Wave control of crane rope-and-mass system

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Abstract
This paper deals with the wave absorbing control of a suspended simple pendulum system by the lateral motion of the support. The control acceleration of the support was derived by the connecting condition of real pendulums to the wave-controlled multiple simple pendulums virtually existing above the support. The wave propagating solution of the homogeneous multiple simple pendulum system was theoretically derived and applied to the virtual pendulum system. The velocity and position feedback control of the support was added to the support acceleration for wave control because the wave control could not fix the support position. This control can place the support at other than the original position. The experiments of a three homogeneous rigid pendulum system and a rope-and-mass system were conducted. The rope-and-mass experimental system has the function of winding up and down of load mass. The effect of the virtual pendulum length on the wave control was also investigated. From the simulation and experiment, our presented control strategy was shown to be effective enough and practical for real crane systems.

Key words: Wave control, Vibration control, Multiple pendulum system, Rope and mass system, Crane rope system

1. Introduction

Manipulation of heavy objects at construction sites, warehouses and shipyards is often accomplished with cranes. The natural sway of crane payloads causes time delays and degradation of positioning accuracy. Hoisting of the payload during transverse motion increases the difficulty of control because the oscillation frequency is time varying. A computer controller that takes into account the rope swing can generate time-optimal commands that result in zero residual vibration. Feedforward and feedback controllers have been proposed in the following studies. Optimal controls with state feedback have been addressed, where the rope length is a parameter or is given as a time-dependent function (Sakawa and Shindo, 1982; Shirai et al., 1993; Murata, et al., 1995). Gain scheduling controllers where the rope length is considered as an arbitrary time function have been proposed (Kaneshige et al., 1997, 1998a, 1998b; Takagi et al., 1998, 2001). Feedforward controllers considering the swing phase control have been reported (Kurimoto and Yabuno, 2009a, 2009b, 2010). Another approach to crane control has been input shaping control, which does not require the feedback mechanism of closed-loop and adaptive controllers (Singhose et al., 2000; Khalid et al., 2006). The above-mentioned works on crane control were based on vibration control, that is, natural-frequency based control.

Wave based approaches in terms of mechanical waves rather than classical control or vibration formulation have been studied (Saigo et al., 1995, 1999, 2003; O’Connor, 2003). Wave control is easier to derive and implement than is the time-optimal control and does not cause control instability. The approach by one of the authors of the present paper was to control the acceleration of the support of pendulum system by the connecting condition to the imaginary multiple simple pendulum system in which the equations of motion were computed online in the controller. This control used the initialization of the imaginary system to prevent the wave reflection at the support of the imaginary
system. Using the imaginary system with initialization brought less control performance and was too deficient in control flexibility to change the parameters adaptively if desired.

In this paper, wave absorbing control without imaginary system initialization is addressed. The newly derived wave propagating solution is used for control of the multiple pendulum system. For the rope-and-mass system, a two degrees of freedom (DOF) homogeneous imaginary system controlled by the wave solution is used. The length of the imaginary pendulum dominates over the control performance in the rope-and-mass system, which is investigated in Section 2.4. The feedback control of the support velocity and the position is introduced in addition to the wave control because the acceleration control of the support is not able to fix the support position. An experiment and a numerical simulation were conducted to confirm the control performances.

The control approach is similar to the previous study on the wave control of the suspended rope (Zheng and Saigo, 2014), which addressed the control strategy to make the boundary condition virtually disappear regardless of the real fixed support. The present study, on the contrary, deals with the wave control of the support (the boundary of multiple pendulum system) with the feedback of the support potion which is able to transfer the support to an arbitrary position.

2. Wave control of multiple simple pendulum system by support acceleration

2.1 Equations of motion

Figure 1 shows a nonhomogeneous n-DOF multiple simple pendulum system with the support acceleration $\ddot{x}_{n+1}$. The mass, length and swing angle of the $i$-th pendulum are $m_i$, $l_i$ and $\theta_i$, respectively.

