Multi-objective optimization for vibration suppression of smart laminated composites

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Abstract

The present paper proposes a multi-objective optimization technique for smart laminated composites to maximize two conflicting objectives. The first objective is the performance of active vibration control of smart composite with piezoelectric (PZT) actuators. The second is the fundamental frequency of smart structures related to the performance of passive vibration control. Both performances of active and passive vibration control are maximized simultaneously. The vibration suppression of smart structures strongly depends on both actuator placements and vibration mode shapes. It is possible to design vibration mode shapes for laminated fibrous composites since their anisotropy for whole thickness is tailorable by arranging fiber orientation angle in each layer. This allows the smart structure with laminated composite to archive higher performance of vibration suppression than those with isotropic materials. However, the optimized structure results in lower natural frequencies than composites with typical fiber orientation angles since an effective input of control force from actuators is realized for the structure with lower stiffness. This reveals that there is a trade-off relation for smart composite structures between the performance of active vibration suppression and natural frequencies. To disclose this relation, the present study applies the effective multi-objective optimization technique, the refined non-dominated genetic algorithm (NSGAI), and obtains Pareto optimal solutions. Calculated results are successfully validated by a comparison with those from the real-time control experiment where a laser excitation technique which is effective to small sized structures is used.

Key words : Smart structure, Laminated composite, Multi-objective optimization, Vibration control, Laser excitation

1. Introduction

The smart structure for vibration control has considerable attention recently in many literatures (Gaudenzi et. al., 2000, Mukherjee et. al. 2002 and Hurlbaush et. al., 2007) since it is one of promising solutions to archive high control performance especially for the micro-sized devices, such as hard disk drive (HDD) storages, printers and digital single-lens reflex cameras. An effective technique of vibration suppression is required to improve performance of these applications and many literatures deal with the optimization of the smart structures (Jianfu and Rogers, 1991, Quek et. al., 2003, Rader et. al. 2007 and Kajiwara et. al., 2009).

Among the types of smart structures, the present study focuses on smart composites consisting of laminated fibrous composites, graphite/epoxy (CFRP) materials, and piezoelectric (PZT) actuators. In addition to the high specific stiffness and strength, the laminated composite has tailorable anisotropy by arranging fiber orientation angle in each layer. The performance of vibration suppression of smart structures strongly depends on actuator placements and vibration mode shapes of structures, and the design of mode shapes is possible for the laminated composite by adjusting the anisotropy arbitrarily for whole thickness. There are some studies for the optimization of smart composites (Jha and Inman, 2003, and Kang and Tong, 2008), however they do not include design of vibration mode

shapes. The authors (Honda et. al., 2011 and 2012) proposed a multidisciplinary design optimization approach for smart composites considering mode shapes affected by the anisotropy of laminated composites. The obtained optimal plate actually indicated higher performance of vibration suppression than the plate with typical lay-ups but resulted in lower natural frequencies compared with others. It is known from this that laminated composites with lower stiffness are selected and survived during the optimization process to make transfer of control force from the actuator to the laminated composite more effectively and to archive higher performance of vibration suppression. That is, there is a trade-off relation for smart composites between the performance of active vibration control and natural frequencies. As is well known, the increase of natural frequencies of structures is a simple strategy to avoid resonance and is associated directly with the performance of passive vibration control of structures. Therefore, the decrease of natural frequency in exchange of improvement of the active control performance is unignorable for the design of smart composite. The present study applies a multi-objective optimization technique to the vibration design of smart composite to reveal above trade-off relation.

Under the assumption that the smart composite is used for the hard disc drive (HDD) head, the small trapezoidal smart composite plate composed of CFRP material and PZT actuator is designed optimally. The smart structures are modeled numerically by finite elements (FEs), and vibration properties including natural frequencies and modal vectors are calculated by finite element analysis (FEA). The calculated fundamental frequency is employed as an objective function in the multi-objective optimization problem. Then, the degree-of-freedom (DOF) of FE model is reduced by using the modal analysis technique. The closed-loop system is designed based on the H_∞ control problem, and the H_∞ norm of controlled response is used as another objective function. Design variables are lay-up configurations of laminated composites, weighting parameters in the control system, and placements of PZT actuators. The refined non-dominated sorting genetic algorithm (NSGAI) method (Deb, 2001) is employed as an optimizer due to its high searching ability for multi-objective optimization problem, and Pareto optimal solutions are derived in terms of active and passive control performance of smart composite.

