Completion of analytical model of active magnetic thrust bearings including asymmetric air gap field between mixed materials

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Abstract

The electrodynamics of magnetic thrust bearings are characterized by an above-average dependency on the bearing materials. Axially directed fields render laminated stators and rotors ineffective. High induced voltages inside the magnetic core evoke eddy currents and opposing fields, which are compensated by an additional magnetizing current causing a significant delay between the measurable coil current and the force-related magnetic flux. The control dynamics are hampered and even though this effect can be reduced by the use of Soft Magnetic Composites (SMC) for non-rotating parts, the thrust disk is usually made out of steel due to its superior tensile strength and saturation flux density. The analytical modeling of mixed-material magnetic thrust bearings reveals new challenges arising from asymmetries and low permeable magnetic core sections, both of which are addressed in this article. In case the air gap is bounded by core sections made of different materials the established analytical models are not applicable and require the presented *asymmetrical air gap wave propagation constant*. Furthermore the consideration of the stator corner reluctances for SMC cores compensates the stationary error of 6 % of the total effective reluctance present in previous works.

Keywords : Active magnetic thrust bearings, Fractional order systems, Eddy currents, Magnetic levitation control, Soft magnetic composites, Analytical modeling

0. Disclosure

The following article is an extended version of the conference paper *Analytical Asymmetric Air Gap Model for Active Magnetic Thrust Bearings of Mixed Materials Including Eddy Currents* by Seifert et al. presented at the 15th International Symposium on Magnet Bearings, 2016. The conference contribution focused on the asymmetric air gap reluctance model, which is required for magnetic bearings consisting of multiple core materials. The low permeability of the deployed soft magnetic composites additionally requires the consideration of the corner elements of the magnetic circuit (Fig. 1b), which have been neglected to date. These considerations are addressed in detail in this article to complete the analytical modeling of active magnetic thrust bearings. However, local saturation effects and frequency dependent leakage fluxes are not considered yet and still subject of further investigations.

1. Introduction

Highly dynamical active magnetic thrust bearings and actuators are driven by multiples of its steady-state voltage (DC voltage leading to maximum coil current in steady-state) to achieve high stiffness and force gradients. The underlying rapid flux gradients cause significant eddy currents inside the magnetic core. Axially directed fields render laminated stators and rotors ineffective and high induced voltages lead to opposing fields originating from these eddy current loops (Fig. 1a). However, the main flux Φ generating the thrust force F is directly dependent on the terminal voltage u and the

opposing fields are compensated by an additional magnetizing current $i'_{\mu\text{Fe}}$. Therefore a significant delay between the measurable coil current and the force-related magnetic flux is observed (Fig. 2). Increasing demand on control dynamics requires to overcome these eddy current effects, which in general are characterized by an above-average dependency on the materials employed.

Fig. 1 Combined radial/thrust bearing: a) Magnetic fields and eddy current loops in cylindrical core parts, e. g. thrust disk b) Magnetic circuit of thrust bearing divided into effective reluctances. c) Cross section
Geometry of test setup: $r_1 = 22.5$ mm, $r_2 = 42$ mm, $r_3 = 49$ mm, $r_4 = 59$ mm, $a_1 = 3.6$ mm, $a_2 = 12.4$ mm, $a_3 = 22$ mm, $b_1 = 14.4$ mm, $b_2 = 5.6$ mm, $d = 5$ mm, $g = 0.5$ mm

As a first approach the application of less electrically conductive materials for stator and rotor, like Soft Magnetic Composites (SMC), can reduce the phase delay drastically compared to conventional solid steel bearings (Fig. 3). In this case as well as for laminated radial bearings the eddy current effects can be modeled simply by considering the eddy current time constant T_{Fe} in the equivalent electric circuit as shown by Bahr et al. (2013). Even for bearings partly made of steel T_{Fe} is generally larger (up to 10 times for $f_{\text{Pulse}} = 20$ kHz) than the actual inverter pulse width and therefore not applicable, as the state of constant current gradient (section B in Fig. 2) is never reached during one switching period. However, the lower saturation flux density and weak mechanical strength ($\sigma = 70$ MPa) currently disqualify SMC as material especially for the rotating components in industrial high-speed applications.