![Fig. 1 Nonhomogeneous multiple simple pendulums system.](image)

The equations of motion are given by Lagrange’s equation of motion. Assuming small swing angles, the kinetic energy $T_k$ and the potential energy $U_k$ of the $k$-th pendulum are

$$T_k = m_k \left( \ddot{x}_k + \sum_{j=k+1}^{n} l_j \dot{\theta}_j \right)^2 / 2 + m_k l_k^2 \dot{\theta}_k^2 / 2 + m_k \left( \ddot{x}_k + \sum_{j=k+1}^{n} l_j \dot{\theta}_j \right) \dot{\theta}_k$$

$$U_k = -m_k g \left( l_k \left( 1 - \theta_k^2 / 2 \right) + \sum_{j=k+1}^{n} l_j \left( 1 - \theta_j^2 / 2 \right) \right)$$

(1)

The Lagrangian $L = \sum_{k=1}^{n} (T_k - U_k)$ gives the following equations of motion,

$$l_i \ddot{\theta}_i + g \left( m_i \frac{1}{m_i + m_2} \dot{\theta}_i - (m_i + m_2) \frac{1}{m_2} \dot{\theta}_i \right) = 0$$

(2)
Equations (3) and (4) show that if \( \ddot{x}_{k+1} \) is the actual support acceleration, as shown in Fig. 2, and satisfies the following relation (5), which is obtained from Eq. (4) by setting \( n = k \), the angle of the \( i(\leq k) \)-th pendulum will propagate to the \( j(> k) \)-th pendulum,

\[
\ddot{x}_{k+1} = g \left[ \sum_{i=1}^{k} m_i \frac{\theta_k}{m_{k+1}} - \sum_{i=1}^{k} m_i \frac{\theta_{k+1}}{m_{k+1}} - \sum_{i=1}^{n} m_i \frac{\theta_n}{m_{k+1}} \right]
\]

Here, we can consider any number of \( n \). Then, Eq. (5) gives the wave absorbing control of the real pendulums if \( \theta_{k+1} \) is obtained. We call this \((n-k)\)-DOF pendulum system an imaginary pendulums system hereafter.

Fig. 2 Real multiple simple pendulums suspended by imaginary multiple simple pendulums.

### 2.2 Wave solution of the homogeneous system

An approximate wave solution of the homogeneous multiple simple pendulum system is derived because it is difficult to derive the wave solution of the nonhomogeneous system.

The equation of motion of the \( k \)-th pendulum of the homogeneous simple pendulum system is expressed by using the pendulum length \( l \) as,

\[
\ddot{\theta}_k + \frac{g}{l} \left[ -(k-1)\theta_{k-1} + 2k\theta_k - (k+1)\theta_{k+1} \right] = 0
\]  

(6)

By taking the Laplace transform of both sides of Eq. (6), written symbolically as \( \Theta(s) = L[\theta(t)] \), we obtain the algebraic equation

\[
(k+1)(g/l)\Theta(s)_{k+1} - \left[ s^2 + 2k\left(\frac{g}{l}\right) \right] \Theta(s)_k = (k-1)(g/l)\Theta(s)_{k-1} = 0
\]

(7)

Assuming the solution of Eq. (7) as
we obtain the characteristic Eq. (9) and the positive propagating solution (10)

\[ \gamma(s)^2 - \left(2 + \left(s/\sqrt{kg/t}\right)^2\right)\gamma(s) + 1 = 0 \]  
\[ \gamma(s) = \frac{1}{4} \left( \sqrt{4 + \left(s/\sqrt{kg/t}\right)^2} - s/\sqrt{kg/t} \right)^2 \]  

The inverse Laplace transform of Eq. (10) gives the time domain expression, Eq. (11)

\[ L^{-1}\left[ \gamma(s) \right] = \frac{2}{t} J_2 \left(2\sqrt{kg/t} \cdot t\right) \]  

where \( J_2 \) is the Bessel function of the first order of the second kind.

Equations (8), (10) and (11) give the transfer function between adjacent pendulum angles in the frequency domain as

\[ \Theta(s)_{k+1} = \frac{k}{k+1} \gamma(s;k) \cdot \Theta(s)_k \quad \Theta(s)_k = \frac{k-1}{k} \gamma(s;k) \cdot \Theta(s)_{k-1} \]  

and in the time domain as

\[ \theta_{k+1}(t) = \frac{k}{k+1} \int_0^t J_2 \left(2\sqrt{kg/t} \cdot (t-v)\right) \theta_k(v) \, dv \]  
\[ \theta_k(t) = \frac{k-1}{k} \int_0^t J_2 \left(2\sqrt{kg/t} \cdot (t-v)\right) \theta_{k-1}(v) \, dv \]  