Obtained numerical results are validated experimentally. There is a difficulty in exciting the small sized structures by an impulse hammer or external exciter due to small area to be excited and their high natural frequencies. An innovative vibration testing method using an impulse excitation invoked by a laser ablation (Kajiwara and Hosoya, 2011 and Hosoya et. al. 2012) is used here. The laser ablation excites structures without physical contacts with high reproducibility since the laser beam is irradiated precisely at the same point of structure with the pre-set power. It is a suitable method for evaluating small sized structures. The experimental results show reasonable agreement with the numerical results and the effectiveness and validity of the present study is successfully confirmed.

2. Analysis and optimization methods

2.1 Modeling of smart structure

The present study assumes an application of a smart composite to the HDD head driver. Figure 1 shows (a) commercial HDD head and (b) the present smart composite used in the experiment and (c) FE model for the numerical calculation. CFRP prepregs are used to fabricate laminated composites and they are limited to symmetric 6-layers $[\theta_1/\theta_2/\theta_3]_s$ to confirm the thickness to the commercial HDD head where θ_i ($i = 1, 2, 3$) is the fiber orientation angle in each layer. The plate is clamped at the longer base of trapezoid, simulating the cantilever boundary condition. The dimensions of composite except for the clamped area are indicated in Fig. 1 (c). A small PZT actuator is installed on the surface of composite.

Fig. 1 (a) Commercial HDD head, (b) present smart composite used in the experiment and (c) FE model for the numerical calculation.

Fig. 2 Block diagram of the present control system.

The smart composite is modeled by FEs as shown in Fig. 1 (c) and vibration properties including natural frequencies and modal vectors are derived by general purpose software (ANSYS11.0) with linear layered structural shell element (SHELL99). The numbers of elements and nodes are 128 and 433, respectively. For simplicity, the stiffness and mass of the PZT actuator with dimension $20 \times 1.0 \times 0.50$ mm (length $\times$ width $\times$ thickness) are ignored in the FE model. The actuator is assumed as a segment of line in the design of control system and its end points are used as input points of control forces which react to rotational DOFs of reduced FE model. An aluminum tip is attached to the smart composite to prevent damage from laser ablation as shown in Fig. 1 (b), and its stiffness and mass are also ignored in the numerical calculation.

The numerical model defined by FE has too many DOFs to deal with it in the design of control system and in the optimization problem requiring a vast number of iterative calculations. The modal analysis technique overcomes this and reduces the DOFs as follows.

Defining the state variable $\mathbf{q}$ with the modal coordinate ξ and its first derivatives, the state-equation of the present system is expressed as

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{A}\mathbf{q} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Lambda} & -\mathbf{\Phi}^T\mathbf{C}_d\mathbf{\Phi} \end{bmatrix} \begin{Bmatrix} \xi \\ \dot{\xi} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^T\mathbf{B}_{1s} \end{bmatrix} \mathbf{w} + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^T\mathbf{B}_{2s} \end{bmatrix} \mathbf{u} \end{aligned} \quad (1)$$

where $\mathbf{\Phi}$ is the modal matrix obtained by FEA, and $\mathbf{\Lambda}$ is the diagonal eigenvalue matrix. $\mathbf{w}$ and $\mathbf{u}$ are the disturbance and control input vectors. $\mathbf{B}_{1s}$ and $\mathbf{B}_{2s}$ are the input matrix for the disturbance and actuator, and they define positions and DOFs of nodes corresponding to the disturbance and actuator inputs. $\mathbf{C}_d$ is the damping matrix.

The feedback signal $\mathbf{y}_2$ are velocities measured by the laser Doppler vibrometer (LDV) in this study, and output equations of the system are given by

$$\begin{aligned} \mathbf{y}_2 &= \mathbf{C}_2\mathbf{q} + \mathbf{D}_{21}\mathbf{w} + \mathbf{D}_{22}\mathbf{u} \\ \mathbf{C}_2 &= G_a\mathbf{C}_0 \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi} \end{bmatrix}, \quad \mathbf{D}_{21} = \mathbf{0}, \quad \mathbf{D}_{22} = \mathbf{0} \end{aligned} \quad (2)$$

where $\mathbf{C}_0$ is the output matrix for the measuring point of LDV and it defines the DOF of node to be measured. G_a is a parameter to determine the magnitude of gain for frequency response, and it is adjusted by comparing results between calculation and experiment. $\mathbf{\Phi}_w$ is the weighted modal matrix and defined by

$$\mathbf{\Phi}_w = [w_1\phi_1 \quad w_2\phi_2 \quad \cdots \quad w_r\phi_r] \quad (3)$$

where w_i and ϕ_i ($i = 1, 2, \dots, r$) are modal weights and modal vectors for each mode with the number of employed modes r . This enables us to weight specific modes arbitrary.

