Simplified deformation model and shape generation of a rhythmic gymnastics ribbon using a high-speed multi-jointed manipulator

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Abstract
In this paper, we propose an entirely new manipulation strategy for dynamic manipulation of a rhythmic gymnastics ribbon, as one example of belt-like flexible objects, with a high-speed multi-jointed manipulator. The manipulation strategy involves manipulating the target object (rhythmic gymnastics ribbon) at a constant, high speed. Then, we can assume that the dynamic behavior of the ribbon can be obtained by performing algebraic calculations of the robot motion using the proposed strategy. Based on this assumption, we derive a simplified deformation model of the ribbon and suggest a simple motion planning method using the proposed model. Finally, we show simulation results and experimental results of shape generations (for example, circle, wave, figure-eight, and crack shapes) of a ribbon based on the proposed method, and we discuss quantitative evaluation of shape generations by image processing.

Key words : Dynamic manipulation, Belt-like flexible object, Constant and high-speed motion, Simplified deformation model, Trajectory planning, Shape generation

1. Introduction

Object manipulation is an extremely important problem to be solved in the robotics field. Recently, the manipulation of flexible objects has attracted attention in order to improve robot manipulation techniques and extend the range of target objects that can be manipulated (Hirai, 1998). Flexible object manipulation, such as knotting with visual feedback reported by Inoue and Inaba (Inoue and Inaba, 1984), knotting using knot theory reported by Matsuno et al. (Matsuno et al., 2001) and knotting with dual-arm robot reported by Katano et al. (Katano et al., 2015), so far has been executed by static or quasi-static manipulation. In addition to knotting manipulations, a task that aligns focusing points to targets position is performed using simple Proportional-Integral-Derivative (PID) controller (Wada et al., 2001). Hashimoto and Ishikawa performed a shape control of a string using a model (Hashimoto and Ishikawa, 2002). Motion planning for performing a deformable linear object manipulation has been achieved (Saha and Isto, 2007). Recently a method to manipulate deformable objects does not require modeling and simulating object deformation has been suggested (Berenson, 2013). Moreover, the efficiency of the manipulation could be significantly improved using the flexible characteristics. Various manipulators have been also developed. For example, Ryan and Michael have constructed a continuum manipulator and analyzed several aspect of the manipulator (Ryan and Michael, 2014), and Hatakeyama and Mochiyama have simulated motion of a developed robot and realized a shooting manipulation inspired by chameleon (Hatakeyama and Mochiyama, 2013).

However dynamic manipulation of flexible objects has not yet been performed actively. If dynamic manipulation of flexible objects could be achieved, high-speed manipulation of flexible objects could also be realized. Thus, in this study, we aimed to achieve the dynamic manipulation of a flexible object. Here, we discuss a theoretical analysis, simplification
of the deformation model, and shape generation of a rhythmic gymnastics ribbon including quantitative evaluation of experimental result by image processing, as shown in Fig. 1 which illustrates overview of this paper. In particular, we expanded the target objects from linear flexible object (ropes) to belt-like flexible object (rhythmic gymnastics ribbons) based on our previous works (Yamakawa et al., 2012b, Yamakawa et al., 2013a). We expect that the proposed method can also be applied to the dynamic manipulation of cloth and paper (Yamakawa et al., 2011, Yamakawa et al., 2012a).

The essential difficulties faced in dynamic manipulation of a flexible object include:

- deformation of the object during manipulation, and
- estimation of the object’s deformation.

In addition, these difficulties involve the following associated issues:

- modeling the object’s deformation, and
- controlling the object’s deformation.

By proposing a method that overcomes these problems, we expect that research on this kind of manipulation will be stimulated. The following solutions can be considered:

- simplifying the deformation model of flexible objects, and
- controlling the shape of flexible objects based on a simplified model.

Therefore, we propose a simple manipulation strategy using a manipulator moving at a constant, high speed to achieve the above methods. The goal of this study is to show that the model and the control method of a rhythmic gymnastics ribbon, which is one example of belt-like flexible objects, can be simplified, enabling dynamic manipulation (Web page: Dynamic Manipulation of a Rhythmic Gymnastics Ribbon with a High-speed Robot Arm, Yamakawa et al., 2013b).

In related works about belt-like object model and manipulation, Wakamatsu et al. have proposed a static model of a belt-like object with angles (Wakamatsu et al., 2009), and Asano et al. have performed deformation path planning for a belt-like object (Asano et al., 2010). However, these methods cannot be applied to dynamic, high-speed manipulation of belt-like flexible objects because they focus on static models. In addition, these studies dealt with a flat cable serving as the belt-like object. Since the flexibility of the target object (rhythmic gymnastics ribbon) in the present study is much higher, it is difficult to achieve dynamic, high-speed manipulation. Therefore, we developed a simple model to achieve dynamic, high-speed manipulation of the ribbon.

