left( AA^+ \right)^{-1} \tau
\]  

(4)

where \( \tau \) is the required torque from the Attitude Control System (ACS) of the spacecraft (Wie, 2008).

4.2 Singularity problem

For the pseudoinverse matrix of \( A \), the singular condition occurs when all individual CMG torque vectors are perpendicular to the required torque direction. This situation is called a ‘singularity’ and, in general, a singularity occurs when matrix \( A \) meets the following condition:

\[
\text{rank} \left( A \right) < 3 \text{ or } \text{rank} \left( AA^T \right) < 3 \iff \text{det} \left( AA^T \right) = 0
\]  

(5)

In the singularity condition, the pseudoinverse does not exist in Eq. (4). In an actual case, the CMGs have to steer its gimbals radically near a singularity, resulting in a failure of the gimbal axis. It is thus essential to avoid the singularity considering this radical motion of gimbals. In general, a singularity measure \( m \) is introduced as (Wie, 2008):

\[
m = \sqrt{\text{det}(AA^T)}
\]  

(6)

4.3 Singular direction avoidance steering law

Several singularity-avoidance steering laws have been presented. Here, as an example, the singular direction avoidance (SDA) steering law (Ford and Hall, 2000) is introduced. The SDA steering law is augmented with a geometric principal rotation of singular vectors and this rotation introduces additional torque direction error to prevent zero gimbal rate output when the commanded torque is nonzero (Meng and Matsunaga, 2011).

In case of SDA, Jacobian matrix \( A \) can be decomposed into the product of three matrices:
\[ \mathbf{A} = \mathbf{U} \Sigma \mathbf{V}^T \]

\[ \Sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix} \quad (7) \]

where \( \sigma_i (i = 1, \cdots, 3) \) is the singular value of \( \mathbf{A} \). Matrices \( \mathbf{U} \) and \( \mathbf{V} \) are orthonormal modal matrices. The pseudoinverse matrix \( \mathbf{A}^+ = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \) can be expanded as:

\[ \mathbf{A}^+ = \mathbf{V} \Sigma^+ \mathbf{U}^T \]

\[ \Sigma^+ = \begin{bmatrix}
1/\sigma_1 & 0 & 0 \\
0 & 1/\sigma_2 & 0 \\
0 & 0 & 1/\sigma_3
\end{bmatrix} \quad (8) \]

When the singular value \( \sigma_i \) becomes zero as the singularity approaches, matrix \( \Sigma^+ \) diverges. To prevent this, matrix \( \Sigma^+ \) is modified by introducing \( \lambda \):

\[ \Sigma_{\text{mod}} = \begin{bmatrix}
1/\sigma_1 & 0 & 0 \\
0 & 1/\sigma_2 & 0 \\
0 & 0 & \sigma_i/(\sigma_i^2 + \lambda)
\end{bmatrix} \quad (9) \]

\[ \lambda = \lambda_0 e^{-\mu \omega} \quad (10) \]

where \( \lambda_0 \) and \( \mu \) are design parameters. SDA is expressed as:

\[ \mathbf{\delta} = \mathbf{A}^+ \mathbf{\tau}_r = \left( \mathbf{V} \Sigma^+_{\text{mod}} \mathbf{U}^T \right) \mathbf{\tau}_r \quad (11) \]

5. Formulation and design procedure

5.1 The set of nominated initial gimbal angles

The \( n \) set of initial gimbal angles nominated to level the gimbal angular displacements is denoted \( \{ \mathbf{a}_1, \cdots, \mathbf{a}_n \} \).