2.2 Design of control system

The block diagram of the control system is shown in Fig. 2. $\mathbf{P}(s)$ is the controlled object and $\mathbf{K}(s)$ is the dynamic compensator working as the controller. The controller $\mathbf{K}(s)$ is designed based on the H_∞ control problem (Kajiwara et. al., 2009) defined as

$$\text{Minimize } \|\mathbf{T}_{y1w}\|_\infty \quad (4)$$

where $\mathbf{T}_{y1w}$ is the transfer function matrix between disturbance $\mathbf{w}$ and controlled variable $\mathbf{y}_1$ where $\|\cdot\|_\infty$ represents the H_∞ norm. $\mathbf{y}_1$ is described using weighting matrix $\mathbf{Q}$ and $\mathbf{R}$ for control response $\mathbf{z}_1$ and control input $\mathbf{u}$ as

$$\mathbf{y}_1 = [\mathbf{Q}^{1/2}\mathbf{z}_1 \quad \mathbf{R}^{1/2}\mathbf{u}]^T \quad (5)$$

Fig. 3 Three step design process of the optimization.

2.3 Multi-objective optimization problem

The present optimization problem is defined to minimize the H_∞ norm of the controlled response H_{z1} and to maximize the fundamental frequency Ω_1 as follows.

$$\begin{aligned} &\text{Min. } H_{z1}(\eta_L, \eta_C, \eta_{AC}) \text{ and Max. } \Omega_1(\eta_L) \\ &\text{Subject to } H_u(\eta_L, \eta_C, \eta_{AC}) \leq H_u^{\max} \text{ and } l_{\min} \leq l(\eta_{AC}) \leq l_{\max} \end{aligned} \quad (6)$$

where η_L , η_C , and η_{AC} are design variables representing lay-up configurations $[\theta_1/\theta_2/\theta_3]$ s, weight parameter $\mathbf{Q}$ in the control problem, and placements of PZT actuators including nodes of both end points of actuator, respectively. Constraints are imposed on H_∞ norm of control input H_u and length of actuator $l(\eta_{AC})$.

Laminated composites with different lay-ups give different natural frequencies and modes but performing the FEA repeatedly in the optimization process results in a heavy calculation load. To get around this, the present optimization process is configured by three-step procedure. Each step is detailed as follows and the flow chart of the optimization is shown in Fig. 3. In the first step, natural frequencies and modal matrixes for all possible lay-ups are calculated in advance and a database containing information of lay-up configurations and modal matrixes associating with lay-up indexes η_L is developed. In the second step, the multi-objective optimization summarized in the next paragraph is performed. During optimization process, a design candidate with the lay-up index η_L refers to the database, and the frequency and modal matrix corresponding to η_L is used to calculate the objective function by skipping the FEA. It is necessary to design the output feedback controller in each step but calculating the feedback control law in every step also results in low calculation efficiency. Thus it is assumed that observation of all states of the structures is possible and a state feedback is available. Finally as the last step, the output feedback system is designed based on the linear matrix inequality (LMI) approach after carrying out the multi-objective optimization.

In the second step of the design process, the multi-objective optimization is performed to obtain Pareto optimal solutions and NSGAI (Deb, 2001) is employed in this study. It is known that the method has an excellent search performance for widely distributed Pareto-optimal solutions with conflicting objectives since it is implemented with an effective sorting method based on individual ranking by non-dominated sorting and a crowded distance metric sorting which evaluates the population density of solutions in the same rank. The flow chart of the NSGAI is briefly indicated in Fig. 4 and each step is summarized as follows.

1. Initial parent population P_0 is generated by random numbers, and the non-dominated sorting is implemented on P_0 where each individual is ranked based on the dominance relation in the objective space.
2. Individual within each rank is sorted again based on the crowded distance metric where the population density is evaluated.

3. Individuals selected by a tournament selection are stored in an intermediate mating pool which has a high probability for occurrence of better ranked and less crowded solutions.
4. In the mating pool, genetic operations such as crossover and mutation generate the child populations Q_t where subscript “ t ” denotes the number of generations.
5. An integrated population R_t is created by combining P_t and Q_t , and fitness values are assigned to all individuals by the non-dominated sorting and crowded distance sorting.
6. Finally, individuals with better fitness are selected by elitist sorting and these become the parent individuals P_{t+1} .
7. Steps 2 to 6 are repeated until $t = t_{\max}$.
8. Individuals with rank 1 among parents at $P_{t_{\max}}$ are Pareto-optimal solutions.