Fig. 3 Response of free vibration of 33-DOF homogeneous simple pendulums with initial conditions

\[ \theta_1 = 1, \quad \theta_{(n=1)}(0) = 0, \quad \theta_{(n=2)} = 0. \quad \Theta_{(n=3-4)} : \text{exact solutions}; \quad \Theta_{(n=3-4)} \text{ : convolution solutions by Eq. (14).} \]

The validity of the solution (10), newly presented in this study, needs to be confirmed by numerical calculation. Figure 3 shows the accuracy of solution (10). The free vibration of a 33-DOF homogeneous multiple simple pendulum system is solved by both the natural-frequency based analytical solution and the convolution integral solution (14), where the non-dimensional time \( \tau = \sqrt{g/t} \) is used. The accuracy of the convolution integral solution from \( \theta(t) \) to \( \theta(t) \) \( \theta_{(k+1)} \) is acceptable, but those from \( \theta(t) \) to \( \theta(t) \) \( \theta_{(k+3)} \) and from \( \theta(t) \) to \( \theta(t) \) \( \theta_{(k-1)} \) are not. The reason may be by that solution (10) was derived from the equation of motion (7) which considered no boundary condition. From this, we can see the wave propagating solution should be applied to more than a 3-DOF homogeneous system. The solution \( \gamma(s;k) \) represents both the propagating relation from the \( k \)-th pendulum to the \( (k+1) \)-th one and that from \( (k-1) \)-th pendulum to the \( k \)-th one.

Figure 4 shows the convolution responses from \( \theta_1 \) to \( \theta_1 \) and from \( \theta_k \) to \( \theta_k \) by using the relation Eq. (14) and Eq. (15), respectively, under the same condition shown in Fig. 3. The convolution solutions are virtually the same. However, the solution by Eq. (15) is more accurate than that by Eq. (14). The difference of accuracy might be attributed to the position of the pendulum from the free boundary. For example, for the solution from \( \theta_1 \) to \( \theta_1 \) using \( k = 4 \) is more accurate than using \( k = 3 \). The difference is smaller for a larger value of \( k \), and both Eq. (14) and (15) will be used practically in the same way.

### 2.3 Wave control of 1-DOF rope-and-mass system by support acceleration
Equation (5) can be applied to the control for more than a 3-DOF homogeneous simple pendulum system just as it is. For a 1-DOF rope-and-mass system, it is appropriate to use a 2-DOF homogeneous simple pendulum system in which the real rope-and-mass system is suspended (imaginary pendulum system). The equations of motion of the real pendulum and imaginary pendulums are

\[
\begin{align*}
\theta_i'' + g\theta_i &= g(-\theta_i + 2\theta_i) \\
I_i\theta_i'' + g(-\theta_i + 4\theta_i - 3\theta_i) &= 0 \\
I_i\theta_i'' + g(-2\theta_i + 6\theta_i - 4\theta_i) &= 0
\end{align*}
\]

(16) (17)

where \( l \) and \( l_i \) are the real pendulum length and the imaginary pendulum length, respectively, and all masses are the same. The right-hand side of Eq. (16) is the control acceleration \( \ddot{x}_i \). The angle \( \theta_i \) in Eq. (17) is solved on-line with the measured value \( \theta_i \) and the estimated value \( \hat{\theta}_i \) which is calculated from Eq. (14) or Eq. (15) with \( \theta_i \). In experiments Eq. (17) is solved by the modified Euler method in the controller. The wave propagation and vibration energy transportation in the homogeneous system will be more rapid than in any nonhomogeneous system, and it will be most appropriate to use the homogeneous system for wave control. A crane rope-and-mass system has, however, hoisting and lowering actions to make it a variable length system, so it is necessary to treat the nonhomogeneous pendulum system. The imaginary pendulum length \( l_i \) is the important parameter for control, and its effect on control is discussed in the next section.

2.4 Wave propagation in nonhomogeneous multiple simple pendulum system

In this section we investigate the wave control of a nonhomogeneous simple pendulum system such as the lowest pendulum and the other pendulums being of different lengths. The equations of motion (16) and (17) expressed in pendulum angles are transformed to expressions in absolute displacements of masses to define the control of forced vibration easily,

\[
\begin{align*}
x_i'' + x_i - x_i &= f_i \\
x_i'' + x_i \left(2\lambda^{-1} + 1\right) x_i - \left(2\lambda^{-1}\right) x_i &= 0 \\
x_i'' - \left(2\lambda^{-1}\right) x_i + \left(5\lambda^{-1}\right) x_i - \left(3\lambda^{-1}\right) x_i &= 0
\end{align*}
\]

(18)

where \((*)'\) denotes the differentiation by non-dimensional time \( \tau = t\sqrt{g/l} \), \( x_i \) is the absolute displacement of the \( i \)-th pendulum, \( f_i \) is the disturbance force to the first (lowest) mass and \( \lambda = l/l_i \).