Fig. 2 Qualitative currents and its asymptotes immediately after voltage pulse - T_{Fe} : Time delay between measurable coil current and thrust force

Fig. 3 Measured step response of the coil current for 48 V voltage step: comparison of thrust bearing with solid SMC stator and either steel or SMC disk

Alternatively, the axial magnetic circuit can be modeled as a reluctance network by analytically solving the underlying differential equations considering the eddy current effects due to the typical high flux gradients. A compensation of the main flux delay within the closed-loop control is theoretically possible, if the model could be simplified and discretized. Thus the control of the magnetic flux density respectively the force generating magnetizing current, instead of the coil current, can be realized. The analytical models of the current and position plants can serve as estimators for the determination of the actual value for the force related air gap flux density B_g from the measured coil current (Fig. 4). Their transfer functions are based on explicit (Monje et al., 2010) or implicit (Riu et al., 2003) fractional-order systems, a research field still in its early stages. The required approximations of the reluctance network and its practical implementation are the focus of further studies.

For e. g. vacuum applications, like turbomolecular pumps, which demand low losses and high dynamics, a combination of both approaches can be applied as in the configuration shown in Fig. 1. The stator is made of SMC and profits from its low electrical conductivity and therefore low eddy current density. For the thrust disk conventional steel (42CrMo4, SAE4140) is used due to its superior tensile strength ($\sigma = 1$ GPa). Even though the air gap geometry is still symmetrical, the magnetic field on the steel side of the air gap suffers from a much higher magnetic skin effect. The reason is its good electric conductivity provoking significant eddy current loops in the outer core regions which prevent the magnetic field from penetrating deeper into the core. This effect is considerable weaker on the SMC side of the air gap (compare Fig. 8). Therefore the assumption of a symmetrical field distribution and identical materials on both sides of the air gap made in previous works (firstly Zhu et al., 2005; later Sun et al., 2009 and Zhu et al., 2010) is invalid for the setup described here.

This paper focuses on the completion of the analytical air gap model also for the underlying case of an *asymmetrical air gap field*, described by the effective air gap reluctance R_{mgi} . In previous works the reluctances of the stator corners (R_{mk} in Fig. 1b) have usually been neglected, but the comparatively low relative permeability of SMC requires they be included, as proposed in this paper. Saturation and frequency dependent leakage fluxes are still neglected though and their consideration will be part of future studies. The results are validated by FE-Analysis (FEA).

Fig. 4 Position control with underlying flux density control loop with measurement of coil current, flux density estimation with fractional-order ($\gamma \in \mathbb{R}$) system $L_{\text{eff}}(s^\gamma) = w^2 / \sum_i R_{\text{mi}}(s^\gamma)$, PID-controller and linearized magnetic bearing model. Parameters: T_B - main field time constant, T_i - inverter delay, w - number of turns, A_g - area of air gap, R_{Cu} - winding resistance, m - rotor mass

2. Advanced Magnetic Circuit Model

The electric conductivity κ of the utilized steel grade 42CrMo4 is three orders of magnitude larger compared to SMC (Somaloy Prototyping Material) causing high eddy currents, which in turn have a great impact on the field distribution within the skin depth δ_α . The analytical model is derived from the diffusion equation, which can be solved for every section (Fig. 1c) of the magnetic core assuming a one-dimensional field propagation, specifically $\vec{B}(\mathbb{R}^3, t) = B(\mathbb{R}^3, t)$ (Stoll, 1974):

$$\Delta B(\mathbb{R}^3, t) = \kappa \mu_0 \mu_r \frac{d}{dt} B(\mathbb{R}^3, t) \quad (1)$$

The spatial derivative indicated by the Laplace operator Δ is independent of time and Eq. (1) can be rewritten for sinusoidal excitation, assuming an initially constant frequency ω :

$$\Delta \underline{B}(\mathbb{R}^3) = j\omega \kappa \mu_0 \mu_r \underline{B}(\mathbb{R}^3) = \alpha^2 \underline{B}(\mathbb{R}^3) \quad (2)$$

which has the form of a *modified Bessel's equation*

$$\frac{\partial^2 \underline{B}_z(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \underline{B}_z(r)}{\partial r} - \alpha^2 \underline{B}_z(r) = 0, \quad (3)$$

in case of axially permeated cylindrical geometries. In general the solution of the diffusion equation for any magnetic actuator is a function of the *wave propagation constant* α (Kucera et al., 1996) containing the square root of the *complex frequency* $j\omega$. By integration one can obtain the magnetic flux Φ and the so-called *effective reluctance* R_{meff} (Tab. 1 - Zhu, 2005; respectively *effective permeability* μ_{reff} introduced by