A termination criterion is the number of generations here. If the number of generations becomes larger than $t_{\max}$ which is the maximum number of generations defined in advance by the designer, the search process is terminated and the obtained individuals are considered as the Pareto-optimal solutions.

Fig. 4 Flow chart of NSGAII.

Fig. 5 Mechanics of pulse excitation by the laser ablation.

Fig. 6 Experimental setup of the present vibration control system.

Fig. 7 Pareto optimal solutions obtained by the present multi-objective optimization.

3. Experimental method

The mechanism of laser ablation is shown in Fig. 5. When a high power laser is radiated to the metal tip, the plume including atoms, molecules and their ions is released explosively. The released plume with mass Δm at velocity v has a momentum Δmv , and this generates impulse to the irradiated object, resulting in the impulse excitation. The ablation occurs instantly (laser width of 5 [nsec] is used here) and ideal impulse force is generated when it is compared with the method with physical contact.

The experimental system is described in Fig. 6. The pulse laser generated by the YAG laser is irradiated to the small smart composite through the collective lens and the velocity response is measured by the LDV. The use of an impact hammer is a typical way to excite structures in a vibration test but it is quite difficult to excite the micro-sized structure with high reproducibility. As shown in Fig. 6, on the other hand, the present method excites and measures the vibration without physical contact. The laser ablation generates the constant impulse power at the required position iteratively. It is therefore available to evaluate vibration characteristics of the micro-sized structures which have small possible areas for excitation and high natural frequencies.

4. Results and discussions

4.1 Numerical results

Material constants obtained by the preliminary tensile test and used in the numerical calculation are $E_1 = 117$ GPa, $E_2 = 8.5$ GPa, $G_{12} = 4.0$ GPa, $\nu_{12} = 0.34$, and $\rho = 1590$ kg/m³. GA parameters for NSGAI are decided by the numerical experiments as follows. Numbers of population and generation are 1000 and 400, and probabilities of crossover and mutation are 0.8 and 0.2. Constraints imposed on the optimization problem (6) are set to $H_u^{\max} = 2.0 \times 10^{-5}$, $l_{\max} = 20.5$ mm and $l_{\min} = 19.5$ mm.

Table 1 Optimal lay-ups, weighting parameters for control problem η_c , H_∞ norms of the controlled response H_{z1} , fundamental frequencies Ω_1 , and reduction amounts of the first and second peaks in the frequency response function (FRF).

Lay-ups	η_c	H_{z1}	Ω_1 [Hz]	Peak reductions [dB]	
				1st	2nd
[0/0/0] _s	3.3×10^7	2.19×10^{-12}	2758	-6.5	-2.25
[0/0/-15] _s	3.2×10^7	2.11×10^{-12}	2751	-6.93	-2.20
[0/-15/0] _s	2.9×10^7	1.73×10^{-12}	2696	-9.52	5.51
[0/-15/-15] _s	2.9×10^7	1.67×10^{-12}	2685	-9.74	6.07
[-15/0/15] _s	2.6×10^7	1.42×10^{-12}	2518	-12.46	-6.95
[-15/0/0] _s	2.6×10^7	1.34×10^{-12}	2511	-13.04	-7.47
[-15/0/-15] _s	2.6×10^7	1.30×10^{-12}	2490	-13.56	-7.6
[-15/-15/15] _s	2.6×10^7	1.18×10^{-12}	2362	-15.94	-8.95
[-15/-15/0] _s	2.6×10^7	1.12×10^{-12}	2345	-16.26	6.29
[-15/-15/-15] _s	2.6×10^7	1.09×10^{-12}	2316	-16.14	-9.5

Fig. 8 Representative AC placement for the optimal solutions.

Figure 7 shows Pareto optimal solutions obtained by the present multi-objective optimization where the vertical and horizontal lines show the values of fundamental frequency Ω_1 and the H_∞ norm of the controlled response H_{z1} , respectively. Each solution is detailed and arranged in descending order of Ω_1 and H_{z1} in Table 1. Some minor differences are found but all solutions result in similar AC placements as shown by a bold line in Fig. 8.