In dynamic manipulation of flexible object, dynamic model of target object is extremely important in order to implement motion planning and sensory feedback of manipulator. However, a modeling and a simulation with the model are very difficult. And several researches have focused on how to model, to simulate and to manipulate these kinds of objects (Khalil and Payeur, 2010). On-line modeling of flexible objects have been studied and acquisition of model parameters of these objects has been discussed (Desbrun et al., 1999). The model of flexible objects is assumed to be mass-spring model (Lang et al., 2002). And the model is analyzed using Finite Element Method (FEM) (Kaufmann et al., 2008). In addition, simulation method using mesh-less model has been proposed (Faure et al., 2011). As discussed above, although some models about flexible object have been proposed, it is difficult to apply to motion planning of manipulator due to model complexity, and it is also difficult to construct sensory feedback control structure. Thus simple model and simple motion planning method for dexterously manipulating flexible object are highly desirable. In this research, we propose a new simple method for achieving these. And we do not use any compensation method and sensory feedback method for dynamic manipulation of flexible object to evaluate only the proposed simple method in this paper.

The rest of this paper is as follows; Section 2 explains an experimental system composed of a high-speed multi-jointed manipulator, a rhythmic gymnastics ribbon to be manipulated, and a real-time controller. Section 3 describes a basic concept of the ribbon manipulation, a model analysis of the ribbon, proposal of a simplified deformation model...
of the ribbon, and trajectory planning for achieving the ribbon manipulation. Section 4 illustrates simulation results and experimental results using the proposed method. Section 5 summarizes the conclusion obtained in this study and future works to be discussed.

2. Experimental System

2.1. High-speed multi-jointed manipulator (Web page: Barrett Technology Inc.)

This robot arm produced by Barrett Technology Inc. is a wire-drive manipulator. The kinematics and a photograph of the manipulator are shown in Fig. 2(a) and (b), respectively. The manipulator has four degrees-of-freedom (4-DOF) consisting of revolving and bending motions and is capable of high-speed movement with a maximum end-effector velocity of 6 m/s and a maximum acceleration of 58 m/s² (= 6G, G is the gravity acceleration). The joint angles of the manipulator can be controlled by Proportional–Derivative (PD) action within 1 ms. The detail specification is shown in Table 1. In this table, the joint 1 is the rotation axis of the base, the joint 2 is the rotation axis of the link 1 which is the same as the shoulder of the human, the joint 3 is the pronosupination axis of the link 1, and the joint 4 is the rotation axis of the link 2 which is the same as the elbow of the human. As can be seen in Table 1, the manipulator we used in this study possesses a high-speed performance.

In this paper the joint angles and the tip position are described by \( q \in \mathbb{R}^4 \) and \( r \in \mathbb{R}^3 \), respectively. The tip position can be calculated by the forward kinematics with the Denavit–Hatenberg description as follows:

\[
r(t) = f(q(t)),
\]

where \( t \) is time.

2.2. Rhythmic gymnastics ribbon

A rhythmic gymnastics ribbon is attached to the end of the manipulator. The rhythmic gymnastics ribbon made by SASAKI SPORTS inc. is used. The length, width, and thickness of the ribbon are 2,000 mm, 50 mm, and about 1 mm, respectively. The mass of the ribbon is about 10 g. Thus this ribbon is very light weight.

2.3. Overall system

The overall experimental system is shown in Fig. 3. The system consists of the high-speed multi-jointed manipulator, the ribbon, attached into the manipulator, to be manipulated, and a real-time controller. The real-time controller performs a trajectory generation as described in Section 3.4, joint angles control of the manipulator, and input-output control. The actual experimental system is illustrated in Fig. 4.

<table>
<thead>
<tr>
<th>Specification of high-speed multi-jointed manipulator.</th>
<th>joint 1</th>
<th>joint 2</th>
<th>joint 3</th>
<th>joint 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduction ratio [-]</td>
<td>35.87</td>
<td>28.21</td>
<td>16.79</td>
<td>17.77</td>
</tr>
<tr>
<td>rated velocity [rad/s]</td>
<td>8.75</td>
<td>11.14</td>
<td>18.71</td>
<td>17.68</td>
</tr>
<tr>
<td>rated torque [Nm]</td>
<td>40.89</td>
<td>64.3</td>
<td>38.28</td>
<td>20.26</td>
</tr>
<tr>
<td>range of movement [deg]</td>
<td>200</td>
<td>240</td>
<td>300</td>
<td>240</td>
</tr>
</tbody>
</table>
3. Concept and Strategy in Dexterous Manipulation of Rhythmic Gymnastics Ribbon