Table 1 Analytical solutions of all partial reluctances for magnetic cores of a single material (Zhu et al., 2005)

R_{mi}	Formula	$R_{\text{meff}}(j\omega) = \sum_i R_{\text{mi}}(j\omega)$
R_{mai}	$\frac{a \cdot \alpha}{2\pi\mu_0\mu_r r_2} \frac{I_0^*(r_2\alpha) K_1(r_1\alpha) + I_1(r_1\alpha) K_0(r_2\alpha)}{I_1(r_2\alpha) K_1(r_1\alpha) - I_1(r_1\alpha) K_1(r_2\alpha)}$	
R_{mao}	$\frac{a \cdot \alpha}{2\pi\mu_0\mu_r r_3} \frac{I_0(r_3\alpha) K_1(r_4\alpha) + I_1(r_4\alpha) K_0(r_3\alpha)}{I_1(r_4\alpha) K_1(r_3\alpha) - I_1(r_3\alpha) K_1(r_4\alpha)}$	
R_{mr}	$\frac{\ln \frac{r_3}{r_2}}{2\pi\mu_0\mu_r b} \cdot \frac{b \cdot \alpha}{\tanh(b \cdot \alpha)}$	
R_{mgi}	$\frac{g \cdot \beta}{2\pi\mu_0 r_2} \frac{I_0(r_2\beta) K_1(r_1\beta) + I_1(r_1\beta) K_0(r_2\beta)}{I_1(r_2\beta) K_1(r_1\beta) - I_1(r_1\beta) K_1(r_2\beta)}$	
with: $\alpha = \sqrt{j\omega \kappa \mu_0 \mu_r} = \delta_\alpha^{-1} \cdot \sqrt{2j}$, $\beta = \sqrt{\frac{2\alpha}{g\mu_r}}$		
* I_ν, K_ν : modified Bessel functions, ν -th order		

Rabinovici et al., 1992). The air gap field represents a special case: even though there are no eddy currents, it is not homogeneous as its width is usually too small (Fig. 5). The only partly homogenization was first considered by Zhu (2005) by applying a reluctance network to the air gap.

Fig. 5 Flux lines Φ within the skin depth δ_α inside a cylindrical iron core with air gap surrounded by a solenoidal excitation coil ($w i_x$ = current linkage) for frequencies a) below, b) around and c) above the threshold frequency f_0 of $R_{mg}(j\omega)$.

They stated that all flux lines in the air gap field region would be either parallel to the air gap surface inside the core or perpendicular through the air gap, which is true for frequencies above a certain material-dependent threshold frequency f_0 (Fig. 5c). It was also assumed that the frequency-dependent width of the air gap field region would not overlap with the adjacent core field region. As shown in Fig. 5b this is not the case for frequencies around f_0 , but the error made was found to be negligible by Seifert (2014). The modeling of the air gap field region with a reluctance network has been discussed in detail in Zhu et al. (2005) and leads to the air gap reluctance R_{mgi} (Eq. (4)). For low frequencies (Fig. 5a) the air gap field is nearly homogeneous as expected, confirmed by the convergence of $R_{mgi}(j\omega)$ to its stationary value for $\omega \rightarrow 0$.

$$R_{mgi}(j\omega) = \frac{g \cdot \beta}{2\pi\mu_0 r_2} \frac{I_0(r_2\beta) K_1(r_1\beta) + I_1(r_1\beta) K_0(r_2\beta)}{I_1(r_2\beta) K_1(r_1\beta) - I_1(r_1\beta) K_1(r_2\beta)} \quad \text{with} \quad \beta = \sqrt{\frac{2\alpha(j\omega)}{g\mu_r}}, \quad I_v, K_v : \text{modified Besselfunctions} \quad (4)$$

However, the solution derived in Zhu et al. (2005) assumes identical materials on both sides of the air gap. In consequence they solely define the *air gap wave propagation constant* β for a *single* material. This paper proposes an approach to determine the *asymmetrical air gap wave propagation constant* β_{12} which also covers the case of *mixed* materials within a magnetic circuit (Sec. 2.1). Assuming a single core material and ignoring the corner elements R_{mk} , all reluctances found by Zhu (Tab. 1) can be summed up to form an electric circuit (Fig. 6) with the *effective inductance* L_{eff} with the number of turns w .

