Control of an AMB to zero static force

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Abstract

Some active magnetic bearing (AMB) applications require that the mean, or static force exerted by the AMB is zero. Examples include hybrid fluid film – AMB bearing pairings, rotor midspan dampers, and statically indeterminate systems with stiff rotors and more than two radial AMBs. When the position of the rotor relative to magnetic center is not known precisely, this zero force requirement can be hard to meet. A low frequency periodic biasing scheme is developed which enables a controller to detect non-zero static force by sensing rotor motion at the bias carrier frequency. A theoretical basis establishes feasibility but also the potential for coupling between control signals and the periodic bias. Simulation demonstrates that an ad-hoc bias adaptation scheme can successfully drive the static component to zero while permitting otherwise conventional control. Experimental results on a small, single DOF test rig further demonstrate the ability to drive the AMB static force to zero while adding damping to the system. Identification of the system stability boundary provides insight on the structural requirements (mechanical stiffness versus magnetic negative stiffness) and the limitations on the control gain for the biasing scheme to work

Keywords : Periodic control, Biasing, AMB damper, Zero static force, Bias linearization

1. Introduction

In general, it is difficult to know the actual static force applied by an active magnetic bearing (AMB) because of uncertainty in shaft position relative to the magnetic center of the AMB. In many applications, this is not a problem because the AMBs are meant to carry the static loads applied to the rotor and the number of AMBs is such that this support is statically determinate. So if the rotor is properly controlled, it is obvious that the AMBs are providing the correct static load and it may also be possible to estimate this load with high confidence. However, some applications of active magnetic bearings require load sharing between the AMB and an adjacent mechanical bearing (Dimitri et al., 2014). In such combination structures, it may be very difficult to ensure that the two adjacent bearings are not applying large opposing forces at zero frequency which produce no net force on the rotor but use up bearing load capacity – the load sharing problem. In other applications, such as magnetic dampers (Kasarda et al., 1989), a specific objective of the control is to ensure zero static force and this may be important either to avoid unintended rotor stress or to conserve AMB load capacity. In yet other applications, more than two AMBs may be applied to a relatively rigid rotor and, as with the mechanical load sharing problem, there is potential for un-constructive competition amongst the bearings (Knospe et al., 1997). A common feature of all of these applications is that, if any one AMB in the system were to be turned off, the rotor would achieve some specific and acceptable equilibrium.

In the open literature where zero static force from the AMB is desired, this goal is approximated by ensuring that the currents applied to opposing AMB quadrants are symmetric about a common fixed value. If the equilibrium position (with AMB turned off) were exactly the magnetic center of the AMB, then this would lead to zero static force from the AMB. However, as mentioned above and discussed in more detail below, it is more common that the mechanical equilibrium is neither at the magnetic center of the AMB nor is it easy to know the extent to which it is different. As a result, to achieve zero static force both the magnetic and mechanical equilibrium positions must be preserved while the AMB is active. The originality of this paper is to address this gap in the literature. The goal of the present work is to estimate the static force response of each AMB in a system and, more specifically, to drive this static force to zero so that, even with the AMB turned on, the non-AMB equilibrium position is preserved.

2. Background

Most active magnetic bearings employ some form of bias linearization. This means that opposing magnetic sectors have currents controlled according to a rule similar to

$$I_{x,1} = I_b + i_c(t) \quad I_{x,2} = I_b - i_c(t) \quad (1)$$

in which I_b is some constant bias current and i_c is a symmetric perturbation applied to realize the bearing force (Schweitzer 2009). The subscript x refers to a direction of force application and the subscripts 1 and 2 denote the two opposing magnetic sectors. In the simplest approximation, the resulting behavior of the bearing is described by

$$f_x(t) = f_{x,0}(I_b, x_0) - k_x(I_b, x_0)(x(t) - x_0) + k_i(I_b, x_0)i_c(t) \quad (2)$$

Here, $f_{x,0}$ is the static force while coefficients k_x and k_i are the open-loop or magnetic stiffness of the sector pair and the actuator gain, respectively. All three are, as shown here, implicitly functions not only of the bias current, I_b , but also of the equilibrium rotor position, x_0 . Even with the very best characterization of these terms, fundamental uncertainty in x_0 can lead to substantial uncertainty in the average value of f_x . In the classes of application considered here, this static position under which the static bearing force should be zero can change with time and machine operating conditions, so there is a need to be able to continuously monitor and regulate the static force.