In this section, we explain the basic concept of the dynamic manipulation of a ribbon and present a method for implementing it. First, we illustrate the basic concept in Section 3.1. Second, we analyze the ribbon mechanics and derive conditions for the robot motion in order to simplify the ribbon dynamics in Section 3.2. Third, we propose a simple deformation model of the ribbon based on the analysis results in Section 3.3. Finally, we suggest a motion planning method that derives the joint angles of the robot from the ribbon configuration by using this model in Section 3.4. The validities of assumptions for model derivation, the simplified deformation model and the motion planning will be verified in the next section.

3.1. Basic concept

Figure 5 shows the basic concept of dynamic ribbon manipulation. When the robot moves at low speed, the ribbon deformation cannot be easily estimated because of the effect of gravity and its interaction with the ribbon deformation, as shown in Fig. 5(a). Also, the ribbon deformation model is complex. On the other hand, when the robot moves at high-speed, the ribbon deforms so as to track the robot trajectory, as shown in Fig. 5(b). As a result, the ribbon deformation model can be simplified.

From the above discussion, the ribbon deformation depends on the motion of the high-speed robot. In addition, we can assume that the ribbon deformation can be derived algebraically from the robot motion. Thus, the ribbon deformation model can be described as a relational expression derived from the robot motion. Also, this makes the model of the ribbon deformation more simple than typical models that use differential equations or partial differential equations (Suzuki et al., 2005, Mochiyama and Suzuki, 2002).

Moreover, if the dynamic manipulation is performed in slow motion, gravity has a non-negligible effect on the ribbon. In that case, the algebraic equation does not hold, and we have to consider a differential equation, making dynamic manipulation in slow motion extremely difficult as a result. Thus, high-speed robot motion is required in order to successfully achieve dynamic manipulation.
3.2. Analysis of manipulation strategy

Here, we derive conditions for the robot motion in order to simplify the deformation model of a rhythmic gymnastics ribbon which is one of the belt-like flexible objects. First, we analyze the ribbon mechanics so as to restrict the ribbon deformation in 2D (two-dimensional) plane. Following this, the conditions for the robot motion can be obtained based on the analysis result.

3.2.1. Dynamic model of rhythmic gymnastics ribbon

In the previous papers (Yamakawa et al., 2012b, Yamakawa et al., 2013a), we considered the rope mechanics in order to obtain the robot motion conditions allowing simplification of the rope model, and we ignored disturbance forces such as gravity and air drag. However, we should consider the effect of such disturbance forces on the ribbon manipulation, because the ribbon behavior is strongly affected by such forces. Therefore, here we discuss the ribbon mechanics including the effects of gravity and air drag. In this analysis, we do not take into account the twist and the flutter of the ribbon.

Consider the equation of motion of the ribbon when the ribbon deformation is restricted to a given curve in 2D plane and a constraint force acts on the ribbon, as shown in Fig. 6 (Goto et al., 1971). The disturbance force is \( F \) [N/m], the constraint force which is acted on the ribbon to restrict the ribbon deformation on the curve is \( R \) [N/m], the length taken along the restricted curve is \( w \) [m] (scalar variable) which represents the ribbon position, the length taken along the ribbon deformation is \( \sigma \) [m] (scalar variable) which represents the position of a certain point on the inside of the ribbon, the density of the ribbon is \( \mu \) [kg/m], the tension is \( T \) [N], and the curvature is \( \rho \) [m].

Considering a length \( \delta \sigma \) [m] of the curve, the difference between the tensions in the tangential direction is \( \delta T \) [N].

Since the angle of the part \( \delta \sigma \) from the center of curvature is assumed to be \( \delta \theta \) [rad], the equation \( \delta \sigma = \rho \delta \theta \) holds. Thus, the tension in the normal direction at both ends of this part can be written as:

\[
2T \sin \frac{\delta \theta}{2} = T \delta \theta = \frac{T \delta \sigma}{\rho}.
\] (2)

The tangential and normal components of the disturbance force \( F \) are \( F_t \) and \( F_n \), respectively. Also, since the force \( R \) is the constraint force, the direction of this force is perpendicular to the curve. Then, the equations of motion in the tangential and normal directions of the part \( \delta \sigma \) become (Goto et al., 1971)

\[
(\mu \delta \sigma) \ddot{w} = \delta T + F_t \delta \sigma,
\] (3)

\[
(\mu \delta \sigma) \dot{w}^2 = \frac{T \delta \sigma}{\rho} + F_n \delta \sigma + R \delta \sigma.
\] (4)