Figure 6 shows an equivalent circuit diagram of a magnetic thrust bearing. It consists of a voltage source u in series with a coil resistance R_{Cu} and an effective inductance $L_{eff}(j\omega)$. The current i flows through the circuit. The effective inductance is given by $L_{eff}(j\omega) = \frac{w^2}{R_{meff}(j\omega)}$. The equivalent circuit equation is:

$$\frac{i(j\omega)}{u(j\omega)} = \frac{1}{R_{Cu} + j\omega L_{eff}(j\omega)} = \frac{1}{R_{Cu} + j\omega \frac{w^2}{\sum_i R_{mi}(j\omega)}} \quad (5)$$

Fig. 6 Equivalent circuit of magnetic thrust bearing with effective inductance

More importantly, the effective inductance allows to define a flux estimator and enables the implementation of a position control with underlying flux density control (Fig. 4), avoiding laborious flux density measurements:

$$\frac{B_g(j\omega)}{i(j\omega)} = \frac{1}{A_g} \frac{\Phi_g(j\omega)}{i(j\omega)} = \frac{L_{eff}(j\omega)}{w A_g} = \frac{w/A_g}{\sum_i R_{mi}(j\omega)} \quad (6)$$

A more stable and dynamic bearing operation is possible (Fig. 4), as the flux density B_i and hence the bearing force f_B is practically independent of the coil current i . Its increase, caused by the eddy currents, as described in Fig. 1a and Fig. 2, has no influence. A flux density control is therefore also insusceptible for eddy currents:

$$f_B(j\omega) = k_B \cdot B_i(j\omega) = k_B \cdot \frac{T_B/w A_g}{1 + s T_B} \cdot u(j\omega) \quad (7)$$

The coil current i only causes a voltage drop over the coil resistance R_{Cu} , which is usually negligible for sufficiently small R_{Cu} . Therefore the current i is solely required for the estimation of the actual flux density B_g in Eq. (6).

2.1. Air Gap Model Considering Asymmetrical Air Gap Field Distribution

Though the current density within the air gap is $J = 0$, an inhomogeneous air gap field implies an *air gap skin depth* δ_β . It corresponds to the skin depth of the current density field $\vec{J}$ at the boundary surface between core and air gap field. If there are different materials on both sides of the air gap (characterized by α_1 and α_2 and respectively β_1 and β_2), there must exist an *asymmetrical air gap wave propagation constant* β_{12} in-between β_1 and β_2 .

The main elements of the magnetic thrust bearing (Fig. 1b) are solid hollow cylinders (R_{mai} , R_{mao}) permeated by an axially directed flux density field $\vec{B}$, changing its direction with frequency ω of its provoking harmonic magnetizing current i . According to LENZ's law a delayed circular current density field $\vec{J}$ is evoked by the change of $\vec{B}$ forming an oscillating electromagnetic wave. For high frequencies ω the J -field can only follow the oscillation close to the surface of the magnetic conductor, which is called the *skin effect* characterized by the *core skin depth* δ_α . In case of a cylindrical conductor the *skin depth* δ is generally defined as the radius r where a field quantity, e. g. the current density, falls to 37 % of its surface value: $\delta = r(J = e^{-1} J_0)$. Within δ_α the current density wave is moving with the propagation speed $v = \omega \cdot \delta_\alpha$. At the boundary between two materials the propagation speed is changing due to the differences in permeability μ and conductivity κ of the materials. However the change is not stepwise as the continuity condition of the normal component of the current density and its evoking flux density $\vec{n} \cdot (B_{n1} - B_{n2}) = 0$ would be violated. Therefore at the boundary point $[r = x_{12}, z = 0]$ the conditions $B_{n1} = B_{n2}$ and $J_{n1} = J_{n2}$ must be satisfied, which also requires the propagation speeds v_1 and v_2 to adopt a mean value v_{12} .

The average v_{12} of multiple speeds in respect to a fixed displacement x_{12} - and hence the proportional skin depth $\delta_{\alpha 12}$ - are generally calculated by the *harmonic mean*:

$$\frac{1}{v_{12}} = \frac{1}{2} \left(\frac{1}{v_1} + \frac{1}{v_2} \right) \xrightarrow{v \sim \delta_\alpha} \frac{1}{\delta_{\alpha 12}} = \frac{1}{2} \left(\frac{1}{\delta_{\alpha 1}} + \frac{1}{\delta_{\alpha 2}} \right) \quad (8)$$

at the boundary $z = 0$. This relationship was verified by FEA in Seifert, 2014 (Fig. 7) for arbitrary materials with the electromagnetic properties κ_x and μ_x . For the wave propagation constant α , proportional to the reciprocal of δ_α , follows the *arithmetic mean*:

$$\alpha = \frac{\sqrt{2j}}{\delta_\alpha} \Rightarrow \alpha_{12} = \frac{1}{2} (\alpha_1 + \alpha_2) . \quad (9)$$