Equation (18) represents the 3-DOF wave controlled simple pendulum system when wave propagating solution is applied to the displacement \( x_i \) by the following expression (19) obtained from Eq. (15),

\[
x_i(\tau) = x_i(\tau) + \frac{2}{3} \int_{\tau}^{\tau} \frac{2}{3} \int_{\tau}^{\tau} J_2 \left[2\sqrt{3/\lambda}(\tau - v)\right] \left[x_i(v) - x_i(v)\right] dv
\]

(19)

The uncontrolled response is obtained by setting \( x_i = 0 \).
Figure 5 shows the controlled responses of the 3-DOF pendulum system to the sinusoidal disturbance on the first (lowest) pendulum mass. Figures 5(a), 5(b) and 5(c) show the gain of the frequency response for $\lambda = 0.2$, 1 and 5, respectively, and Fig. 5(a), 5(b) and 5(c) show the phase for $\lambda = 0.2$, 1 and 5, respectively. In the gain charts, the time domain controlled responses sufficiently correspond to the frequency responses. Figure 5(b) shows almost perfect wave control over the entire frequency range. On the other hand, Figs 5(a) and 5(c) show the resonance-like response peak near the first and the third natural frequencies. The phase curves in Fig. 5(a) and 5(c) are also close to the uncontrolled responses and indicates the decreasing of damping.

Fig. 5 Controlled responses of the 3-DOF nonhomogeneous pendulum system Eq. (18) to the disturbance on the first pendulum mass. Figures 5(a), 5(b) and 5(c) show the gain of frequency response of the first pendulum X1(controlled) and UX1(uncontrolled) for $\lambda = 0.2$, 1, 5, respectively. The time domain responses X1s correspond to the frequency responses. Figures 5(a), 5(b) and 5(c) show the phase charts for $\lambda = 0.2$, 1, 5, respectively.

Figures 6(a), 6(b) and 6(c) show the free vibration control for $\lambda = 0.2$, 1 and 5, respectively. The parameter $\lambda \neq 1$ provides lower performance for the free vibration control and is the same as that of the forced vibration. Figure 6(c) shows very poor control performance, especially for the 3rd natural-frequency vibration; Figure 6(a) shows less control performance than does Fig. 6(b) in the first natural-frequency vibration. The control depends on the wave propagation, which can be seen by the vibration mode shown in Figs. 6(a), 6(b) and 6(c).

The parameter $\lambda > 1$ has the limit value for stable wave control. The approximate limit value is given by the relation that the maximum natural frequency $\Omega_n$ of the $n$-DOF pendulum system should be less than the wave propagation limit $\Omega$ as

$$\Omega_n \leq \Omega' = 2\sqrt{n\lambda^2}$$  \tag{20}$$

The above wave propagation limit is obtained by the condition that gives the vanishing of the phase delay of wave solution, Eq. (10). Figure 7 shows the natural frequencies and the relation (20) for $n = 3$. The limit value $\lambda_{\text{lim}}$ is nearly 5, which is confirmed in Figs. 6(c) and 6(c).

The wave control on the more than 4 DOF simple nonhomogeneous pendulum system has the other unfavorable phenomenon, mode localization (Pierre, 1988). Figure 8 shows the natural frequencies of 5-DOF simple pendulums of which the $k$-th length $(2 \leq k \leq 5)$ is $\lambda$ and the first length is 1. The 5-th natural frequency and the 4-th natural frequency have closed near $\lambda \approx 5$, which is far lower than the propagation limit value of $\lambda \approx 9$; this means mode localization occurred. The wave controlled responses for $\lambda = 5$ and 7 are shown in Figs. 8(b) and 8(b), respectively.
The controlled performances deteriorated near natural frequency $\approx 1.5$ (we represent this mode localization frequency as $\Omega_{nl}$). The mode localization might disturb the wave propagation.