Equation (3) leads to:

\[
\mu \ddot{w} = \frac{dT}{d\sigma} + F_t.
\] (5)

Furthermore, the equation below can be obtained from Eq. (4):

\[
\mu \dot{w}^2 = \frac{T}{\rho} + F_n + R.
\] (6)

In addition, as disturbance forces, we consider the effects of gravity \( g \) [m/s^2] and air drag \( c \dot{w}^2 \) [N/m]. The tangential and normal components of gravity and air drag can be calculated as follows:

\[
g_t = \mu g \sin \phi, \quad g_n = -\mu g \cos \phi,
\] (7)

\[
D_t = c_t \dot{w}^2, \quad D_n = c_n \dot{w}^2.
\] (8)
where $c_t$ and $c_n$ [kg/m²] are the tangential and normal coefficients of the air drag, and $\phi$ [rad] is the angle of the robot velocity vector and can be calculated from the robot trajectory. Therefore, the disturbance forces in the tangential and normal direction are given by

$$ F_t = g_t + D_t, \quad F_n = g_n + D_n. $$  

As a result, the equations of motion for the ribbon can be replaced by the following equations:

$$ \mu \ddot{w} = \frac{dT}{d\sigma} + g_t + D_t, $$  

$$ \mu \frac{\dot{w}^2}{\rho} = \frac{T}{\rho} + g_n + D_n + R. $$

Eqns. (10) and (11) represent the ribbon dynamics under the condition that the ribbon behavior is restricted to the given curve.

Next, we derive conditions for the robot motion in order to simplify the ribbon dynamics.

### 3.2.2. Conditions of high-speed manipulator motion for manipulation of rhythmic gymnastics ribbon

In this analysis, the retractility of the ribbon is ignored. Since the constraint force $R$ is considered to be zero in the real world, $R = 0$ is required. The condition that $R = 0$ be satisfied can be obtained from Eq. (11):

$$ \mu \dot{w}^2 = \frac{T}{\rho} - \mu g \cos \phi + c_n \dot{w}^2. $$

Additionally, we introduce an assumption that the condition about the tension ($T = 0$) is held. In this assumption, we consider the tension at the middle part and the free end of the ribbon. Also we ignore the tension at the beginning end of the ribbon (connecting to the tip of the manipulator) for the simplicity of the analysis, because the tension at the beginning end is not equal to zero. Thus, we can suppose that the tension at the part described above may be equal to zero. This assumption leads to

$$ (pc_n - \mu) \dot{w}^2 = \rho \mu g \cos \phi. $$

As a result, the velocity of the ribbon can be given by the following equation:

$$ \dot{w}^2 = \frac{\rho \mu g \cos \phi}{pc_n - \mu}. $$

This means that the ribbon behavior in the normal direction does not depend on the motion of the robot with the velocity given by Eq. (14) and the ribbon behavior can be restricted to the given curve (namely, the robot trajectory).

Next, we consider the condition in the tangential direction. Substituting Eq. (14) into Eq. (10) yields:

$$ \mu \ddot{w} = \mu g \sin \phi - c_t \frac{\rho \mu g \cos \phi}{pc_n - \mu}. $$

Since the ribbon is very light weight, the density of the ribbon $\mu$ is also very small. So $pc_n \gg \mu$ holds. Therefore, we have

$$ \mu \ddot{w} = \mu g \sin \phi - \mu g \frac{c_t}{c_n} \cos \phi. $$

The relationship between the air drag coefficients is $c_t/c_n = \tan \phi$. Substituting this equation into Eq. (16) yields the following equation:

$$ \mu \ddot{w} = \mu g \sin \phi - \mu g \tan \phi \cos \phi = 0. $$

As a result,

$$ \dot{w} = \text{const.} $$

holds.

There is an inconsistency between the conditions in Eqns. (14) and (18). Thus, the condition Eq. (14) is relaxed as follows:

$$ \dot{w}^2 = \frac{\rho \mu g}{pc_n - \mu}. $$
In this case, we assume that the maximum velocity is given by Eq. (14). Although the velocity of the robot motion is approximated by Eq. (19), we show that this approximation is sufficiently appropriate in the experimental results described in Section 4.

This result means that, when the ribbon moves along the ribbon reference configuration, the velocity in the tangential direction of the ribbon is constant, and a uniform force is applied to each point on the ribbon. On the contrary, if the condition that the velocity and tension of the ribbon be constant is satisfied, the ribbon can move along the reference configuration of the ribbon. Manipulating the ribbon at constant velocity can be achieved by moving the manipulator in the tangential direction at a constant velocity. It is impossible to control the tension to be constant in the case of the free end of the ribbon. However, assuming that the ribbon is sufficiently long and that the ribbon tracks the reference configuration, the condition that the tension be constant approximately holds.