In the context of magnetic actuators and bearings the direct boundary between two core materials is usually not of interest. Rather the transition between two materials bridging an air gap is relevant (Fig. 8), like in the underlying case of a magnetic thrust bearing consisting of a SMC stator paired with a steel-made thrust disk. It can be assumed that the flux takes the shortest path over the air gap between both stator paths. Consequently it flows solely in axial direction and the normal component of the flux density B_n is constant. Therefore, one can assume that above relations remain valid despite the air gap.

Fig. 7 Exemplary eddy current density J inside a cylindrical iron core of two different arbitrary materials for the hypothetical case of a direct boundary without air gap

Fig. 8 Exemplary eddy current density J inside a cylindrical iron core (of different material combinations: $\kappa_{\text{SMC}} \ll \kappa_{\text{Steel}}$) with air gap surrounded by a solenoidal excitation coil (w = number of turns, current i_x of arbitrary frequency). Core skin depth δ_α : equal for each material in all three cases. Air gap skin depth: δ_β depends on both materials.

By calculating the air gap wave propagation constant β by its known formula (Tab. 1) and inserting it into Eq. (9) it becomes apparent that indeed the air gap g appears on both sides of the equation and β_{12} persists independently of it:

$$\beta = \sqrt{\frac{2\alpha}{g\mu_r}} = \sqrt{\frac{2\mu_0}{g}} \sqrt{\frac{j\omega\kappa}{\mu_0\mu_r}} \Rightarrow \alpha = \frac{\beta^2 \cdot g\mu_r}{2} \xrightarrow{\text{into Eq. (9)}} \beta_{12}^2\mu_{r12} = \frac{1}{2}(\beta_1^2\mu_{r1} + \beta_2^2\mu_{r2}). \quad (10)$$

Furthermore a dimensional analysis of the air gap propagation constant $[\beta] = 1/\text{m}$ suggests the relation

$$\beta \sim \frac{1}{\delta_\beta} \quad \text{and therefore} \quad \delta_\beta \sim \sqrt[4]{\frac{\mu}{\kappa}} \quad \text{and} \quad \delta_\beta = \frac{c}{\beta} \quad \text{with} \quad c = \sqrt[8]{-2}. \quad (11)$$

The novel definition of the proportionality constant c was found empirically by a FEA (e. g. Fig. 9, Seifert, 2014) with no mathematical proof yet. However, it appears to be exact for high frequencies leading to the case of convergence in Fig. 5c, matching the general validity of the reluctance model.

Fig. 9 Skin depth of current density ($\delta = r(J = J_0/e) \Rightarrow$ contour line $J = J_0/e = J(r_2)/e$) in stator and air gap ($g = 0.5 \text{ mm}$) for an arbitrary frequency f_x , permeability μ_x and conductivity κ_x where it applies $\delta_\alpha \sim \sqrt{\mu\kappa}$ and $\delta_\beta \sim \sqrt[4]{\mu/\kappa}$
a) if $\kappa = \text{const.} \Rightarrow \delta_\alpha = f(1/\mu)$, $\delta_\beta = f(\mu)$ b) if $\mu = \text{const.} \Rightarrow \delta_\alpha = f(1/\kappa)$, $\delta_\beta = f(1/\kappa)$ c) if $\mu/\kappa = \text{const.} \Rightarrow \delta_\beta = \text{const.}$

The *average permeability* μ_{r12} is a mathematical auxiliary quantity which can be calculated recursively from the special case in Fig. 9c. It becomes apparent that no matter how much the core skin depths $\delta_{\alpha1}$ and $\delta_{\alpha2}$ diverge, the air gap skin depth δ_β is equal for any material combination as long as the proportion μ/κ is equal for both materials, which can be used as boundary condition

$$\delta_{\beta1} = \delta_{\beta2} = \delta_{\beta12} \Rightarrow \frac{c}{\beta_1} = \frac{c}{\beta_2} = \frac{c}{\beta_{12}} \Rightarrow \beta_{12}^2 = \beta_1^2 = \beta_2^2 = \beta^2 \quad (12)$$

to obtain the solution for μ_{r12} from Eq. (10), leading to the arithmetic mean of μ_{r1} and μ_{r2} :

$$\beta^2\mu_{r12} = \frac{1}{2}(\beta^2\mu_{r1} + \beta^2\mu_{r2}) \Rightarrow \mu_{r12} = \frac{1}{2}(\mu_{r1} + \mu_{r2}) \quad (13)$$

