Experimental analysis of a water-pump driving mechanism using an orthogonal double-slider joint

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Abstract
Slider-crank mechanisms are frequently used to convert between linear and rotational motion. When a slider-crank mechanism creates linear piston motion, a side force occurs between the cylinder sides and the piston head. The side force can be reduced using a Scotch yoke mechanism. However a Scotch yoke mechanism requires two parallel opposed sliders, therefore it is difficult to keep a precision and structure complicated each machine elements. This side force causes various problems, so authors have proposed an orthogonal double-slider joint mechanism to reduce the side force acting on the piston. We build three types of water-pump to investigate efficiency differences among the driving mechanism types, namely, a slider-crank mechanism with a crosshead, a Scotch yoke mechanism, and the orthogonal double-slider joint mechanism. We measure the input torque needed to drive a water-pump under same conditions for stroke, cylinder cross-section, and crank rotational speed. To investigate the influence of sliding frictional resistance acting on the crosshead, we compare results between the cases of driving by the slider-crank mechanism with a crosshead and the orthogonal double-slider joint mechanism. To investigate the influence of structural differences, we compare results between the cases of driving by the Scotch yoke mechanism and the orthogonal double-slider joint mechanism. We find that among the three mechanisms the orthogonal double-slider joint mechanism can drive the water-pump with the least input torque.

Key words: Slider-crank mechanism, Orthogonal double-slider joint mechanism, Scotch yoke mechanism, Water-pump, Dynamics

1. Introduction

Slider-crank mechanisms are used in applications such as engines, compressors, and water-pumps. When a slider-crank mechanism creates linear piston motion, a side force perpendicular to the direction of motion occurs between the cylinder sides and the piston head, causing problems such as vibration and heat generation. The side force increases with discharge pressure, so a crosshead that guides the piston in its linear motion is needed in applications such as diesel engines for large ships. The side force is caused by joint force transmitted via the connecting rod, because the joint force acts in a direction oblique to the cylinder axis. Therefore, several trials have been performed to understand the phenomenon of frictional resistance caused by the side force (Aritomi et al., 2012; Fei et al., 2013; Nakshima et al., 2012; Noda et al., 2013).

Water-pumps are classified into two categories: positive displacement pumps and turbo pumps. Turbo pumps are applied to various devices because they can transport fluid without pressure fluctuations and can be easily miniaturized (Yamoto et al., 2011; Yumoto and Shinshi, 2012; Pai et al., 2008). These pumps are suitable in cases where the discharge quantity is large and the discharge pressure is small. High-precision machine tooling is needed to manufacture the impeller in a turbo pump (Honda et al., 2014; Yamaguchi and Takahashi, 2013). Positive displacement pumps use a slider-crank mechanism. While they generate large pressure fluctuations, the
slider-crank mechanism creates large discharge pressures. These pumps require few components, and their manufacture and assembly are relatively simple. However, pumps using a slider-crank mechanism are unsuitable for transporting large amounts of fluid because the above-described side force precludes high-speed crank motions.

The side force is related to the ratio of the length of the connecting rod and the crank length, so this type of pump is difficult to miniaturize. However, solving the problems related to side force and the connecting rod length would allow use of positive displacement pumps in applications where only turbo pumps have been conventionally used. The side force can be reduced using a Scotch yoke mechanism. However a Scotch yoke mechanism requires two parallel opposed sliders, therefore it is difficult to keep a precision and structure complicated each machine elements.

In a previous study (Yoshizawa et al., 2012), we proposed an orthogonal double-slider joint mechanism to reduce the side force acting on the piston. Previous studies have shown reduced electricity consumption in comparison with the slider-crank mechanism when driving air compressors by the proposed mechanism. We have also attempted to elucidate characteristics of the mechanism by using statics experiments and dynamics analysis, but static mechanical characteristics showed only slight improvements, which were insufficient to demonstrate lowered electricity consumption.

