Parameterization, statistical measurement and numerical modeling of fluctuated meso/micro-structure of plain woven fabric GFRP laminate for quantification of geometrical variability

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Abstract
This paper proposed a stochastic multiscale modeling method by parameterization and quantification of the fluctuated meso/micro-structure of plain woven fabric glass fiber reinforced plastics (GFRP) fabricated by hand layup. The experimental results of static tensile tests of 25 specimens made by 5 persons showed large scattering and the macroscopic parameters could not enough explain the variability. Hence, the meso/micro-structure based stochastic multiscale method was presented first, and its geometrical modeling technique was deeply studied in this paper. By the observation of the specimens, fluctuated geometrical features were parameterized. The defined parameters were statistically measured. The generated 2D models could express the realistic meso/micro-structures. It should be noted that the nesting in laminated composites and the distributed fiber volume fraction were included in the proposed modeling method.

Key words: Plain woven fabric GFRP laminate, Hand layup, Meso/micro-structure, Geometrical variability, Nesting, Distributed fiber volume fraction

1. Introduction

Applications of fiber reinforced plastics (FRP), especially CFRP (carbon FRP), are growing, and the need for cheaper molding process than the autoclave technique is increasing such as the resin transfer molding (RTM) for continuous fiber reinforcement or press molding for long or short fiber reinforcement. In those cheaper processes, the problem lies in the variability in the mechanical properties such as stiffness and strength, due to unexpected geometrical variability at the microscale. There are many studies on the variability and/or uncertainty in RTM process (Li, et al., 2006), and numerical simulation such as the direct fiber simulation (Yamanoi, et al., 2010) is expected to play an important role to predict and control the variability. Also, the influence of the microstructural variability on the mechanical properties will be analyzed numerically by a sort of stochastic finite element method (Sakata, et al., 2010, Kubo, et al., 2016).

A multiscale computational method is essential to analyze the uncertainty propagation from micro- to macro-scales. The micromechanics approach is widely used (Lomov, et al., 2001, Venaerschot, et al., 2013, Zeng, et al., 2015), but one of the authors has utilized the asymptotic homogenization method (Takano, et al., 2000) and validated in the
multiscale modeling of textile thermoplastic composites. This method was extended to consider the uncertainty in physical parameters, i.e., the mechanical properties of constituent materials at the microscale, to analyze the probability density function of the macroscopic or homogenized properties (Sakata, et al., 2010, Basarrudin, et al., 2013, Zhou, et al., 2015). In the stochastic multiscale modeling, there are two categories of parameters. The one is physical parameter such as the Young’s modulus of constituent materials. Another one is geometrical parameter of the microstructure. The geometrical modeling of the microstructure is very difficult when it is fluctuated or it includes uncertainty.

From the viewpoint of microstructure of FRP, the textile composite is much more difficult than uni-directionally fiber aligned case. This paper focuses on the plain woven fabric composite that is the most commonly used textile composite. We can find many publications on the plain woven composite and numerical modeling including interfacial damage (Naik, 1994, Qu, et al., 2011, Bizeul, et al., 2011). In the laminated plate, the nesting is greatly influential on the damage propagation (Olave, et al., 2015). However, the numerical modeling of the nesting had many limitations in the published studies due to simplifications (Prodromou, et al., 2011), which resulted in the differences between numerical models and reality. We can also find papers that showed measured statistical data of some typical geometrical parameters (Summerscales, et al., 2004, Vanaerscot, et al., 2016), but limited number of parameters could not express the fluctuated real microstructures.

Therefore, to analyze the influence of variability at the microscale of plain woven fabric composite laminate by stochastic homogenization method, all necessary geometrical parameters to express the fluctuation due to molding process are proposed and measured statistically. This study employed simplest hand layup process that can include inter-individual differences. Not only the nesting but also the distributed fiber volume fraction in both meso- and microscales is presented in this paper.

In the following, static tensile test of 25 specimens made by 5 persons is presented in Chapter 2, where the macroscopic parameters could not enough explain the variability. The multiscale modeling framework is shown in Chapter 3. In Chapter 4, the proposed parameters and statistical data are shown so that anyone can append new data in the presented database. Some numerical models are shown in Chapter 5 followed by concluding remarks.