Finally for the *asymmetrical air gap wave propagation constant* β_{12} we can write

$$\beta_{12} = \sqrt{\frac{2 \cdot \alpha_{12}}{g \cdot \mu_{r12}}} = \sqrt{\frac{2 \cdot (\alpha_1 + \alpha_2)}{g \cdot (\mu_{r1} + \mu_{r2})}} = \sqrt{\frac{\beta_1^2 \cdot \mu_{r1} + \beta_2^2 \cdot \mu_{r2}}{\mu_{r1} + \mu_{r2}}}. \quad (14)$$

The *asymmetrical air gap skin depth* δ_{12} relates to the symmetrical cases as follows:

$$\frac{\mu_{r1} + \mu_{r2}}{\delta_{\beta12}^2} = \frac{1}{2} \left(\frac{\mu_{r1}}{\delta_{\beta1}^2} + \frac{\mu_{r2}}{\delta_{\beta2}^2} \right). \quad (15)$$

Figure 10 shows that the reluctance R_{mgi} for the asymmetric case fits well in-between the symmetric cases. The maximum error compared to the FEA is only 4 % for the latter and 2 % for the first case. It arises due to the overlap of R_{mgi} and R_{mai} close to the respective threshold frequencies as explained in Fig. 5. Obviously for the asymmetric case the error is halved, as the overlap just occurs on one side of the air gap for a certain frequency.

Fig. 10 Bode plot of the air gap reluctances and adjacent axial core reluctances $G(j\omega) = 20 \cdot \lg(|R_{mgi}(j\omega) + 2R_{mai}(j\omega)| \cdot V_s/A)$ for the three cases, f_{mgi} : threshold frequencies of air gap reluctances R_{mgi}
Example geometry (compare Fig. 8): $r_1 = 22.5$ mm, $r_2 = 42$ mm, $a_1 = a_2 = 11$ mm, air gap: $g = 0.5$ mm

2.2. Consideration of corner reluctances

The corner reluctances R_{mk} (Fig. 1b) were usually neglected (Zhu et al., 2005; Sun et al., 2009) because of their low impact in case of high permeable steel cores. But for SMC stators with $\mu_r < 600$, disregarding them will cause an error of up to 6.3 % of the stationary value of the total effective reluctance R_{meff} , which should be avoided. Due to the appearance of elliptic integrals there is no analytical field solution of the diffusion equation (2) for this geometry and other approaches (Flax et al., 1966) are too cumbersome to be used in this case. However, FEA have shown that the determination of the stationary value at $f = 0$ of R_{mk} is sufficient as illustrated in Fig. 13. Indeed the effective reluctance would rise for high frequencies due to the skin effect, but on the other hand the flux path becomes shorter so both effects cancel each other out for common corner geometries. In this section two approaches to determine the stationary values of the corner reluctances are presented (Fig. 11). The first approximates the corners by a toroid with a quarter-elliptic cross-section, the second assumes folded linear flux paths.

Fig. 11 Simulated flux lines for low frequencies and assumptions for the approximation of the stator corners with toroidal (a) or linear (b) flux paths

a) Stationary reluctance of a toroid with quarter-elliptic cross-section

As a starting point of the derivation serves a toroid with circular cross-section (Fig. 12a), where the local radius ρ is assumed to be constant: $\rho = r_o - r_i$. Here, r_o and r_i correspond to the outer and inner radii of the overall bearing geometry.

$$R_{mko} : \rho = \rho_o = r_4 - r_3 \stackrel{!}{=} b \text{ with } r = r_o = r_3 \quad R_{mki} : \rho = \rho_i = r_2 - r_1 \stackrel{!}{=} b \text{ with } r = r_i = r_2 \quad (16)$$