In this study we experimentally investigate advantages of the proposed mechanism from the viewpoint of dynamical characteristics. In the experiments, dynamic load is applied to the mechanism by driving the water-pump to investigate differences in efficiency among three driving mechanisms: a slider-crank mechanism with a crosshead, a Scotch yoke mechanism, and the orthogonal double-slider joint mechanism. We measure the input torque needed to drive the water-pump under same conditions for stroke, cylinder cross-section, and crank rotational speed.

![Fig. 1 Side view of the orthogonal double-slider joint mechanism. The mechanism has sliders moving along the X- and Y-axes. One characteristic of this mechanism is that input rotational motion is converted into two orthogonal linear motions. The X-axis coincides with the direction of the piston motion, and the Y-axis is perpendicular to it in the cross-section of the crankshaft axis.](image)

![Fig. 2 Perspective view of an orthogonal double-slider joint mechanism. The mechanism uses a crank-connecting plate to transmit power from the crankshaft to the piston instead of to the connecting rod. This plate is jointed to an L-shaped double slider, which resolves circular crank motion into orthogonal linear motions along the X- and Y-axes.](image)
Fig. 3  Assembling drawing of an orthogonal double-slider joint mechanism created using 3D CAD. In this figure, two linear rails are used to connect the L-shaped double slider.

Fig. 4  Force transmitting through the connecting rod. (a) The magnitude of input torque needed depends on the magnitude of output force. (b) The joint force acts in the oblique direction of the cylinder axis. The vertical component of the joint force corresponds to the side force. The magnitude of the side force increases with the angle between the connecting rod and the cylinder axis.

Fig. 5  Crosshead added to a slider-crank mechanism. (a) In this schematic drawing, the crosshead is attached to the piston. The crosshead supports the side force instead of the piston. (b) In this assembly drawing, the crossheads are the green parts.
2. Kinematic Configuration

Figure 1 shows a side view of the orthogonal double-slider joint mechanism. The X-axis coincides with the direction of piston motion and the Y-axis is perpendicular to it in the cross-section of the crankshaft axis.

Figure 2 shows a perspective view of the orthogonal double-slider joint mechanism. The mechanism with the orthogonal double-slider joint uses a crank-connecting plate to transmit power from the crankshaft to the piston instead of the connecting rod. This plate is jointed to an L-shaped double slider, which resolves circular crank motion into orthogonal linear motions along the X- and Y-axes.

One characteristic of this mechanism is that input rotational motion is converted into two orthogonal linear motions. The plane inclusive of the centerline of the piston and perpendicular to the axis of the crankshaft is defined as the reference plane. All components of the mechanism are included in this plane. The plane perpendicular to this reference plane is defined as the ground plane. An X-axis slider joint located on the ground plane beyond the crank diameter, under the crankshaft, connects the ground plane and the L-shaped double-slider. A Y-axis slider joint connects the crank-connecting plate and the L-shaped double slider.

The L-shaped double slider is driven along the X-axis in the crank diameter according to the rotational motion of the crank-connecting plate, which is free to rotate on the crankshaft with radius equal to the offset. Simultaneously, this plate is connected with the L-shaped double-slider through the Y-axis slider joint, which constrains the relational rotating motion between the plate and the L-shaped double slider. In this structure, the side force does not act on the piston. The output motion can be gained as a pure cosine function, where rotational motion is given as the mechanism input.

The plane transmits power supplied as input from the crankshaft as output to the piston, which is perpendicular to the ground plane defined as the rectangular transmission plane. The rectangular transmission plane, which is parallel to the section of the piston that includes the Y-axis slider joint perpendicular to the reference plane, transmits power accompanying the motion along the X-axis.

In our mechanism, the ground plane includes the X-axis so that output motion is transmitted along the X-axis (Fig. 1). The output motion can also be transmitted along the Y-axis when the ground plane includes the Y-axis.

![Slotted yoke](image1)
![Slider-bar](image2)

Fig. 6 Assembling drawing of a Scotch yoke mechanism created using 3D CAD. A Scotch Yoke mechanism requires a slotted yoke and two parallel opposed slider-bars. This slotted yoke and two parallel opposed slider-bars are arranged perpendicular to each other. Thus it needed high precisions machining and assembly.