3. Control of the final position of support

The abovementioned wave control strategy has no control effect on the final position of the system because the control is based on the acceleration of the support. When the control acceleration becomes zero due to no swing angles of all pendulums, the control finishes regardless of any constant velocity as a rigid body. The responses in Fig. 6 for the initial angle without initial angular velocities settled nearly to the original position. However, the responses for the initial angular velocity do not settle to the starting point and keep traveling (figures are not shown). So, we introduce position control in addition to pendulum angle control. In this study, we used the support control acceleration by adding the feedback control of support velocity and position as

$$a = K_s \dot{x}_i - K_v \dot{x}_i - K_p (x_i - x_f),$$

(21)

where $K_s$, $K_v$, and $K_p$ are constants to be configured in some way and $x_f$ is the final support position. By setting $x_f$ to any value other than zero, we can transfer the pendulum system to the target position $x_f$. The above feedback terms

Fig. 6 Controlled free vibrations of 3-DOF nonhomogeneous pendulum system for $\lambda = 0.2, 1, 5$. Figure 6(c) for $\lambda = 5$ shows very poor control performance.

Fig. 7 Wave propagation limit of 3-DOF nonhomogeneous simple pendulums of length $\lambda (k = 2, 3)$ and $1(k = 1)$. $\Omega_i$: $i$-th natural frequency; $\Omega^*$: propagation limit given by expression (20). No mode localization occurs.
are introduced to realize the 1-DOF damped free vibration characteristics as
\[
\ddot{x}_i + K_x \dot{x}_i + K_p (x_i - x_f) = 0
\]

Feasible values of the feedback constants were investigated by numerical calculations of the evaluation value \( J(t) \) by setting \( x_f = 0 \),
\[
J(t) = \sqrt{\sum_{i=1}^{n} \theta_i(t)^2 + \{x_i(t)/l\}^2}
\]
\[(22)\]

Apparently, the evaluation value depends on the initial conditions and changes with time, so it is difficult to find the optimum feedback constants for general conditions. Here, we just investigated the tendency of \( J(t) \) for the 3-DOF pendulum model, which is the same as the experimental apparatus with initial conditions \( \theta_i = 0.1 \) and \( \dot{\theta}_i = 1.0 \). Because our target system is a rope-and-mass system, the control performance for the disturbance to the first pendulum was considered. Figure 9 shows the evaluation values at \( t = 8 \) s; Figs. 9(a-i) and 9(a-j) are for \( \lambda = 0.2 \), 9(b-i) and 9(b-j) for \( \lambda = 1.0 \), and 9(c-i) and 9(c-j) for \( \lambda = 1.5 \); Figs. 9(a-i), 9(b-i) and 9(c-i) are for \( K_u = 0.6 \), and 9(a-j), 9(b-j) and 9(c-j) for \( K_s = 1.0 \). From these figures, we can see that the suitable feedback constants are nearly the same regardless of the value of \( \lambda \); that is, \( 4 \leq K_x \) is suitable and \( K_p \) has little effect on the control. Also, it can be seen that the absolute values of \( J(8) \) depend on the value of \( \lambda \), and the larger value of \( \lambda \) has better control performance. Some combinations of \( K_x \) and \( K_p \) for the case of \( K_s < 1 \) give better control performance than for the case of \( K_s = 1 \). We confirmed the above tendency for the initial conditions \{ \( \theta_i = 0.1 \), \( \dot{\theta}_i = 0 \) \} and \{ \( \theta_i = 0 \), \( \dot{\theta}_i = 1.0 \) \}, and for the case of \( J(6) \), too.