As a result, the robot motion conditions necessary to simplify the ribbon model are as follows:

1. Constant-velocity motion — the ribbon deformation can be restricted to the reference trajectory of the robot arm.
2. High-speed motion — the effects of gravity can be reduced.

Thus, the ribbon can deform so as to track the robot motion by manipulating the ribbon with this strategy. Since the robot is moved at a constant speed, each point on the ribbon tracks the robot motion with a constant time delay. This time delay depends on the location of the point.

When we can neglect the gravity effect and the air drag in the ribbon mechanics, we can apply the method proposed in the previous papers (Yamakawa et al., 2012b, Yamakawa et al., 2013a).

3.3. Simplification of deformation model of rhythmic gymnastics ribbon

A flexible object is frequently modeled by a distributed parameter system (partial differential equation). As another model, a flexible object is approximated by a multi-link system, and the equation of motion (ordinary differential equation) is derived for each joint of the multi-link system. It is difficult to utilize these model to the robot trajectory generation and the robot control for the manipulation.

In this research, we apply the multi-link system to the ribbon model. Then, the equation of motion can be replaced by an algebraic equation under the constraint of constant, high-speed motion of the robot.

3.3.1. Discrete deformation model of rhythmic gymnastics ribbon

In a continuous deformation model of the ribbon, the coordinate at the point located within a distance \( \sigma \) from the tip position of the ribbon can be given by,

\[
b(w, \sigma) = r(w) + \int_0^\sigma e(w + \sigma')d\sigma'
\]  

where \( r(w) \) is a reference trajectory of the robot manipulator, and \( b(w, 0) = r(w) \) is satisfied at the starting point of the ribbon. \( e(w + \sigma) \) is an unit tangential vector at a minimal range \( d\sigma \).

In case that the tip position of the ribbon is moved with the velocity \( \dot{\lambda}(= \dot{w}) \) along the reference trajectory, the continuous deformation model of the ribbon can be rewritten as follows:

\[
b(w, \sigma) = b(\lambda t, \sigma) = r(\lambda t) + \int_0^\sigma e(w + \sigma')d\sigma'
\]  

where the right second term means continuous coordinates of the ribbon. Discretizing the continuous coordinates about the distance \( \sigma \), this term can be represented by summation of the distance between the particles (the link length) \( \Delta l \) and an unit vector \( e_j \) to the next particle, as shown in Fig. 7.

As a result, we can obtain a discrete deformation model of the ribbon described by the following equation.

\[
b_i(t) = r(\lambda t) \quad (22)
\]

\[
b_i(t) = b_1(t) + \sum_{j=2}^{N} \Delta l e_j \quad (i = 2, 3, \ldots, N) \quad (23)
\]

where \( b_i \) is a coordinate at \( i \)th particle numbering from the starting point of the ribbon, \( \Delta l \) is the distance between the particles (the link length), \( e_j \) is the unit vector to the next particle, and \( N \) is the number of the particles of the ribbon.

In general, the robot control is executed in discrete time system, and the cycle time of the control system is assumed to be \( \Delta T \). Here, \( \Delta l \) is set so as to \( \Delta l = \lambda \Delta T \) and \( \Delta T \) is sufficiently small. As a result, the above assumption (Eqns. (22) and (23)) can be rewritten by

\[
b_{i+1}(k \Delta T) \approx b_i((k - 1) \Delta T), \quad (24)
\]
where \( k \) is a control step number. This equation means that the location of the particle at the cycle \( k \) is same as the location of the neighbor particle at the cycle \( k - 1 \). In the proposed method, assuming that the reference trajectory in terms of time is provided by the particle location that is discretized with a constant distance and the tip position of the robot arm is controlled, we can control the ribbon deformation so as to trace the reference trajectory.

### 3.3.2. Simple deformation model of rhythmic gymnastics ribbon

As a more simple model based on the above discussion (Eqns. (22 – 24)), we propose a new model of the ribbon deformation represented by the following equation:

\[
\begin{align*}
  b_i(t) = & \begin{cases} 
  r(t - (i - 1)\Delta T) & \text{(under non-gravity)} \\
  r(t - (i - 1)\Delta T) + \left[ 0 \ 0 \ -\frac{1}{2}gt^2 \right]^T & \text{(under gravity)}
  \end{cases} \\
\end{align*}
\]

(25)

where \( t \) is time; \( i \) is the joint number of the ribbon \( (i = 1, 2, \ldots, N) \); \( b_i \in \mathbb{R}^3 \) is the \( i \)th joint coordinate of the ribbon; \( r \) is the tip position of the manipulator; \( g \) is gravitational acceleration; and the superscript T means the transpose. The second term represents the effects of gravity. Since the robot moves at high speed, the effects of gravity can be approximated as \(-\frac{1}{2}gt^2\). The first joint is set at the position grasped by the robot. As a result, the time delay \( d_1 \) is equal to zero.