Fundamentally, this uncertainty in static force is created by uncertainty in the rotor position relative to the magnet array effective magnetic center – the rotor position where equal currents in all coils leads to no net force. In AMBs with conventional bias, offset of the rotor position leads in an obvious way to a static force equal to the position offset times the negative magnetic stiffness associated with the biasing. AMBs with no bias that are able to adequately function activating only one of a pair of opposing coils at a time are still susceptible to an uncertain static force. This can be seen by assuming

$$f = \lambda_2 i_2^2 - \lambda_1 i_1^2 \quad (3)$$

in which rotor offset from magnetic center leads to $\lambda_1 \neq \lambda_2$. For such a bearing, if we assume for the sake of argument that the two currents can approximate

$$i_1 = \sqrt{\max(0, A \sin \omega t)} \quad i_2 = \sqrt{\max(0, -A \sin \omega t)} \quad (4)$$

then the force history is

$$f(t) = \lambda_2 \max(0, -A \sin \omega t) - \lambda_1 \max(0, A \sin \omega t) \quad (5)$$

If $\lambda_1 = \lambda_2 = \lambda$ then the force is just $f(t) = -\lambda A \sin \omega t$. But if $\lambda_1 \neq \lambda_2$, then the average force may be computed to be

$$\bar{f} = -\frac{\omega}{2\pi} \int_0^{2\pi/\omega} [\lambda_1 \max(0, A \sin \omega t) - \lambda_2 \max(0, -A \sin \omega t)] dt = -\frac{A}{\pi}(\lambda_1 - \lambda_2) \neq 0 \quad (6)$$

Thus, even absent a static biasing scheme, rotor offset relative to magnetic center leads to a non-zero static force.

3. Approach

To get around the problem of uncertain static load with uncertain rotor–magnetic center misalignment, consider the consequences of using a bias which is a combination of constant and periodic (sinusoidal) terms:

$$I_{x,1}(t) = (1 + \beta)(I_{b,0} + I_{b,s} \sin \omega_b t + i_p(t)) \quad I_{x,2}(t) = (1 - \beta)(I_{b,0} + I_{b,s} \sin \omega_b t - i_p(t)) \quad (7)$$

Here, the nominal constant bias is $I_{b,0}$ but a periodic component with amplitude $I_{b,s}$ and frequency ω_b is added. The relative magnitudes of these persistent biases are not equal between the two sectors: β defines the relative magnitude. As before, active control will be accomplished by manipulating the now–potentially asymmetric perturbation current, $(1 \pm \beta)i_p(t)$. Further, assume that the position of the rotor relative to the magnetic center may be described by knowing the effective air gap lengths, g_1 and g_2 , associated with the two opposing sectors so that without loss of generality,

$$g_1 = g_0(1 + \gamma) \quad \text{and} \quad g_2 = g_0(1 - \gamma) \quad (8)$$

in which the instantaneous journal position relative to magnetic center is $x = g_0\gamma$. Obviously, uncertainty in nominal position x_0 translates to corresponding uncertainty in γ .

With these assumptions, we can compute the force generated by an opposed pair of magnets:

$$f_x(t) = -\lambda \left(\frac{I_1^2}{g_1^2} - \frac{I_2^2}{g_2^2} \right) = -\lambda \frac{(1+\beta)^2 (I_{b,0} + I_{b,s} \sin \omega_b t + i_p(t))^2}{g_0^2 (1+\gamma(t))^2} + \lambda \frac{(1-\beta)^2 (I_{b,0} + I_{b,s} \sin \omega_b t - i_p(t))^2}{g_0^2 (1-\gamma(t))^2}$$

$$= -\frac{4\lambda}{g_0^2 (1-\gamma^2)^2} \left((1-\gamma\beta)(\beta-\gamma)(i_b^2 + i_p^2) + (1+\beta^2 - 4\gamma\beta + \gamma^2 + \beta^2\gamma^2) i_b i_p \right) \quad (9)$$

in which λ is a magnetic constant that depends on pole area and coil turns and $i_b(t) \equiv I_{b,0} + I_{b,s} \sin \omega_b t$. In the case that $i_p = 0$, it is clear that the force goes to zero when $\beta \rightarrow \gamma$. Further, if it is possible to achieve $\beta = \gamma$ then