4. Experiment

4.1 Experimental apparatus and procedure

Figure 10 shows the experimental apparatus for the 3-DOF homogeneous pendulum system and the rope-and-mass system. The horizontal movement of the pendulum support is actuated by a ball screw and nut system which is controlled by a servo motor. The servo motor is controlled by a DSP system. The angles of the pendulums are measured...
The rope-and-mass system is measured by the angle of a small arm on the axis of the support with the potentiometer to which the end of the rope is connected. The support position is obtained by the integration of the control acceleration feedback constants experimentally and used desirable feedback constants obtained in the previous section had not brought stable control, we searched suitable (not direct measurement). The sampling time of the control is 20 ms and the convolution integral terms are 80. The change the rope length at any speed. The mass of the pulse motor unit is utilized as the load mass. The angle of the pendulum). In the rope-and-mass system, the winding up/down device is constructed by a pulse motor. This system can by potentiometers; the uppermost is for control and the others are for monitoring the wave propagation of the angles of the 3-DOF pendulum system. The length of the homogeneous pendulum is 0.24 m (the equivalent length as a simple pendulum). In the rope-and-mass system, the winding up/down device is constructed by a pulse motor. This system can change the rope length at any speed. The mass of the pulse motor unit is utilized as the load mass. The angle of the rope-and-mass system is measured by the angle of a small arm on the axis of the support with the potentiometer to which the end of the rope is connected. The support position is obtained by the integration of the control acceleration (not direct measurement). The sampling time of the control is 20 ms and the convolution integral terms are 80. The 2-DOF imaginary system is computed by using the modified Euler method for the rope-and-mass system. Because the desirable feedback constants obtained in the previous section had not brought stable control, we searched suitable feedback constants experimentally and used \( K_a = 0.25 \) or \( 0.5 \), \( K_v = 0.5, 1.0 \), or \( 2.0 \), \( K_p = 0.5, 1.0 \) or \( 2.0 \). The reasons why the numerical simulation results were impractical might be due to the control delay caused by mechanical inertia of the ball screw driving system, the measuring error of the angle, and the error of pendulum modeling as simple pendulums for the 3-DOF system.

4.2 Experimental results

4.2.1 3-DOF homogeneous pendulum system
Figure 11 shows the experimental responses for the initial angle \( \theta_1 \approx 0.4 \text{ rad} \) and \( x_r = 0 \), where the angle \( \theta_1 \) is the computed value in the controller. Figures 11(a), 11(b) and 11(c) are controlled responses by using the imaginary
system same as the real system (λ=1), and the feedback constants \{K_x = 0.25, K_y = K_p = 0.5\}, \{K_x = 0.5, K_y = K_p = 0.5\}, and \{K_x = 0.5, K_y = K_p = 1.0\}, respectively. Figure 11(d) shows free vibration of the 3 pendulum system, which shows the damping in the experimental apparatus. The relative angular movements were damped away in a few seconds and vibrated as a single pendulum. The controlled response demonstrated the effectiveness of the presented wave control strategy. Figures 11(a) and 11(b) have confirmed that the feedback constant \(K_x\) has a control effect on the angle suppression; Figures 11(b) and 11(c) showed that the support travel is shorter for larger feedback constants \(K_y\) and \(K_p\), though the duration of the support oscillation is longer. This might be due to the control inaccuracy, which is discussed in Section 5 by comparing the experimental results with the simulation.

### 4.2.2 Rope-and-mass system

Figures 12(a), 12(b) and 12(c) show the experimental responses of the rope-and-mass system during winding up at velocity 0.16 m/s, winding down at velocity 0.16 m/s, and at constant length, respectively; The imaginary pendulum length \(l_c\) is 0.12 m and the rope length varies roughly between 0.9 m and 0.12 m, that is, parameter \(\lambda\) varies between 0.13 and 1. The time of winding is about 5 s. The 2-DOF imaginary pendulums are solved by the modified Euler method. The angles \(\theta_1\), \(\theta_2\) and \(\theta_3\) in Fig. 12 are the computed values in the controller. The mechanical control switch is turned on after the DSP controller starts, so all values are recorded before the system is actually controlled, and the starting support position is not zero for the initial pendulum angle \(\theta_1\). Feedback constants used are \(K_x = 0.25, K_y = 1.0\) and \(K_p = 1.0\).

![Experimental responses of rope-and-mass system](image)

**Fig. 12** Experimental responses of rope-and-mass system. (a): during winding up at velocity 0.16 m/s; (b): winding down at velocity 0.16 m/s; (c): at constant length; the imaginary pendulum length \(l_c\) is 0.12 m and the rope length varies between 0.9 m and 0.12 m. Feedback constants are \(K_x = 0.25, K_y = 1.0\) and \(K_p = 1.0\). \(\theta_i\): \(i\)-th pendulum angle; Pos: support position.