2. Conventional characterization of variation in static tensile test

Plain woven fabric reinforced GFRP laminates with 4 layers were fabricated by hand layup by 5 persons, A to E. E-glass robing cloth WR 570 C-100 (Nittobo) and unsaturated polyester resin were used. Each person made 1 laminate, from which 5 specimens for static tensile test and 2 specimens for SEM observation were taken as shown in Fig. 1(a). In this fabrication process, the warp and weft directions were defined as shown in Fig. 1(a). Figure 1(b) shows the results of static tensile tests for totally 25 specimens. The Young’s modulus, knee-point stress and final fracture stress were measured. The mechanical properties were very largely scattered. Micro-cracks appeared first and delamination was observed after the knee-point. When the stress value became flat, the inter-laminar delamination propagated and the strain value measured by strain gage increased rapidly, whilst the load did not change.

![Fig. 1 Results of static tensile tests of 4 layered plain woven fabric GFRP for 25 specimens made by 5 persons, A to E](image-url)
When the thickness of each specimen and above measured properties were plotted against the fiber volume fraction of the specimen as shown in Fig. 2, the variability can be explained in some part. This fiber volume fraction of the specimen is called macroscopic fiber volume fraction, $V_f^{\text{macro}}$, in this paper. It was measured by calcination method. Concerning the thickness, very clear dependency on $V_f^{\text{macro}}$ was seen as shown in Fig. 2(a), which is a reasonable result. The Young’s modulus in Fig. 2(b) is basically increased as the increase of $V_f^{\text{macro}}$. The inter-individual difference among 5 persons can be explained by $V_f^{\text{macro}}$. This implies that the variability in one lot is not so large. This variability in one lot can be called the intra-individual difference. On the other hand, for the knee-point stress and final fracture stress, $V_f^{\text{macro}}$ could not enough explain the characteristics of fabricated specimens as shown in Figs. 2(c) and (d). Therefore, meso- and microscopic geometrical features were parameterized first, and then measured statistically.

![Thickness vs. Fiber Volume Fraction](image1)

![Young's Modulus vs. Fiber Volume Fraction](image2)

![Knee-point Stress vs. Fiber Volume Fraction](image3)

![Fracture Stress vs. Fiber Volume Fraction](image4)

**Fig. 2** Mechanical properties plotted against fiber volume fraction at macroscale

### 3. Multiscale modeling framework for stochastic homogenization analysis

For the measurement, as shown in Fig. 1(a), the cross-sections in both warp and weft directions were imaged by SEM. The typical examples are shown in Fig. 3. The specimen C showed lower fracture stress as shown in green in Fig. 1(b), and the specimen E showed highest strength as shown in yellow in Fig. 1(b). The differences between specimen C and specimen E can be roughly explained by the macroscopic fiber volume fraction as described in Fig. 2. However, to explain the scattering for 5 specimens, A to E, meso- and microscopic microstructures should be examined in more detail. To characterize the geometrical features, three scales were defined as shown in Fig. 4. The goal of this study is to build realistic finite element models for the stochastic homogenization analysis based on the statistical measurement of the geometrical parameters.

The framework of the stochastic multiscale modeling and simulation is illustrated in Fig. 5. The microstructural parameters are classified into physical and geometrical ones. The former means the parameters to express the
uncertainty in the elastic moduli of fiber, fiber bundle and matrix resin. One of the authors has so far presented the formulation of first-order perturbation based stochastic homogenization (FPSH) method and its applications to porous material (Miyauchi, et al., 2015, Wen, et al., 2015) and particulate embedded composite materials (Wen, et al., 2016). The main contribution of this paper lies in the parameterization of geometrical features of plain woven fabric composite laminate made by hand layup and the model generation algorithm based on the statistically measured data. Since this work is based on the cross-sectional observation, one limitation is that only 2D model is discussed.

As shown in Fig. 5, temporal unit cell model is defined first with a normalized coordinate system $\xi$ and $\eta$. The parameters to layup this temporal unit cell model and the morphological parameters in the temporal unit cell model are defined. Finally, a nesting function is defined to generate realistic RVE (representative volume element) models of woven fabric composite laminate. The fiber bundles are subdivided into sub-regions that have different fiber volume fraction called microscopic fiber volume fraction, $V_{\text{micro}}$. Each fiber bundle have different fiber volume fraction called
mesoscopic fiber volume fraction, $V_{f,\text{meso}} = \int V_{f,\text{micro}} \, dA$. Finally, many microstructure models $X_j (j=1, \ldots, n)$ can be generated. Each parameter is explained in the next chapter.