This equation can be also obtained from Eqns. (22 – 24). Equation (24) can be satisfied for all particles of the discrete model of the ribbon. Thus, Eq. (24) can be rewritten by

\[
\begin{align*}
  b_i(k\Delta T) & \approx b_{i-1}((k - 1)\Delta T) \\
  b_{i-1}((k - 1)\Delta T) & \approx b_{i-2}((k - 2)\Delta T) \\
  & \vdots \\
  b_1((k - (i - 1))\Delta T) & \approx r((k - (i - 1))\Delta T).
\end{align*}
\]

(26)

Considering these equations from the final particles of the ribbon to the start particles of the ribbon which is the same as the tip position of the robot arm, we can obtain

\[
\begin{align*}
  b_i(k\Delta T) & \approx b_{i-1}((k - 1)\Delta T) \approx b_{i-2}((k - 2)\Delta T) \approx \cdots \approx b_1((k - (i - 1))\Delta T) \\
  & \approx r((k - (i - 1))\Delta T).
\end{align*}
\]

(26)

Consequently, we can get the following equation,

\[
\begin{align*}
  b_i(t = k\Delta T) & \approx r(k\Delta T - (i - 1)\Delta T) = r(t - (i - 1)\Delta T).
\end{align*}
\]

(27)

This equation is the same as Eq. (25). Therefore, we can replace the discrete model as described by Eqns. (22 – 24) to the more simple model as represented by Eq. (25).

Figure 8 shows an overview of the proposed simple model. As shown in Fig. 8, the ribbon deforms so as to track the tip of the manipulator.

Since the proposed model does not include an inertia term, Coriolis and centrifugal force terms, or a spring term, it is not required to estimate the dynamic model parameters. The advantage of the proposed model is that the number of model parameters is lower than in typical models. Therefore, the proposed model itself is robust. Moreover, since the ribbon model can be algebraically calculated, the simulation time becomes much shorter.
3.4. Motion planning with simple deformation model of rhythmic gymnastics ribbon

This section explains the motion planning for deriving the joint angles of the manipulator from the ribbon configuration. Typically used motion planning methods are very complex, and the calculation time becomes longer. On the other hand, the proposed motion planning method is very simple, because this method can be carried out using only algebraic calculations, making the calculation time much shorter.

The procedure of the motion planning method is described in the following.

First, we give the number of links of the multi-link system of the ribbon. The desired ribbon configuration \((b)\) is graphically defined in a three-dimensional space by the user. Here, there exists a case where the link distance between the two joint coordinates on the given ribbon configuration is not equal to \(l\). Therefore, the ribbon configuration is described using polar coordinates. Motion planning in only a two-dimensional plane is considered in the experiments.

Second, the ribbon configuration \((b = (b_1, \cdots, b_i, \cdots, b_N))\) is converted so as to match the manipulator kinematics to avoid problems such as a singular point.

Third, gravity compensation is performed as follows (Fig. 9). Since the robot moves at high speed and the action time is significantly short, the effect of gravity can be approximated by \(\frac{1}{2}g\Delta T^2\) (\(\Delta T\) is the motion time). Gravity compensation is performed using the following equation:

\[
 b_{ri} = \begin{cases} 
 b_i \quad \text{(under non-gravity)} \\
 b_i + \begin{bmatrix} 0 & 0 & \frac{1}{2} - \frac{1}{N} \end{bmatrix}^T (\text{g} \Delta T^2) \quad \text{(under gravity)} 
\end{cases}
\]  

(28)

where \(i\) is the joint number, \(b_i\) is the \(i\)th joint coordinate of the given ribbon configuration, \(b_{ri}\) is the \(i\)th joint coordinate of the ribbon configuration with gravity compensation (the calculated robot trajectory in Fig. 9), \(N\) is the number of joints, \(\Delta T\) is the motion time, and the superscript \(T\) indicates the transpose. Here, since gravity compensation is not needed in the neighborhood of the grasped position, the value of the gravity compensation depends on the joint location.