Figures 13(a), 13(b) and 13(c) show the experimental responses of rope-and-mass system during winding up at velocity 0.16 m/s, winding down at velocity 0.16 m/s, and at constant length, respectively. The imaginary pendulum length \(l_c\) is 0.9 m and rope length varies between 0.9 m and 0.12 m, and the parameter \(\lambda\) varies between 1 and 4.5. The other experimental conditions are the same as those in Fig. 12. From Figs. 12 and 13, it is confirmed that the presented control strategy works very well on the rope-and-mass system. Figures 12(c) and 13(c) show the typical control performance of parameter \(\lambda\) shown in Fig. 9. The settling time is longer when \(\lambda < 1\) than when \(\lambda > 1\). On the other hand, the support travel is shorter when \(\lambda < 1\) than when \(\lambda > 1\). This characteristic will be useful to design the small-space crane system at the expense of settling time. The better controllability of winding down rather than winding up might arise from the damping in the experimental apparatus or the experimental apparatus imperfection, because the simulation responses do not present such tendencies, as described in Section 5. The deterioration of control of angle \(\theta_1\) between 7 s and 15 s in Fig. 13(a) could be caused by the phenomena investigated in Section 2.4; that is,
Figure 14 shows the experimental responses of the rope-and-mass system without control corresponding to the responses in Fig. 12 and Fig. 13. This figure shows clearly the control effectiveness in Fig. 12 and Fig. 13. The response in Fig. 14(a) was not a plane motion but a spherical one when the rope length was fairly short in wind-up action, and did not show adequately the uncontrolled free vibration corresponding to the controlled response.

Figures 15(a) and 15(b) show the support position control of the 0.2 m traverse. The pendulum lengths are $l = l_c = 0.9 \text{ m}$ and the feedback constants are $K_u = 0.25$, $K_c = K_p = 1.0$ and $K_u = 0.25$, $K_c = K_p = 2.0$, respectively. In these figures, we can see that the position control in Fig. 15(b) is better than that in Fig. 15(a), but the

Fig. 13 Experimental responses of rope-and-mass system. (a): during winding up at velocity 0.16 m/s; (b): winding down at velocity 0.16 m/s; (c): at constant length; the imaginary pendulum length $l_c$ is 0.9 m and the rope length varies between 0.9 m and 0.12 m. Feedback constants are $K_u = 0.25$, $K_c = 1.0$ and $K_p = 1.0$. $\theta_i$: $i$-th pendulum angle; Pos: support position.

parameter $\lambda$ becomes near the limit value for control.

Figure 14 shows the experimental responses of the rope-and-mass system without control corresponding to the responses in Fig. 12 and Fig. 13. This figure shows clearly the control effectiveness in Fig. 12 and Fig. 13. The response in Fig. 14(a) was not a plane motion but a spherical one when the rope length was fairly short in wind-up action, and did not show adequately the uncontrolled free vibration corresponding to the controlled response.

Figures 15(a) and 15(b) show the support position control of the 0.2 m traverse. The pendulum lengths are $l = l_c = 0.9 \text{ m}$ and the feedback constants are $K_u = 0.25$, $K_c = K_p = 1.0$ and $K_u = 0.25$, $K_c = K_p = 2.0$, respectively. In these figures, we can see that the position control in Fig. 15(b) is better than that in Fig. 15(a), but the
angle control in Fig. 15(a) is better than that in Fig. 15(b). The feedback constants have a control effect on the position control, but a negative effect on the angle control. The optimal combination of $K_a, K_r$ and $K_p$ depend on the operational condition and should be obtained by simulations, as shown in Fig. 9. The experimental apparatus does not have enough traverse distance and a detail experiment has not been carried out; however, it is confirmed that the present control strategy works well for the traverse crane system.

Fig. 15 Experimental response of traverse control of rope-and-mass system. $l=0.9$ m, $l_i=0.9$ m, $x_j=0.2$ m; $K_a =0.25$, (a): $K_v = K_r =1.0$, (b): $K_v = K_r =2.0$; $\theta_i$: $i$-th pendulum angle; Pos: support position.

5. Discussion

By comparing the experimental results to the numerical simulation, we first investigate why the larger feedback constants in the experiment produced a poor position control performance. Figure 16 shows the simulation results of the 3-DOF pendulum system corresponding to the experiment shown in Fig. 11. Because the simulation model has no damping, high-frequency vibrations occur. By neglecting these high-frequency vibrations, the control responses of the angles are similar to the experimental results, but the control responses of the support position are not similar. The simulation results show fairly less support position fluctuation than do the experimental results. However, the simulation results also show the small low-frequency support fluctuation. This might cause the larger support fluctuation in the experiment due to the servo system inaccuracy. The tendency of the support displacement fluctuation

Fig. 16 Numerical control simulation of 3-DOF homogeneous pendulums with experimental apparatus dimensions for initial angle $\theta_i = 0.3$ and feedback constants are, (a): $K_a = 0.25$, $K_r = K_p = 0.5$, (b): $K_a = 0.5$, $K_r = K_p = 0.5$, (c): $K_a = 0.5$, $K_r = K_p = 1.0$. $\theta_i$: $i$-th pendulum angle; Pos: support position.
is similar in the experiment and the simulation; the larger wave control constant brought the larger position displacement, and the larger feedback constants brought the higher frequency fluctuation.