The above mentioned FPSH can calculate the homogenized elastic tensor $D^{\mu}_j$, for each RVE model. When the random parameter $\alpha$ denotes the normal distribution of the elastic tensor of constituent material as $D(\alpha)$, the homogenized elastic tensor of the composite materials is also a function of $\alpha$. Let $f(D^{\mu}_j)$ denote the probabilistic density function, and the following equations express the stochastic modeling of the elastic tensor of composite and constituent materials (Basaruddin, et al., 2013, Miyauchi, et al., 2015). Note here that different type of distribution can be calculated using the characteristic displacements, which is specific in the conventional homogenization theory.

1. \[
f(D^{\mu}_j) = f(D^{\mu}_j (X_j, D(\alpha))) = f(D^{\mu}_j (X_j, \alpha)) + f(D^{\mu}_j) f(\alpha)
\]
2. \[
f(D(\alpha)) = D^{\mu} + D^{\mu} f(\alpha)
\]

The homogenized elastic tensor is finally obtained by the mixture distribution in Eq. (3) in which the probability of each model $f(X_j)$ is multiplied. Usually a flat probability is used for $f(X_j)$ as prior probability (Wen, et al., 2015). In order to perform the stochastic homogenization analysis, the generation of random and realistic microstructure
models is the most important. Hence, in this paper, all necessary geometrical parameters and measured statistical data to generate many microstructure models are presented.

\[ f(D^n) = \sum_{j=1}^{m} f(X_j) f(D^n_{X_j}) \]  

(3)

4. Parameterization and statistical measurement

4.1 Parameterization

The macroscopic geometrical information of microstructure is defined by the following parameters in Eq. (4), in which the morphology \( A \) in each unit cell that was shown in Fig. 5 is in more detail defined by Eq. (5).

\[ X = \left( V_f^{macro}, L_{surf-mat}, L_{\xi}, A, s(\xi), L_{int-mat}, \phi \right) \]  

(4)

\[ A = \left( L_{surf}, \xi, \eta, V_f^{meso}, L_{major}, L_{minor}, V_f^{micro} \right) \]  

(5)

The meaning of each parameter is listed below. Please also see Fig. 5.

- \( t \): Thickness (mm)
- \( V_f^{macro} \): Fiber volume fraction at macroscale
- \( L_{surf-mat} \): Thickness of surface matrix layer (mm)
- \( L_{\xi} \): Size of temporal unit cell in \( \xi \) direction (mm)
- \( \phi \): Lamination misalignment
- \( s(\xi) \): Nesting function
- \( L_{int-mat} \): Thickness of inter-laminar matrix layer (mm)
- \( \xi \): Position fluctuation of fiber bundle from standard position in horizontal direction measured at the center of fiber bundle (mm)
- \( \eta \): Position fluctuation of fiber bundle from standard position in vertical direction measured at the center of fiber bundle (mm)
- \( V_f^{meso} \): Fiber volume fraction at mesoscale
- \( L_{major} \): Length of major axis at fiber bundle (mm)
- \( L_{minor} \): Length of minor axis at fiber bundle (mm)
- \( V_f^{micro} \): Fiber volume fraction at microscale

Note here that \( \xi \) and \( \eta \) denote the local coordinate system defined for each unit cell model. The unit cell size in \( \eta \) direction is note defined in the above parameter list, but is defined by the morphological parameters in Eq. (5) according to the configuration in Fig. 5. \( L_{int-mat} \) is not necessary if the application is limited to the laminate made by hand layup shown in Fig. 3, but is defined for future application to HPRTM (high pressure resin transfer molding) process. In the morphological parameters, the position and the size of fiber bundles are parameterized. However, the inclination was neglected. The fiber volume fraction at the microscale is defined in a digitized manner by subdividing the fiber bundle into 24 sub-regions in later measurement, assuming that, in finite element model, the cross section of fiber bundle is divided into 6 elements as shown in Fig. 5.

4.2 Statistical data of dimensions, misalignment and nesting

The dimension of temporal unit cell was determined and measured first for warp and weft directions as plotted in Fig. 6. The total number of measured samples was 171. It seems natural that the unit cell size in warp direction is larger than that in weft direction. Hereafter, according to the above definition, all parameters were measured.

The position of the center point in each fiber bundle is distributed first. Its fluctuation from a standard position in each temporal unit cell, \( \xi \) and \( \eta \), was measured statistically as shown in Fig. 7. Note that the absolute values were presented in Fig. 7. Also note that \( |\eta| \) is strongly dependent on the location to take the cross-section, but is partially fluctuated by the deformation of the fiber bundles and by the nesting. Therefore, in 3D microstructure modeling, \( |\eta| \) should not be directly used but should be just referred statistically to model the deformation of the cross-sections and the nesting. In this paper, was directly used in 2D modeling. The position fluctuation was larger in the in-plane direction than that in the lamination direction. The number of measured samples was 308, in which the center position

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was clearly identified.