![Schematic drawings of three pump-driving mechanisms. In each drawing, the center of gravity of the link is located by parameters \( l_{gi} \) and \( \phi_i \). (a) Type SC is a slider-crank mechanism with crosshead. In this figure, Link 3 indicates the crosshead. (b) Type SY is a Scotch yoke mechanism. In this figure, Link 3 moves along the cylinder axis. (c) Type XY is the orthogonal double-slider joint mechanism. In this figure, Link 3 indicates the L-shaped double-slider.](image3)
3. Kinematic model

Figure 7 shows kinematic models for the water-pump driving mechanisms. We respectively refer to the slider-crank mechanism with crosshead, the Scotch yoke mechanism, and the orthogonal double-slider joint mechanism as Type SC, Type SY and Type XY. In Figs. 7(a)–(c), Link 1 denotes the crank, and \( \theta_i \), \( a_i \), and \( m_i \) respectively denote the input angular displacement, and the length and the mass of \( i \) \((i = 1–3)\). Also, \( l_{gi} \) and \( \phi_i \) respectively denote the radius and the angle to locate the center of gravity of Link \( i \). In Fig. 6(a), Link 2 and Link 3 denote the connecting rod and the crosshead, respectively. In Fig. 7(b), Link 3 denotes a slider connected to parallel guides on the frame. Link 2 denotes the slider that moves along the line perpendicular to parallel guides on the frame. In Fig. 7(c), Link 3 denotes the orthogonal double-slider in the L-shaped form.

![Fig. 7](image)

Fig. 8 Experimental devices for driving the pump. In these figures, the diameters of the cylinders and lengths of the cranks are the same. (a) The experimental device with the slider-crank mechanism. In this device, the crosshead is attached to the piston. (b) The experimental device with the Scotch yoke mechanism. Link 2 is realized as slider 1 in the slot. (c) The experimental device with the orthogonal double-slider joint mechanism. Linear slider guides connect the L-shaped slider.
4. Experimental equipment

Figure 8 shows the experimental devices used to measure the input torque required to drive the water-pump using the slider-crank, Scotch yoke, and orthogonal double-slider mechanisms. This experiment is conducted to among three mechanisms compare the input torques. Table 1 shows the kinematic constants used when designing the devices. In Fig. 9, the piston intakes and disgorges water from the left wall, which has non-return valves shown in Fig. 10. The motor is connected to the crankshaft through a torque transducer (KYOWA TP-500GMCB), allowing input torque to be measured according to changes in rotational speed.

Although the speed of the piston head of the slider-crank mechanism differs from the others, the rotational speed of the motor is same in this experiment, each of water flow rates is same value, we assume that the load acting on the piston head of the slider-crank mechanism is almost same as the load of the others.
Table 1. Kinematic constants

<table>
<thead>
<tr>
<th>Type</th>
<th>( m_i ) [kg]</th>
<th>( a_i ) [m]</th>
<th>( l_{Gi} ) [m]</th>
<th>( \phi_i ) [rad]</th>
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</thead>
<tbody>
<tr>
<td>Link 1</td>
<td>0.0261</td>
<td>0.0220</td>
<td>0.0110</td>
<td>0.0000</td>
</tr>
<tr>
<td>Link 2</td>
<td>0.00624</td>
<td>0.0640</td>
<td>0.0304</td>
<td>0.0000</td>
</tr>
<tr>
<td>Link 3</td>
<td>0.0293</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Type SY</td>
<td>( m_i ) [kg]</td>
<td>( a_i ) [m]</td>
<td>( l_{Gi} ) [m]</td>
<td>( \phi_i ) [rad]</td>
</tr>
<tr>
<td>Link 1</td>
<td>0.0261</td>
<td>0.0220</td>
<td>0.0110</td>
<td>0.0000</td>
</tr>
<tr>
<td>Link 2</td>
<td>0.0628</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Link 3</td>
<td>0.0202</td>
<td>0.610</td>
<td>0.0652</td>
<td>1.209</td>
</tr>
<tr>
<td>Type XY</td>
<td>( m_i ) [kg]</td>
<td>( a_i ) [m]</td>
<td>( l_{Gi} ) [m]</td>
<td>( \phi_i ) [rad]</td>
</tr>
<tr>
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<td>0.0220</td>
<td>0.0110</td>
<td>0.0000</td>
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<tr>
<td>Link 2</td>
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<td>0.0140</td>
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<tr>
<td>Link 3</td>
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<td>0.0458</td>
<td>0.0352</td>
<td>0.875</td>
</tr>
</tbody>
</table>