The control responses during winding up and down are investigated by the numerical simulation. Figure 17 shows the simulation responses corresponding to the experiment shown in Fig. 13. The tendency of the simulation response during winding up is similar to that of the experiment; the support displacement fluctuation at the end of winding up can be seen. This phenomenon would be amplified in the experiment by the experimental apparatus inaccuracy, as mentioned above. On the other hand, no clear differences appear in the simulation between winding up and down processes in the control of angles, as shown in Figs 13(a) and 13(b), so these differences in the experiment might be caused by the experimental apparatus inaccuracy, too.

Figure 18 shows the simulation of the traverse and wave control. Figure 18(a) is the case of the same condition shown in Fig. 15(b). The control tendencies of the simulation are similar those in the experiment, and so the control performance of the presented method for the simultaneous control for the wave and traverse were confirmed. Figures 18(b) and 18(c) are the longer traverse case \( x_f = 0.9 \) m for winding up and down, respectively. These results showed that the presented control strategy can perform the traverse and vibration control simultaneously and very simply.

Figure 19 shows the mode localization characteristics of a larger degree-of-freedom system than that shown in Fig. 8, where the lowest pendulum length is 1 and the other pendulums lengths are \( \lambda \). The mode localization frequency is about 1.5 Hz, which corresponds to \( \Omega_{\text{ml}} \) for the case of 5-DOF system. The natural frequency of this mode could not be wave controlled. This means that we could not avoid the uncontrollability of wave propagation by extending the degrees of the freedom of pendulum system. We would like to investigate these phenomena in detail in the near future.

The experimental results showed better control performance than did the results using imaginary system and initialization, even though the experimental apparatus was not identical (Saigo et al., 2003).

6. Conclusion

We presented the wave control strategy for a crane rope-and-mass system by using the wave-controlled imaginary pendulum system as the support acceleration control in combination with the feedback control for the support position. The wave control is simply performed with the newly developed wave solution. The control strategy is able to control for both pendulum angle and position of support during winding up and down at the same time. The experiments and numerical simulations showed the validity of the presented control strategy. The pendulum system should be treated as a nonhomogeneous system when winding up/down the load mass, and then mode localization occurs. The mode localization brings the wave control limitation other than the usual wave propagation limit. At present we could not find a method of wave control for the mode localization system. We will investigate this phenomenon in detail in the near future.

Fig. 17 Numerical control simulation of rope-and-mass system winding up (a) and down (b) with feedback constants \( K_w = 0.25 \), \( K_p = K_{p_1} = 1.0 \), and \( l_f = 0.9 \) m. Rope length varies between 0.9 m and 0.12 m. The time of winding is 5 s. \( \theta_i \): \( i \)th pendulum angle; Pos: support position.
Fig. 18 Numerical simulation of transverse and wave control rope-and-mass using $K_u = 0.25$, $K_v = K_p = 2$ and $l_c = 0.9$ m. (a): constant length $l = 0.9$ m to position $x_l = 0.2$ m; (b): winding up to position $x_l = 0.9$ m; (c): winding down to position $x_l = 0.9$ m. $\theta_i$: $i$-th pendulum angle; Pos: support position; L: rope length. Simulation result (a) corresponds with the experimental result in Fig. 15(b).

Fig. 19 Mode localization of 13-DOF nonhomogeneous simple pendulums of which $k$-th length is $l_k(l_k \neq 1)$ and $l(1 = 1)$. (a): natural frequencies of 13-DOF system; (b1): frequency response of lowest pendulum for $\lambda = 8$; X1 and UX1: controlled and uncontrolled response of the lowest pendulum; (b2): frequency response of uppermost pendulum for $\lambda = 8$; X13 and UX13: controlled and uncontrolled response of the uppermost pendulum. Mode localization occurs for $\lambda \approx 5$ at natural frequency $\approx 1.5$.

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