Fig. 12 Geometric derivation of elliptical approach

The cross-section $A(\rho, \vartheta)$ permeated by the magnetic flux Φ is not constant, as it has the shape of a circular ring at the boundary $A(\rho, \vartheta = 0)$ connected to core sections with axially directed flux paths (R_{mao} , R_{mai}), but is rectangular-shaped at the boundary $A(\rho, \vartheta = \pi/2)$ connected to core sections with radially directed flux paths (R_{mr}). Considering the whole interval $\vartheta \in [0, \pi]$ leads to the geometric analogy of a truncated cone (Fig. 12c) with the lateral surface:

$$A(\rho, \vartheta) = 2\pi r \cdot \rho + \pi \rho^2 \cdot \cos(\vartheta) \implies A(\rho, \vartheta = 0) = \pi(r + \rho)^2 - \pi r^2 \quad \text{and} \quad A(\rho, \vartheta = \frac{\pi}{2}) = 2\pi r \cdot \rho. \quad (17)$$

In case of a toroid with a quarter-elliptic cross-section, the local radius ρ varies with ϑ and the corner reluctance R_{mk} can be obtained by integration of infinitesimally small reluctance elements of length ∂l :

$$R_{mk} = \int_{\vartheta} \partial R_{mk} \quad \text{with} \quad \partial R_{mk} = \frac{\partial l}{\mu_0 \mu_r A(\vartheta)} = \frac{1}{\mu_0 \mu_r} \cdot \frac{\frac{1}{2} \rho(\vartheta) \cdot \partial \vartheta}{2\pi r \cdot \rho(\vartheta) + \pi \rho(\vartheta)^2 \cdot \cos(\vartheta)}. \quad (18)$$

However, the integral can only be solved for the simple cases of a constant $\rho(\vartheta) = \rho_o$ in the interval $\vartheta \in [0, \pi/2]$ and $\rho(\vartheta) = \rho_i$ in $\vartheta \in [\pi/2, \pi]$ leading to

$$R_{mko}(\rho_o) = \frac{1/\pi \mu_0 \mu_r}{\sqrt{4r^2 - \rho_o^2}} \cdot \arctan \frac{2r - \rho_o}{\sqrt{4r^2 - \rho_o^2}} \quad \text{and} \quad R_{mki}(\rho_i) = \frac{1/\pi \mu_0 \mu_r}{\sqrt{4r^2 - \rho_i^2}} \cdot \arctan \frac{2r + \rho_i}{\sqrt{4r^2 - \rho_i^2}}. \quad (19)$$

The elliptic representation of $\rho(\vartheta)$

$$\rho(\vartheta) = \sqrt{b^2 \cdot \sin^2 \vartheta + \rho^2 \cdot \cos^2 \vartheta} \quad (20)$$

has no analytical solution but can be approximated by an arithmetic series as visualized in Fig. 12b:

$$R_{mk} = \frac{k_{corr}}{\pi \mu_0 \mu_r} \sum_{n=1}^N \left[\frac{1}{\sqrt{4r^2 - \rho_n^2}} \arctan \frac{(2r - \rho_n) \cdot \tan\left(\frac{\vartheta}{2}\right)}{\sqrt{4r^2 - \rho_n^2}} \right]_{n_0 + \frac{n-1}{N}}^{n_0 + \frac{n}{N}} \quad \text{with} \quad \begin{cases} R_{mko} : n_0 = 0, r = r_3 \\ R_{mki} : n_0 = 1, r = r_2 \end{cases} \quad (21)$$

and

$$\rho_n = \sqrt{b^2 \sin^2 \vartheta_n + r^2 \cos^2 \vartheta_n}, \quad \vartheta_n = \frac{\pi}{2} \left(n_0 + \frac{n - \frac{1}{2}}{N} \right), \quad k_{corr} = 0.72. \quad (22)$$

The arithmetic series converge and the limit can be approximated numerically for sufficiently large N .

In the real magnetic core the flux path is shortened compared to an ideal quarter-ellipse (Fig. 11a), which requires the consideration of a correction factor k_{corr} determined by FEA (Fig. 13). There is no analytical proof known for k_{corr} , but in case of common magnetic bearing geometries with sufficiently large a and $\Delta h = (r_3 - r_2)$ as well as $\rho/3 < b < 3\rho$ it can be assumed to be an adequate approximation.

b) Linear approximation: Expansion of adjacent geometries

In the stationary case described a homogeneous field distribution can be assumed and the shortest possible, average flux path crosses the corner elements, linearly connecting the centerlines of the respective adjacent core elements. Shown in Fig. 11a, the adjacent core parts next to every corner element can be extended by a quarter of the corresponding corner dimension leading to simplified expressions for the corner reluctances R_{mk} :

$$R_{mko} = \frac{\ln \frac{3r_3 + r_4}{4r_3}}{2\pi\mu_0\mu_r b} + \frac{b}{4\mu_0\mu_r \cdot \pi(r_4^2 - r_3^2)} \quad \text{and} \quad R_{mki} = \frac{\ln \frac{4r_2}{3r_2 + r_1}}{2\pi\mu_0\mu_r b} + \frac{b}{4\mu_0\mu_r \cdot \pi(r_2^2 - r_1^2)}. \quad (23)$$