The major and minor lengths of fiber bundle, \( L_{\text{major}} \) and \( L_{\text{minor}} \), are shown in Fig. 8. The number of measurable samples was 403, and \( \mu \) and \( \sigma \) stand for average and standard deviation, respectively. There must be a certain correlation between two parameters considering the deformation of fiber bundle in molding process. Therefore, the measured lengths were plotted in Fig. 9, and the upper and lower bounds were defined as shown by the red dashed lines for further model generation. They corresponded to the fiber volume fraction of 35.5 % and 62.0 % approximately in each fiber bundle at the meso-scale.

![Statistical data of width of unit cell (n=171)](image)

![Statistical data of position fluctuation of fiber bundle](image)

![Statistical data of shape of fiber bundle](image)
The misalignment in the lamination of temporal unit cell was summarized in Fig. 10, which was used in a function to describe the nesting in Eq. (6). In this equation, the interlayer cross-sectional line is formulated with respect to the normalized coordinate system for temporal unit cell, where the subscript shows the layer number and every coefficient factors have the statistical data. Using this function and the statistical data, it is possible to generate the interlayer cross-sectional line as shown in Fig. 11. The green dotted lines show the measured interlayer. The black solid line and yellow band show the generated interlayer using the average and double of standard deviation, respectively.

\[
s(\xi) = \frac{1}{2} \left[ B_i \cos \left( \frac{2\pi}{L_i} \xi \right) + B_{i+1} \cos \left( \frac{2\pi}{L_{i+1}} \xi - \theta \right) \right], \quad B_i = \frac{L_{\text{minor}}}{2}, \quad L_i = \frac{L_{i+1}}{2}, \quad \theta = 4\pi \phi, \quad i = 1 \sim 3
\]  

(6)

Fig. 9 Correlation between \( L_{\text{major}} \) and \( L_{\text{minor}} \)

Fig. 10 Statistical data of lamination misalignment (\( n=24, \mu=0.350, \sigma=0.210 \))

Fig. 11 Statistically generated nesting model and comparison with measured inter-layer shape in real specimen. Black line and yellow area shows average and double of standard deviation in the generated model. Green dashed lines show the measured inter-layer shape.
4.3 Statistical data of fiber volume fraction

The fiber volume fraction of tensile specimens, $V_f^{\text{macro}}$, ranged from 57.1 % to 64.3 % as shown in Fig. 2, but it could not explain fully the variability in mechanical properties. Therefore, its variability was defined and measured hierarchically in the meso- and microscales as shown in Fig. 4. The mesoscopic fiber volume fraction, $V_f^{\text{meso}}$, is the averaged value for each cross-sectional area of fiber bundle. $V_f^{\text{micro}}$ at the microscale is defined in subsectioned small region $q$ in each cross-sectional area of fiber bundle by Eq. (7), where $n_{pq}$ denotes the number of fibers in the region $q$, $S_q$ denotes its area, and $d_{\text{fiber}}$ denotes the averaged diameter of single fiber. The relation between mesoscopic fiber volume fraction and microscopic one is described by Eq. (8) when the number of subsectioned small region is 24 in this study.

Figure 13 shows the statistical data of mesoscopic fiber volume fraction. The average number of fibers in fiber bundle was 2015, and was used in Eq. (7) to obtain Fig. 13. The variability at microscale is displayed in Fig. 13(a), and the detailed statistical data normalized by the mesoscopic fiber volume fraction of each cross-sectional area of fiber bundle is shown in Fig. 13(b). It was surprising that the fiber volume fraction was largely scattered at the mesoscale ranging from 48 to 76 %. The maximum and minimum fiber volume fraction at the microscale had ± 30 % deviation. The variation at microscale seems to have no specific rule and to be quite random in Fig. 13(a).