When the spacecraft is not making observations in orbit, the CMGs can change the set of initial gimbal angles through null motion without disturbing the attitude of the spacecraft. Several sets of gimbal angles that CMGs can achieve through null motion in a certain period of time are defined. Null vector \( \mathbf{n} \) satisfies the equation (Wie, 2008).

\[ \mathbf{A} (\gamma \mathbf{n}) = 0 \]

\[ \mathbf{n} = [C_1, C_2, C_3, C_4]^T : \text{Jacobian null vector} \]

\[ C_i = (-1)^{i+1} M_j : \text{Jacobian cofactor} \ (i = 1, 2, 3, 4) \]

\[ M_j = \det (\mathbf{A}_j) : \text{Jacobian minor} \]

\[ \mathbf{A}_i : \mathbf{A} \text{ with } i\text{-th column removed} \]

\[ \gamma : \text{Scaling factor} \]

Jacobian matrix \( \mathbf{A} \) is replaced with Eq. (13).

\[ \mathbf{A}' = \frac{\mathbf{A}}{I_{c_s} \omega_{\text{reg}}} = \begin{bmatrix}
-c\delta_1 c\beta & s\delta_1 & c\delta_1 c\beta & -s\delta_1 \\
-s\delta_1 & -c\delta_2 c\beta & s\delta_2 & c\delta_1 c\beta \\
c\delta_1 s\beta & c\delta_1 s\beta & c\delta_1 s\beta & c\delta_1 s\beta
\end{bmatrix} \quad (13) \]

Scaling factor \( \gamma \) is defined as follows.

\[ \gamma = \begin{cases}
m'^6 & \text{for } m' \geq 1 \\
m'^{-6} & \text{for } m' < 1
\end{cases} \quad (14) \]
\[ m' = \sqrt{\det(A'A')} \]  
(15)

The norm of the null vector is modified according to a gimbal angular velocity limit:

\[
n' = \begin{cases}  
n & \text{for } \|n\| \geq k\dot{\delta}_{\text{limit}} \\
n' & \text{for } \|n\| < k\dot{\delta}_{\text{limit}} 
\end{cases}
\]  
(16)

where \( k \) is a positive constant scalar, \( k = 2 \) in this study. \( \dot{\delta}_{\text{limit}} \) is the gimbal angular velocity limit. By calculating Eq. (16) sequentially, profile of gimbal angular velocity of null motion can be generated. Within this null motion profile, the set of initial gimbal angles when starting to maneuver are selected in advance. During null motion, the \( n \) set of nominated initial gimbal angles are extracted at constant intervals of \( \Delta t \).

The \( j \)-th nominated set of initial gimbal angles is expressed as:

\[ \alpha_j = \sum_{p=1}^{n} n'(t_0 + p\Delta t)\Delta t \]  
(17)

where \( n'(t) \in R^{n} \) are gimbal angular velocities during null motion. Suppose that \( t_0 \) is the time to start null motion under a non-observing condition, \( t_1 \) is the time to end null motion, and \( t_1 - t_0 \) is time interval of generating null motion. Number of times \( j \) is expressed as \( j = (t_1 - t_0)/\Delta t \).

5.2 Design of the evaluation function

The evaluation function considers not only the manipulability which is referred in the authors’ previous method (Nanamori and Takahashi, 2008), but also the gimbal angular displacements of the four CMGs. The proposed method evaluates the effect of suppression against radical motion of the gimbals according to the manipulability. Moreover, the proposed method evaluates the effect of leveling from the average and variance of the gimbal angular displacement. Definition of the symbols in relation to multiple attitude maneuvers is shown in Fig. 4.

First, the evaluation term for the manipulability is formulated. Suppose that the upcoming maneuver of the spacecraft is the \( k \)-th from the first maneuver, \( t_2 \) is the time to start the \( k \)-th maneuver, \( t_3 \) is the time of the end of \( k \)-th maneuver, and the infinitesimal time \( \Delta t_s \) as the sampling time during the off-line preliminarily calculation, number of times \( q_s \) is expressed as \( q_s = (t_3 - t_2)/\Delta t_s \). The manipulability \( m(t) \) over a constant threshold \( m_\alpha \) is evaluated as an integral value, by utilizing the relation that the radical motion of gimbals would occur when the CMGs is near singularity, or manipulability is near zero. The evaluation term is defined to have a close correlation with the radical motion. The manipulability evaluation value \( S_{\alpha j} \) is defined as:

\[ S_{\alpha j} = \sum_{p=1}^{q_s} (m_\alpha - m(t_2 + p\Delta t_s))\Delta t_s \quad \text{for } m_\alpha > m(t) \quad j = 1, \ldots, n \]  
(18)