As listed in Table 1, ten different kinds of Pareto optimal solutions are obtained in the optimization. The solution with the lowest H_{z1} (the highest active control performance) and the lowest Ω_1 is [-15/-15/-15]_s. This shows the same lay-up and AC placement with the solution obtained in the single-objective optimization (Honda et. al., 2012). On the other hand, the solution with [0/0/0]_s lay-up gives the highest Ω_1 in the obtained solutions. This is physically reasonable since the orthotropic plate stiffened in the longitudinal direction has the highest Ω_1 for cantilever plates. Other solutions indicate intermediate performances between two and thus it is known that the present optimization approach gives widely distributed Pareto optimal solutions ranging from the highest performance with active vibration control to the highest fundamental frequency. This also makes it clear that there are trade-off relations between two vibration characteristics considered here and this fact emphasizes the importance of the multi-objective optimization for the vibration design of smart laminated composite.

4.2 Experimental results

Experimental results are given for three smart composites with different lay-ups. The composite with [-15/-15/-15]_s lay-up possesses the highest active control performance in the numerical results, [0/0/0]_s has the highest fundamental frequency, and [0/90/0]_s is the reference model with the typical lay-up referred as cross-ply. Frequency response functions of velocity calculated based on the optimization results are shown in the left side in Fig. 9 with the mobility [(m/sec)/N] in dB scale in the vertical line and frequency [Hz] in the horizontal line. The right side is measured power spectra of velocity in the experiment where the obtained results are not divided by the input force since their direct measurement is not easy for the laser excitation technique. However, it is possible to calibrate excitation forces in advance since irradiated direction, strength, and position of laser are highly reproducible. Details for force measurements are described in (Hosoya et. al. 2012) where excitation forces are calibrated with a rigid block and Newton's second law. For the simplicity, the calibration is not conducted in this study.

There are some differences between numerical and experimental results. The second and higher natural frequencies of the experiment are much lower than the numerical ones. The second modes are slightly suppressed in the numerical results due to active control but no reduction is found in the experiment. These are caused by ignoring the mass and stiffness of the PZT actuators and aluminium tips for laser irradiated points, fabrication errors of hand lay-ups for small sized laminated plates, and imperfection in the realization of the boundary condition for the experiment.

The reduction amount in the dB scale of the first peak is listed in Table 2. It has no mean to compare the magnitude of reduction between the numerical calculation and experiment since each value is calculated from FRF and power spectra. However, they have common points that the smart composite with $[-15/-15/-15]_s$ gives the largest reduction among the three lay-ups for both sets of results and $[0/0/0]_s$ indicates the highest fundamental frequency. The reference model of $[0/90/0]_s$ is superior to other two neither the active control performance nor fundamental frequency. This leads the conclusions that the present multi-objective optimization technique gives valid results and is effective in the vibration design of smart laminated composites.

$[0/0/0]_s$ (the optimal solution with the maximum Ω_1)

$[0/90/0]_s$ (Reference model, cross-ply)

$[-15/-15/-15]_s$ (Optimal solution with the minimum H_{z1})

Calculated FRF

Measured Power spectra

Fig. 9 Calculated FRF and measured power spectra for three smart composites with $[0/0/0]_s$, $[0/90/0]_s$, and $[-15/-15/-15]_s$.

Table 2 Suppression amounts in dB scale and fundamental frequencies in Hz of the first peaks for smart composites with three lay-ups, [0/0/0]_s, [0/90/0]_s, and [-15/-15/-15]_s.

Lay-ups		[0/0/0] _s	[0/90/0] _s	[-15/-15/-15] _s
Experiment	[dB]	-13.68	-12.36	-15.28
	[Hz]	2391	2069	1925
Numerical calculation	[dB]	-9.08	-7.45	-17.15
	[Hz]	2758	2421	2316

5. Conclusions

The present study proposes the multi-objective optimization technique for the smart laminated composites to reveal the trade-off relation between the performance of active vibration control by the PZT actuator and the passive performance involved by the fundamental frequency. The refined non-dominated genetic algorithm (NSGAI) is applied to minimize the H_{∞} norm of controlled response and to maximize fundamental frequency. The design variables are lay-up configurations of laminated composites, weighing parameters in control systems, and actuator placement. Based on the obtained numerical results, test specimens are fabricated and the experiment of real time control is conducted with the laser excitation technique.

The calculated Pareto optimal solutions cover various designs. The solution with the highest active control performance agrees with the result from single-objective optimization, and the orthotropic plate stiffened in the longitudinal direction is found as the solution with the highest fundamental frequency. The experimental results partly give differences from the numerical results but both results agree well in terms of control performance for each lay-up. From these, it is concluded that the present multi-objective optimization technique is effective for the vibration design of smart laminated composites.

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