Fourth, the trajectory of the tip position \((r)\) of the manipulator is calculated from the converted ribbon configuration \((b_r)\). From the assumption that the ribbon deformation depends on the high-speed arm motion, the trajectory \((r)\) of the manipulator can be obtained to track the given coordinate of each joint of the ribbon. Namely, we have the following equations:

\[
 r(t = 0) = b_rN, \quad r(t = \Delta T) = b_{r1}.
\]  

(29)

The trajectory is determined so as to linearly move from the \(N\)th link to the 1st link during the motion time \(\Delta T\).

Finally, the joint angles \((q)\) of the manipulator can be obtained by solving the inverse kinematics \((q = f^{-1}(r))\).

3.5. Applicability limit of proposed method

Because of the model simplification and the assumptions in the derivation of the simplified model of the ribbon, there exist applicability limits of the proposed method as follows;

(1) The shape generation is limited to 2D plane in the proposed method.
(2) The high speed of the robot is required under gravity. However, the proposed method is available under non-gravity and on the table (namely the motion of the ribbon in \(z\) axis is under restraint).
(3) It is difficult to make the desired shape generation in the beginning end, which is the same as the tip position of the manipulator, and the free end of the ribbon.
About (1), we plan to expand the proposed method to 3D (three-dimensional) space in the future.

4. Shape Generation of a Rhythmic Gymnastics Ribbon

In order to verify the effectivenesses of the simplified deformation model and the motion planning proposed in Section 3, this section demonstrates shape controls of a rhythmic gymnastics ribbon. Circle, wave, figure-eight, and crank shapes of the ribbon were achieved based on the proposed method. In these situations, the starting configuration was with the ribbon hanging straight down. Here, we describe simulations and experiments conducted to realize shape control of the ribbon to produce the circle, wave, figure-eight, and crank shapes. Although the calculus for the motion planning based on the simplified model was computed off-line in these experiments, real-time motion planning could be achieved with our method because the motion planning was performed with algebraic calculations. In the experiments, we do not introduce the gravity compensation described in the lower equation in Eq. (28) for evaluating the effectiveness of only the proposed simple method.

4.1. Circle shape

First, the circle shape was produced. Figure 10 and Fig. 11 respectively show the simulation results with the simplified model and the experimental results (a composite photograph and a continuous sequence) of producing the circle shape. In Fig. 11(a), the dotted white line depicts the reference shape of the ribbon (that is, the robot trajectory), and these pictures were taken at intervals of 0.067 s. As can be seen from these figures, the circle shape of the ribbon was successfully produced.

**Shape Error**

Based on the continuous photographs of the experiment, we calculate a shape error between a reference motion of the robot arm, namely the reference ribbon shape, and the actual ribbon shape as follows;

\[
error_{exp}(t) = \frac{1}{N} \sum_{i=1}^{N} \sqrt{e_{ix}(t)^2 + e_{iy}(t)^2},
\]

where \(i\) is the number of a corresponding point of the ribbon, \(N\) is the number of the total corresponding points of the ribbon. \(e_{ix}\) is equal to \(Ref_{ix} - I_{ix}\) and \(e_{iy}\) is also equal to \(Ref_{iy} - I_{iy}\). \(Ref_{ix}\) and \(Ref_{iy}\) are \(x, y\) coordinates of the \(i\)th corresponding point of the reference ribbon shape (the robot motion). \(I_{ix}\) and \(I_{iy}\) are \(x, y\) coordinates of the \(i\)th corresponding point of the actual ribbon shape. And \(x\) and \(y\) describe the coordinates in the lateral and longitudinal directions, respectively. Since \(N\) is depending on the ribbon part in the image, \(N\) is different in each frame. This equation means that a summation of a distance between two corresponding points of the actual ribbon shape and the reference ribbon shape (the trajectory of the robot arm). Since this evaluation method is the same as the references (Yamakawa et al., 2012b, Yamakawa et al., 2013a), it can be considered that this method is sufficiently valid in the evaluation of the result about shape generation of the ribbon.

And, based on the sequential photographs of the experimental result as shown in Fig. 11(a), this calculation is performed every time interval. As can be seen this calculation, when the value of this error is small, we can obtain the approximate reference ribbon shape.

The result of the error given by Eq. (30) is shown in Fig. 11(b). It can be seen this figure that the error between 0–0.4 s is small. But as time advances, the error becomes larger due to the gravity. And since the maximum error was around 250 mm for the ribbon length (2,000 mm) (the maximum number of the corresponding points \(N\) is about 5500 in this error calculation.), it is considered that the error is sufficiently small. Thus the circle shape can be considered to be obtained.