Second, the evaluation term of the gimbal angular displacement is formulated. Gimbal angular displacement \( \delta^{sum}(k) \) accumulated in the \( k \)-th maneuver is defined as:

\[ \delta^{sum}(k, j) = \sum_{p=1}^{q_s} \|\delta(k, (t_2 + p\Delta t_s))\|\Delta t_s \]  
(19)

The average and variance of the gimbal angular displacements of the four CMGs of the \( k \)-th maneuver in Eq. (19) are defined as:

\[ \delta^{ave}(k, j) = \frac{1}{4} \sum_{i=1}^{4} \sum_{p=1}^{q_s} \|\delta(k, j_i(t_2 + p\Delta t_s))\|\Delta t_s \]  
(20)

\[ \delta^{var}(k, j) = \frac{1}{4} \sum_{i=1}^{4} \sum_{p=1}^{q_s} \|\delta(k, j_i(t_2 + p\Delta t_s))\|\Delta t_s \]  
(21)
The second term of the evaluation function \( S_2(j) \) is defined considering tradeoff between the average and variance of the gimbal angular displacements of the four CMGs:

\[
S_2(j) = \nu_1 \delta^{av} + \nu_2 \delta^{sv} \quad j = 1, \ldots, n
\]  

(22)

where \( \nu_1 \) and \( \nu_2 \) are arbitrary positive scalar parameters, which are determined to set the first term’s order equal to the second one. From Eqs. (18) and (22), evaluation function is defined as:

\[
S(j) = K \frac{S_1(j)}{S_1} + \frac{S_2(j)}{S_2} \quad j = 1, \ldots, n
\]  

(23)

where \( K \) is arbitrary positive weighting scalar parameter, \( \bar{S}_1 \) and \( \bar{S}_2 \) are the average value of \( S_1(j) \) and \( S_2(j) \). From Eq. (23), the set of gimbal angles for which the evaluation value is smallest is selected as the suitable set of initial gimbal angles \( \alpha_{su} \) among the \( n \) set nominations:

\[
\alpha_{su} = \min\left[S(j)\right] \quad j = 1, \ldots, n
\]  

(24)

Gimbal angles transit from the present gimbal state to suitable gimbal angles \( \alpha_{su} \) without disturbing the attitude of spacecraft. Afterward, the CMGs steer its gimbals using the SDA in Eq. (11) and realizes the maneuver. During this time, the Attitude Control System (ACS) as a feedback control system compensates against unknown disturbance and several errors.

5.3 Formulation and judgment of the execution timing of the proposed method

Considering the extra load by null motion, the minimum number of maneuvers before applying the proposed method \( r_{min} \) is determined. Suppose that the situation that the spacecraft repeats attitude maneuvers \( r \) times after applying null motion in the \( k \)-th maneuver. Utilizing Eq. (19), the accumulated gimbal angular displacement from the \( k \)-th maneuver is expressed as:

\[
\delta^{sum}(k, j, r) = \delta^{null}(k, j) + \sum_{k' = k}^{k+r} \delta^{sum}(k', j)
\]  

(25)

where \( \delta^{null}(k, j) \) is the gimbal angular displacement accumulated by null motion and \( k' \) is number of times. \( \delta^{null}(k, j) \) is expressed as:

\[
\delta^{null}(k, j) = \left[ \sum_{j=1}^{q_{null}} n_j'(t_0 + l \Delta t) \right] \mathbf{T}_j \ldots \left[ \sum_{j=1}^{q_{null}} n_j'(t_0 + l \Delta t) \right] \mathbf{T}_j^T
\]  