In spite of the simplifications the linear approximation agrees well with the results of the FEA (Fig. 13). No common correction factor k_{corr} can be found, as the approximation errors differ for R_{mko} and R_{mki} .

c) Comparison

Both approaches yield satisfying results with approximation errors of less than 2 %, favored by the coexistence of both, outer and inner corner reluctances, whose approximation errors almost cancel each other out. The phase shift φ of the sum of all corner reluctances ΣR_{mk} persists at less than 1° and is negligible. Which approach to prefer highly depends on the geometric dimensions of the magnetic actuator or bearing: for compact geometries with dominant corners the elliptic representation will lead to more exact results while the linear approximation is advisable in case of long axial $a \gg b$ and radial $\Delta h \gg h$ core elements. Although in that latter case the impact of the corner reluctances is comparatively small.

Fig. 13 Absolute values and phase shift of the effective corner reluctances $R_{mk}(j\omega)$ for both approximations compared to the FEA-results, underlying geometry equals test setup of Fig. 1

3. Conclusion and outlook

The established analytical models for electromagnetic actuators and magnetic thrust bearings are not applicable, if multiple materials are employed within a single magnetic circuit, like in the case of a SMC-made stator and a steel-made thrust disk as shown in Fig. 14. In this paper the air gap reluctance network by Zhu et al. leading to R_{mgi} is generalized and now covers the case of different materials on each side of the air gap. Over a wide frequency range the reluctance model agrees well with the FEA (Fig. 14) and there occur only minor deviations in the mid-frequency range around f_{mgi} due to the unavoidable overlap of part reluctances.

For high permeable materials ($\mu_r > 1000$) the reluctances of the stator corners can be neglected as done in previous works. However, the application of SMC ($\mu_r = 435$) leads to errors of the stationary value $R_{\text{meff}}(0 \text{ Hz})$ of up to 6.3 %. The presented consideration of the corner reluctances R_{mk} can reduce that error to negligible 0.3 %.

Having a closer look at the frequency response (Fig. 14) of the total effective reluctance $R_{\text{meff}}(j\omega)$ an unusual gradient of 10 dB/decade for high frequencies becomes apparent, indicating a *half-order system* $G(s^{1/2})$, which was expected as all reluctances R_{mi} in Tab. 1 are proportional to the wave propagation constant α and the square root of the complex frequency $\sqrt{j\omega}$. Our future goals are aimed at showing how *fractional-order* systems, controllers and estimators can be used for a highly dynamic magnet bearing control.

Fig. 14 Logarithmic magnitude plot $G_{\text{eff}}(j\omega) = 20 \cdot \lg(|R_{\text{meff}}(j\omega)| \cdot \text{Vs/A})$ and absolute values $|R_{\text{meff}}(j\omega)|$ of the total effective reluctance of the magnetic circuit of the test setup geometry; FEA with neglected saturation and leakage fluxes compared to the completed analytical model and the previous model of Zhu, 2005 (only applicable in case of single core material); f_{mgi} : threshold frequency of air gap reluctance R_{mgi}

A major drawback of all of the current actuator and bearing models is that saturation is not taken into account. Although low saturation can be easily achieved by design for magnetic thrust bearings, the skin effect drives the core surface - especially of the steel-made thrust disk - into saturation (Fig. 15). Other than global saturation caused by static bias fluxes, this so called local saturation also provokes the main flux to bridge the thrust disk on the whole. Both effects are highly frequency dependent and correlated, which makes them challenging to model. Rodriguez et al. (2007) present an approach to include global saturation effects in the frequency domain for cylindrical iron cores based on the Modified Scalar Preisach Model. In future works we intend to show how their model can be adapted for magnetic thrust bearings and for the more complex case of local saturation. Sun et al. (2009) already expanded the reluctance network of Zhu et al. (2005) by parallel leakage reluctances, which can be included into the model presented here as a next step.

Fig. 15 Flux density distribution showing local saturation in thrust disk for SMC and STEEL and for the dynamic case 1 ms after 48-V-step, when coil reaches rated current (bias flux fully compensates control flux on right stator side)

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