5. Experimental results

Figure 11 shows the results of the experiment, plotting the maximum value of the input torque in ordinate and the rotational speed of the crank in abscissa. Values in this figure are average maximum values over 300 cycles. And as shown in Fig.12, input torques increase linearly as the rotational speed increases in three cases. Rates of increase in this experiment are almost same.

![Fig. 11](image-url)

**Fig. 11** Maximum input torque needed to drive the water-pump according to the rotational speed of the crank. Blue bars denote the case of the slider-crank mechanism, red bars the Scotch yoke mechanism, and green bars the orthogonal double-slider joint mechanism. Except when the rotational speed of the crank is equal to 150 [r.p.m], the input torque using the orthogonal double-slider joint mechanism is least among the mechanism types.

![Fig. 12](image-url)

**Fig. 12** Slope of the input torque against rotational speed of the crank. The blue line denotes the case of the slider-crank mechanism, the red line the Scotch yoke mechanism, and the green line the orthogonal double-slider joint mechanism.
6. Dynamics analysis

Figures 13(a)-(c) respectively show free-body diagrams of the slider-crank mechanism and the orthogonal double-slider joint mechanism. In these figures, $F_{ij}$ denotes the force transmitted from Link $i$ to Link $j$ through the joint. Figures 7 (a)-(c) show the position of the center of gravity for each moving link. In the figures, $M_1$ denotes the input torque about the crankshaft. The center of gravity of Link $i$ is indicated by the radius $l_{G_i}$ and the angle $\phi_i$. $m_i$ denotes the mass of Link $i$, $G_i$ denote the center of gravity of Link $i$. $\mathbf{r}_{G_i}$ is the position vector drawn from O to $G_i$. $l_{G_i}$ denotes the moment of inertia around $G_i$. $F_L$ denotes the load acting on the piston, whose direction coincides with the piston motion.

In the case of Type SC, the slider-crank mechanism, the point of application of load $F_L$ is B. In case of Type SY and Type XY, D is the point of application of load $F_L$. The equations of motion for Link $i$ ($i = 1–3$) in the mechanisms were previously derived (Yoshizawa, et al., 2012). By solving these equations simultaneously, the input torque $M_1$ and the reacting force $F_{34y}$ from the cylinder inner wall (the stationary link; the slider guide along the direction of piston motion) are obtained as follows.

In the case of Type SC (the slider-crank mechanism), the input torque $M_1$ to drive the water-pump and the side force $F_{34y}$ when the load $F_L$ acts on the piston are respectively expressed as