(26)

where \( \mathbf{n}'(t) = \begin{bmatrix} n_1'(t) & \ldots & n_{q_{null}}'(t) \end{bmatrix}^T \) is the null vector in Eq. (16). \( t_j \) is the time to set the \( j \)-th nominated gimbal angles, and number of times \( q_{null} \) is expressed as \( q_{null} = (t_j - t_0) / \Delta t \). The accumulated gimbal angular displacement from the \( k \)-th maneuver \( \delta^{sum}(k, j, r) \) should be reduced compared to the case of not applying null motion. The condition is expressed as:

\[
\delta^{sum}_{total}(k, j, r) < \sum_{k' = k}^{k+r} \delta^{sum}(k', 1) \quad \Rightarrow \quad r \geq r_{min}
\]  

(27)

On the other hand, because the effect of the proposed method is not permanent, null motion should be applied with proper timing over multiple maneuvers. The maximum number of maneuver for the proposed method \( r_{max} \) is determined considering gaps of gimbal angles between the initial condition and the terminal ones analytically. Conclusively, the number of maneuvers before applying the proposed method \( r \) is set within the range defined as:

\[ r \leq r_{max} \]
6. Numerical simulation

6.1 Simulation condition

In general, actual earth observing satellites maneuver around the roll axis. In the simulation, the target angle is 30 degrees around the roll axis, including two rest-to-rest maneuvers (0 to 30 degrees and then back to 0 degrees around the roll axis). The maximum maneuver rate of the spacecraft is 4 deg/s and the average rate is 2 deg/s. The SDA is applied to the steering law, and the proposed method (changing the set of initial gimbal angles and maneuvering according to the SDA) and the previous method (maneuvering according to the SDA only) are thus compared for the same feedback control system and the same mission.

Simulation parameters of the satellite and CMGs are given in Table 1. The feedback controller of the ACS is a Proportional Derivative (PD) controller.

![Fig. 4 Definition of the symbols in relation to multiple attitude maneuvers.](image)

\[ r_{\text{min}} \leq r \leq r_{\text{max}} \] (28)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>gimbal axis moment of Inertia ( I_{Cx} )</td>
<td>0.19 kgm²</td>
</tr>
<tr>
<td>wheel axis moment of Inertia ( I_{Cy} )</td>
<td>0.11 kgm²</td>
</tr>
<tr>
<td>maximum angular velocity of gimbal axis</td>
<td>( \pm 1.0 ) rad/s</td>
</tr>
<tr>
<td>maximum angular acceleration of gimbal axis</td>
<td>3.0 rad/s²</td>
</tr>
<tr>
<td>wheel speed</td>
<td>6000 rpm</td>
</tr>
<tr>
<td>angular velocity of Z axis</td>
<td>0 rad/s</td>
</tr>
<tr>
<td>angular acceleration of Z axis</td>
<td>0 rad/s²</td>
</tr>
<tr>
<td>skew array angle ( \beta )</td>
<td>54.7 deg</td>
</tr>
</tbody>
</table>
| satellite moment of inertia \( I_s \)          | \[
\begin{bmatrix}
1.50 \times 10^3 & 0 & 0 \\
0 & 1.50 \times 10^3 & 0 \\
0 & 0 & 1.50 \times 10^3 \\
\end{bmatrix}
\text{kgm}^2
\]  |
6.2 Simulation results

6.2.1 Generating the set of nominated initial gimbal angles

The $n$ sets of nominated gimbal angles are generated by continuing null motion defined in Eq. (16). The time interval of generating null motion $t_{n} - t_{0}$ is supposed to be 5 seconds, and the constant intervals for extracting the nomination of $\Delta t$ is set as 0.5. Therefore the number of extracted sets is $n = 10$. Time histories of null motion are shown in Fig. 5. When null motion starts, gimbal angles are supposed to be zero without loss of generality in this numerical simulation. However if they have arbitrary angles, CMGs start null motion from their angles. The off-line preliminarily calculation is done for each set of extracted initial gimbal angles. From the result of the off-line preliminarily calculation, the suitable set of initial gimbal angles is selected.