4.2. Wave shape

Second, the wave shape was produced. Figure 12 and Fig. 13 respectively show the simulation results and the experimental results of producing the wave shape. The pictures in Fig. 13 were taken at intervals of 0.167 s. It can be seen from these results that the wave shape was successfully produced.

**Shape Error**

Figure 13(b) shows the result of the error given by Eq. (30) in wave shape. As time advances, the shape error becomes larger because of the gravity effect. In addition, the maximum error was around 220 mm for the ribbon length
4.3. Figure-eight shape

Third, the figure-eight shape was produced. Figure 14 and Fig. 15 respectively show the simulation results and the experimental results of producing the figure-eight shape. The pictures in Fig. 15 were taken at intervals of 0.133 s. From these results, the figure-eight shape of the ribbon was successfully produced.

Shape Error

The result of the error calculated by Eq. (30) is shown in Fig. 15(b). It can be seen this figure that the error does not increase in about 0–1.0 s. However the error increased by the gravity after the time about 1.0 s. Since the maximum error was about 380 mm for the ribbon length (2,000 mm) (the maximum number of the corresponding points \( N \) is about 3600.), it is considered that the error is small. But the accuracy of the shape generation become worse compared to the circle and wave shapes productions.

4.4. Crank shape

Finally, the crank shape was produced. Figure 16 and Fig. 17 respectively show the simulation results and the experimental results of producing the crank shape. The pictures in Fig. 17 were taken at intervals of 0.067 s. From these results, the crank shape of the ribbon was also successfully produced.

Shape Error

The result of the error given by Eq. (30) is shown in Fig. 17(b). From this result, the error increases as time advances.
In particular, the error drastically increased in the robot motion in the lateral direction (the time is between about 0.25–0.4 s). Furthermore, the maximum error was around 380 mm for the ribbon length (2,000 mm) (the maximum number of the corresponding points $N$ is about 3300.). It is considered that this error is very small for the image size. As a result, we can consider that the crank shape can be obtained.

4.5. Summary of results

As the results show, the effectiveness of the proposed method was verified by these simulations and experiments. In the experiments, shape errors occurred since the robot was initially moved from the steady state. However, if the robot moves in cyclic motion for a sufficiently long time, a ribbon shape that is closer to the reference shape can be obtained. Also, although gravity compensation was not performed in these experiments, shape control with better accuracy can be achieved by using gravity compensation.

Moreover, as described in the model analysis in Section 3.2, it is impossible to control the tension to be constant at the beginning end of the ribbon. As a result, it is difficult to perfectly control the ribbon shape using the proposed method. However, if the ribbon is sufficiently long in the ribbon manipulation about the shape generation, the condition that the tension be constant approximately holds and the shape generation can be achieved.

5. Conclusions

5.1. Summary of this paper

The goal of the work described in this paper was to achieve dynamic manipulation of a ribbon by simple method including simplified model and motion planning based on the simple model. First, we described the basic concept of the dexterous ribbon manipulation using the high-speed motion of the robot. Second, by analyzing the ribbon mechanics,
we revealed that the dynamic model of the ribbon can be reduced to a simple algebraic model by assuming constant, high-speed motion of the robot. Next, we proposed an algebraic ribbon model calculated from the robot motion based on analysis. Then, we suggested a motion planning method based on the proposed model. Finally, we showed simulation and experimental results of the shape control of the ribbon, confirming the validity of the proposed model. And we discussed the numerical error between the reference trajectory of the robot arm and the actual ribbon shape, and confirmed the production of the ribbon shape from the viewpoint of the error considerations.

5.2. Future work

As future works we will discuss the following elements;

(1) Productions of various ribbon shapes such as rectangle and character,

(2) Experiments under different conditions, for example shape control on the table such that the gravity effect cannot be affected, and

(3) Introduction of visual feedback to the system in order to robustly produce ribbon shapes.

We discussed four shape generations of the ribbon in this paper. On (1), we will realize various ribbon shapes different from the four shapes. On (2), we will make experiments of the ribbon shape production on the table in which the gravity effect is not affected to the ribbon deformation. As a result, more high accuracy shape control of the ribbon will be achieved. On (3), we will apply a high-speed visual feedback with a high-speed vision (1,000 frame per second) to the ribbon shape control in order to compensate the gravity effect on the ribbon deformation. By introducing the visual feedback, we will realize ribbon shape productions robustly.
black: manipulator
gray: ribbon

![Fig. 16 Producing a crank shape (simulation).](image)

![Fig. 17 Producing a crank shape (experiment).](image)

**References**


Inoue, H. and Inaba, M., Hand-eye Coordination in Rope Handling, Robotics Research: The First International Sympo-