$$M_1 = F_L a_1 (\cos \theta \tan \phi + \sin \theta) - \frac{m_2 g a_1 \cos \theta}{a_2 \cos \phi} (l_{G_2} \cos (\phi_2 - \phi) - a_2 \cos \phi)$$
$$+ \frac{d\phi}{d\theta} \left\{ m_2 a_1 \left\{ l_{G_2} \cos \theta \sin \phi_2 \cos \phi + l_{G_2} \sin (\phi_2 - \phi - \theta) - a_2 \sin (\theta + \phi) \right\} \right.$$  
$$+ \frac{d^2\phi}{d\theta^2} \left\{ a_1 \cos \theta (l_{G_2} - m_2 l_{G_2} \cos \phi) + m_2 a_1 l_{G_2} \sin (\phi_2 - \phi) \cos \theta \tan \phi + \sin \theta \right\}$$
$$+ m_2 a_1 a_2 \cos (\theta + \phi) \right\}$$
$$+ \frac{d^2}{d\theta^2} \left\{ \frac{m_2 l_{G_2} a_1 \cos \theta}{a_2 \cos \phi} \sin (\phi_2 - \phi) - a_1 m_2 \sin \theta - a_1 m_3 (\cos \theta \tan \phi + \sin \theta) \right\}$$
$$+ m_1 g l_{G_1} \cos (\theta + \phi_1). \quad (1)$$
\[ F_{34y} = F_L \tan \phi - \frac{m_2gl_2 \cos(\phi_2 - \phi)}{a_2 \cos \phi} - m_3g + \left( \frac{d\phi}{d\theta} \right)^2 m_2l_2 \frac{\sin \phi_2}{\cos \phi} + \frac{d^2\phi}{d\theta^2} \left[ \frac{1}{a_2 \cos \phi} (l_2g - m_2l_2^2) + m_2l_2 \{ \sin(\phi_2 - \phi) \tan \phi - \cos(\phi_2 - \phi) \} \right] + \frac{d^2s}{d\theta^2} \left[ -m_3 \tan \phi \right] + \frac{m_2l_2 \sin(\phi_2 - \phi)}{a_2 \cos \phi}. \]  

(2)

Similarly, in the case of Type SY (the Scotch yoke mechanism) and Type XY (the orthogonal double-slider joint mechanism) the input torque \( M_1 \) to drive the water-pump and the side force \( F_{34y} \) when the load \( F_L \) acts on the piston are respectively expressed as

\[ F_{34y} = -m_3g \]  

(3)

\[ M_1 = a_1 \{ m_3a_1 \cos \theta \sin \theta + F_L \sin \theta + m_2 g \cos \theta \} + m_1g l_{g1} \cos(\theta + \phi_1). \]  

(4)

Here, the components of the force and torque are expressed as

\[ M_1 = M_1 \mathbf{k} \quad F_{34} = F_{34x} \mathbf{i} + F_{34y} \mathbf{j}, \]

where \( \mathbf{k} \) denotes the direction and orientation of the Z-axis. The moments acting on the slider are expressed as follows:

\[ M_2 = l_{g2}m_2a_1 \left( \frac{d\theta}{dt} \right)^2 \sin(\theta - \phi_2) - l_{g2}m_2g \cos \phi_2 \]  

(5)

\[ M_3 = l_{g2}m_2 \left\{ a_1 \left( \frac{d\theta}{dt} \right)^2 \sin(\theta - \phi_2) - g \cos \phi_2 \right\} - l_{g3}m_3 \left\{ a_1 \cos \theta \sin \phi_3 \left( \frac{d\theta}{dt} \right)^2 + g \cos \phi_3 \right\} \]

\[ + (a_3 + a_1 \sin \theta)m_3a_1 \cos \theta \left( \frac{d\theta}{dt} \right)^2 + a_1 \sin \theta F_L \]  

(6)

As shown in Fig.14 to calculate the load acting through the piston, the pressure at the piston is calculated using Bernoulli’s principle and equation of continuity, as described in the following. The equation of continuity is expressed as

\[ A_1v_1 = A_2v_2 \]  

(7)

Bernoulli’s principle is expressed as

\[ P_1 = \frac{\rho}{2} (v_2^2 - v_1^2) + \rho g \Delta h + P_2 \]  

(8)

The load on piston \( F_L \) is calculated as

\[ F_L = P_1A_1 \]  

(9)

Variables used in the above equations are listed below:
$A_1$: Cross-sectional area of cylinder [m$^2$]  
$A_2$: Cross-sectional area of pipe [m$^2$]

$v_1$: Flow velocity in the cylinder [m/s]  
$v_2$: Flow velocity in the pipe [m/s]

$P_1$: Pressure in the cylinder [Pa]  
$P_2$: Pressure in the output nozzle [Pa]

$\rho$: Density of water (=1000) [kg/m$^3$]  
$g$: Acceleration of gravity (= 9.81) [m/s$^2$]