6.2.2 Selection of the suitable set of initial gimbal angles by evaluation function

The set which evaluation value is the smallest is selected as the suitable set of initial gimbal angles. Here, the constant threshold $m_{th}$ relating to the manipulability is set as 1.5. As for weighting factors, $\nu_{1} = 1.0$, $\nu_{2} = 2.0$ are set in Eq. (22). By changing weighting factor $K$ of the evaluation function $S(j)$ in Eq. (23), the effect of the term on manipulability against the term on gimbal angular displacement is analyzed. The selected nomination number $j$ and the suitable sets of initial gimbal angles $a_{su}$ in each case are given in Table 2. Figure 6 (a) shows the time histories of manipulability defined in Eq. (6) in each case. Figure 6 (b) shows the average $\delta^{avw}(j)$ and variance $\delta^{aw}(j)$ of the gimbal angular displacement. As shown in Fig. 6 (a), when the evaluation function does not consider the manipulability in Case 1 ($K=0$), this steering is not preferable for the next maneuver because the terminal value of manipulability is low. On the other hand in Case 2 ($K=2.00$), the well-balanced steering is realized from the viewpoint of manipulability and gimbal angular displacement compared to the Case 3 ($K=0.100$). From such case studies, weighting factor $K$ is set as $K = 2.00$ in this study. From this setting, the suitable set of initial gimbal angles is selected according to Eq. (24) as:

$$a_{su} = a(j \mid j = 7) = [-0.283 \ 0.283 \ 1.33 \ -1.33]^{T}$$ (29)

6.2.3 Results of a single attitude maneuver

The results of the attitude maneuver starting from the selected suitable set of initial gimbal angles in Case 2 are shown in Fig. 7, which is the 30 deg maneuver around the roll axis, not passing the singularity. Figure 8 shows the gimbal angular displacements after the one maneuver, which contains (a) the previous method, (b) the proposed method, and (c) null motion applied in the proposed method. Figure 9 shows (a) the average of the gimbal angular displacements and (b) the variance of those, in case that the spacecraft repeats the same type of maneuver. This simulation assumes that the proposed method applies null motion once before starting multiple maneuvers. Figure 9 shows the time histories of gimbal angular acceleration.

Figure 7 reveals that the control performance of the proposed method is approximately the same as that of the previous method. In addition, the numerical simulation confirmed that the proposed method was able to level the gimbal angular displacements (as shown in Fig. 8). As shown in Fig. 8 (a), if the spacecraft repeats the same type of maneuver in which the CMGs do not pass the singularity, the gimbal angular displacement is biased toward specific CMGs (CMG 1 and CMG 3 in this simulation) when employing the previous method. Repeating this type of steering would amplify this bias. In this respect, as shown in Fig. 8 (b), the proposed method succeeds in leveling the gimbal angular displacement. From Fig. 8, $\delta^{aw} = 2.03$, $\delta^{aw} = 4.35$ for the previous method and $\delta^{aw} = 1.61$, $\delta^{aw} = 1.04$ for the proposed method after the one maneuver. From Fig. 6 (a), the proposed method (Case 2) enables CMGs to keep the manipulability greater, or farther from the singularity than the previous method. This means that the proposed method suppresses radical motion of gimbals when the CMGs are near singularity. However as shown in Fig. 9, this effectiveness is not so remarkable because the CMGs does not pass the singularity in this case. This effectiveness would be more apparent in case that the CMGs pass singularity during maneuvers.
6.2.4 Results of the multiple attitude maneuvers