\[
V_f^{\text{meso}} = \frac{n_{pq} \times \pi (d_{\text{fiber}})^2}{S_q}
\]

\[
V_f^{\text{meso}} = V_f^{\text{meso}}(L_{\text{major}}, L_{\text{minor}}, V_f^{\text{micro}}) = \frac{1}{V} \int V_f^{\text{micro}} dV = \frac{1}{24} \sum_{i=1}^{24} V_f^{\text{micro}}
\]

\[
\text{Frequency (\%)}
\]

\[
\text{V_f^{macro} (\%)}
\]

\[
48 \quad 52 \quad 56 \quad 60 \quad 64 \quad 68 \quad 72 \quad 76
\]

\[
\text{Frequency (\%)}
\]

\[
\text{V_f^{macro}}
\]

\[
\text{(a) Color display of variability of fiber volume fraction}
\]

\[
\text{(b) Statistical data (n=96, \mu=0, \sigma=0.140)}
\]

Fig. 12 Statistical data of mesoscopic fiber volume fraction ($n=391, \mu=62.51, \sigma=5.11$)

\[
\text{Frequency (\%)}
\]

\[
\text{V_f^{macro} normalized by V_f^{macro}}
\]

\[
\text{Frequency (\%)}
\]

\[
\text{V_f^{macro}}
\]

\[
\text{(a) Color display of variability of fiber volume fraction}
\]

\[
\text{(b) Statistical data (n=96, \mu=0, \sigma=0.140)}
\]

Fig. 13 Statistical data of microscopic fiber volume fraction
5. Finite element modeling for stochastic homogenization analysis

Based on the measured statistical data from Fig. 6 to Fig. 13 and one correlation data in Fig. 9, this chapter shows the generated 2D model for stochastic homogenization analysis. The thickness $t$ and the lamination misalignment $\varphi$ were determined first. To show some examples, the parameters in Table 1 were used. Note that the assumed thickness is close to the upper and lower bounds of specimens as shown in Fig. 2(a). 6 typical RVE models are shown in Fig. 14 for each thickness case. When compared with the cross-sections of real specimen shown in Fig. 3, we can find similarity in the lenticular shape of the cross-section or waviness of fiber bundles or the nesting situation. Due to the randomness, it is hard to quantify the similarity, but it is a matter of course because the generated models are based on the measured database for the specimens.

In the finite element meshing, the lenticular cross-section of fiber bundle was subdivided into 6 elements, because the database of the fiber volume fraction was built by dividing into 24 sub-sections as shown in Fig. 13(a). The distribution of fiber volume fraction was shown in Fig. 13(b), but a selected value of the fiber volume fraction was assigned to an element randomly. This is because no feature was found in this assignment in this study. It must be correlated to the deformation of the fiber bundle in the hand layup process, which should be further investigated in the future. Once the fiber volume fraction is determined, it is easy to assign the physical parameters as an orthotropic material model using, for instance, the homogenization theory. To perform the nonlinear analysis in the future, the fracture strength of the fiber bundle must be assigned as a function of the fiber volume fraction.

In this paper, the RVE model generation is limited to a 2D model. The database will be able to be applied to a 3D model, but the generation algorithm has not been fixed at this moment and should be further studied.

Fig. 14 Examples of 2D FEM model
(a) Model names: (top) 1.5-1, 1.5-2, 1.5-3, 1.5-4, 1.5-5, 1.5-6 (bottom)
(b) Model names: (top) 1.7-1, 1.7-2, 1.7-3, 1.7-4, 1.7-5, 1.7-6 (bottom)
6. Conclusion

This paper presented new parameters to describe the randomness in the meso/micro-structures of plain woven fabric GFRP laminate made by hand layup. They were measured statistically by careful observation of 25 specimens made by 5 persons. The presented database can be used in the stochastic finite element modeling. The generated 2D RVE models in this paper were close to the cross-sectional images of real specimens. It should be noted that the nesting was easily described and that the distribution of fiber volume fraction in the fiber bundles was investigated hierarchically. The database must be verified by comparison with additional specimens made by other persons in the future. This paper contributed to allow anyone to append new data in the presented database.

The development of model generation algorithm for 3D RVE models has not been fixed at this moment, but the presented parameters and database can be referred also in the 3D modeling. Also, the correlation between the process parameter such as the pressure given by the worker is a matter of concern in order to modify the process and built a sort of recipe. It is expected to clarify the correlation between the mechanical properties with the help of stochastic multiscale finite element analysis in the near future.

Although the presented database was obtained by the specimens made by hand layup, the proposed parameters can be utilized for other molding process such as RTM (resin transfer molding) process. The thickness of inter-laminar matrix layer, $t_{\text{mat-intL}}$, was not so important in the case of hand layup, but it can hopefully express the differences due to the inlet pressure in the HP-RTM (high pressure RTM) process.

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