$\Delta h = \frac{v_2^2}{2g}$: Head loss

$\xi$: Loss coefficient of the abrupt contraction pipe (=0.824)

\[ \text{The flow velocity in the cylinder is calculated from the piston velocity equation. In other words, the flow velocity in the cylinder is taken to be equal to the piston head velocity. In the case of the slider-crank mechanism, the piston head velocity is expressed as follows:} \]

\[ v_1 = a_1 \omega \sin \theta + \frac{a_1^2 \omega \cos \theta \sin \theta}{a_2 \sqrt{1 - \frac{a_1^2 \sin \theta^2}{a_2}}} \]  \hspace{1cm} (10)

\[ \text{In the case of the Scotch yoke mechanism and the orthogonal double-slider joint mechanism, the piston head velocity is expressed as follows:} \]

\[ v_1 = a_1 \omega \sin \theta \]  \hspace{1cm} (11)

\[ \text{Figure 15 shows the piston velocities calculated by Eqs. (10) and (11). In this figure, the piston velocity } v_1 \text{ is the same for the Scotch yoke mechanism and the orthogonal double-slider mechanism. The value for the slider-crank mechanism is larger.} \]

\[ \text{Fig. 14 Schematic diagram for the pressure calculation. The flow velocity in the cylinder is assumed to be equal to the velocity of the piston. The value of } \xi \text{ is calculated from the cross sectional area of contraction.} \]

\[ \text{Fig. 15 Functional relationship between the velocity of the piston and the crank angle. The velocities of the Scotch yoke mechanism and the orthogonal double-slider joint mechanism are the same. Piston velocities for cases where the rotational speed of the crank is equal to 150 [r.p.m] and 300 [r.p.m] are shown.} \]
Fig. 16 Calculated input torque values. Blue bars denote the case of the slider-crank mechanism, red bars the Scotch yoke mechanism, and green bars the orthogonal double-slider mechanism. The input torques of the Scotch yoke mechanism and the orthogonal double-slider mechanism are nearly the same.

Figure 16 shows the results of calculation of input torque according to the rotational speed of the crank. In this graph, the values of input torques for the Scotch yoke mechanism and the orthogonal double-slider mechanism are nearly the same.

7. Discussion

As shown in Figs. 11 and 16, the experimentally and analytically determined input torque is larger for the slider-crank mechanism with crosshead than for the other mechanisms. Comparing the experimental results shown in Fig. 11 with the numerical results shown in Fig. 16 qualitatively demonstrates that the calculated value is larger than experimental value. This suggests that Bernoulli’s principle should be used for laminar flow, but under experimental conditions the water behavior exhibits turbulent flow condition, possibly causing the differences between the results.

8. Conclusion

In the experimental results, the proposed mechanism was advantageous over the slider-crank mechanism with a crosshead in that input torque decreased by 15%. The Scotch yoke mechanism has a similar advantage in that input torque decreased by 10%. Therefore, the orthogonal double-slider joint mechanism is able to be driven by a smaller torque compared with other mechanisms. The experimental results showed that the proposed mechanism has advantageous over these mechanisms in that input torque.

However, in the analytical results, while the input torque of the proposed mechanism was smaller than that of the slider-crank mechanism with crosshead, it was nearly the same as that of the Scotch yoke mechanism. A detailed investigation of the cause of these differing results between experiment and analysis is an important topic for future research.

We used the same equations for dynamics analysis of the slider-crank mechanism and the orthogonal double-slider mechanism. From the viewpoint of output motion, it can be assumed that the Scotch yoke mechanism and the orthogonal double-slider joint mechanism have the same kinematic characteristics. Despite that assumption, in the experimental results the input torque of the orthogonal double-slider joint mechanism was smaller than that of the Scotch yoke mechanism. This was likely caused by structural differences between those mechanisms.

Generally, in the Scotch yoke mechanism the prismatic pair is realized by inserting the slider into the slot, while in the orthogonal double-slider joint mechanism the prismatic pair is realized using liner guides. Another future task is investigating the effects of these structural differences on the dynamical characteristics. Note that the Scotch yoke mechanism requires high-precision assembly, and this may be one reason why analytical values for input torque were not
attained in the experiment. For future research, we will investigated precision assembly needed to arrange the prismatic pair.

References