The minimum number of maneuvers before applying the proposed method $r_{\text{min}}$ is set. Figure 10 shows the average and variance of the accumulated gimbal angular displacement after repeating the same patterns of maneuvers. From Fig. 10 (b), the proposed method keeps the variance lower than the previous method even after repeating the same type of maneuver. However, the extra load is added by applying null motion $\delta^{\text{avr}} = 1.59$ before starting the maneuvers, null motion should be applied between every predefined number of maneuvers in the advantage of the gimbal angular displacement. From Fig. 10(a), it is preferable to apply the proposed method about every four maneuvers in this case.

Next, the maximum maneuvers before applying the proposed method $r_{\text{max}}$ is set. Figure 11 (a) shows that the gaps of gimbal angles from the initial condition just after applying null motion. Figures 11 (b) and (c) show that the time histories of manipulability and gimbal angular displacements of the four CMGs respectively in each number of maneuver $(r=1, 100, 200, 300, 400, 500)$. By confirming the gaps of the initial condition about the set of gimbal angles and manipulability compared to the condition of $r=1$, $r_{\text{max}} = 300$ is set in this study. Therefore, the number of maneuvers before applying the proposed method $r$ is set within the range defined as:

$$4 \leq r \leq 300$$  \hspace{1cm} (30)

7. Conclusions

This study presented a new steering law for a pyramid configuration of four Control Moment Gyros (CMGs), which attempts to suppress the radical motion of gimbals near singularity and level the load for each CMG. The advantage of the proposed method is to suppress the motion of gimbals that leads to the failure of the CMGs, by adding the null motion to the conventional steering law before starting to maneuver. Additionally, analytical findings on the execution timing of the proposed method in relation to multiple attitude maneuvers were newly presented. The proposed method utilizes the characteristic that each gimbal angular displacement of a CMG depends on a set of initial gimbal angles. An off-line preliminarily calculation is carried out and a suitable set of initial gimbal angles is selected at the start of the maneuver according to a defined evaluation function. The evaluation function considers not only the manipulability, but also the gimbal angular displacements of the four CMGs. The proposed method confirmed the effect of suppression against radical motion of the gimbals, and reduction and leveling of gimbal angular displacement. Assuming the spacecraft repeats the same pattern of the maneuver repeatedly, the number of maneuvers before applying the proposed method $r$ is determined considering the extra load by null motion and the accumulated gaps of gimbal angles between the initial condition and the terminal ones. The steering patterns of the CMGs are analyzed and the execution timing of null motion is presented. The dynamics of a typical pyramid configuration of four CMGs was modeled and a numerical simulation in case CMGs pass through singularity was carried out. Through analysis and numerical simulation, it was shown that the proposed method, in comparison with the previous method, leveled the gimbal angular displacements of the four CMGs without degrading the performance of attitude control and keeps the manipulability greater. To verify performance of the proposed method in case of CMGs passing near singularity during large angle maneuver is future work.
Fig. 5  Time histories of null motion.

(a) Time histories of manipulability.  
(b) Each term on evaluation function.

Fig. 6 Case study of parameter $K$.

Fig. 7 Time histories of Euler angle (Case 2).
(a) The previous method.  
(b) The proposed method (Case 2).  
(c) Null motion (Case 2).

Fig. 8 Gimbal angular displacements, related to term $S_1$ in the evaluation function.

(a) The previous method.  
(b) The proposed method (Case 2).

Fig. 9 Time histories of gimbal angular acceleration.

(a) Average.  
(b) Variance.

Fig. 10 Transition of the average and variance of the gimbal angular displacements according to the numbers of maneuvers.
(a) Gimbal angle error against the suitable initial gimbal angles according to the numbers of maneuvers.

(b) Time histories of manipulability.

(c) Gimbal angular displacements.

Fig. 11 Results of the multiple maneuvers.

References