$$f(\beta = \gamma) = -\frac{4\lambda}{g_0^2} i_b(t) i_p(t) \quad (10)$$

Define an alternate control variable $i_c(t)$ through

$$i_p(t) = \frac{I_{b,0}}{i_b(t)} i_c(t) = \frac{I_{b,0}}{I_{b,0} + I_{b,s} \sin \omega_b t} i_c(t) \quad (11)$$

to produce

$$f(\beta = \gamma) = -\frac{4\lambda I_{b,0}}{g_0^2} i_c(t) \quad (12)$$

so that the signal to force relationship $i_c \rightarrow f$ is linear and time invariant. The alternate control variable $i_c(t)$ simplifies interaction between the plant with the periodic bias control scheme by making the plant appear to be linear time invariant from the perspective of a controller acting through i_c , at least under the simplification that $\beta = \gamma$. In this way, it is possible to work with other LTI control schemes while producing minimal harmonic distortion. Note that, as long as $I_{b,s} < I_{b,0}$, then the inverse required by Eq. (11) always exists and only depends on known parameters.

The value of this result is that, if $i_c(t)$ has no components at zero frequency or at ω_b , then any force applied by the AMB at zero frequency will be matched by a component at a frequency of ω_b with a proportional magnitude. With this, the control strategy to achieve zero static force is to detect rotor response at ω_b using a fixed frequency high-Q filter and drive this response to zero by choice of β . Because the response at ω_b (i.e.: the signed Fourier response component) is monotonic in β , the control process is a simple search on β . Beyond this, the remainder of control of the AMB is as with any conventional AMB: the force is linear in $i_c(t)$ so conventional control algorithms will work as expected with the caveat that they must be conditioned to have a zero average value and to avoid the frequency ω_b .

In the present work, the heuristically justified control for β starts by generating the convolution signal $c(t)$:

$$c(t) = \int_{t-T}^t (x(\tau) + x_0) \sin \omega_b \tau \, d\tau \quad : \quad T = \frac{2N\pi}{\omega_b} \quad (13)$$

in which N is some positive integer on the order of 5. This signal does not capture the phase of the displacement relative to the bias perturbation signal: only the in-phase component is measured. In a more general development, it may be necessary to measure the phase but in the present work, the frequency ω_b is low enough that the mechanical response of the suspended mass is essentially in-phase with the perturbation induced by the periodic bias. If a higher frequency were used for ω_b , then it might become important to track the phase and use this information in controlling β .

With this signal in hand, evolution of β is controlled by

$$\beta(t) = \int_0^t K_\beta c(\tau) \, d\tau \quad (14)$$

Here, the coefficient K_β is a tunable control parameter. Given the highly nonlinear nature of this problem, proof of convergence of $\beta \rightarrow \gamma$ is non-trivial and beyond the scope of the present work but, as will be illustrated in the sequel, this control is sufficient for at least some range of the system parameters. The full system is depicted in Fig. 1.

Of course, this time-invariant linearization assumes that it is actually possible to ensure that $\beta = \gamma$ but since γ reflects the rotor position and is therefore time varying, it is prudent to assume that this is only an approximation and will affect the actual performance achieved. In light of this limitation, the ensuing simulations and experimental investigation will explore the consequences of this inevitable coupling.

Figure 1: System block diagram. Numbers in parentheses refer to equations defining the block action.

4. Simulation

The first simulation explores the control of β when the AMB must cooperate with an adjacent mechanical bearing, as indicated in Fig. 2. The adjacent mechanical bearing is indicated by the spring and dashpot in the diagram and by the parameters K_m (stiffness) and C (damping). In this simulation, the choice is made that $i_p(t) = 0$ so the only objective is to ensure that the AMB applies no force to the rotor even though it is biased (gap fluxes are not zero) and the rotor is not located at the magnetic center. Of course, in the simulation, the control mechanism for achieving $\beta = \gamma$ has no intrinsic knowledge of γ but, rather, only sees the output of a convolution of x with $\sin \omega_b t$: $c(t)$ as defined by Eq. (13). Lacking conventional feedback control, this process is only stable as long as there is something in the mechanical structure to counteract the destabilizing influence of what is ordinarily referred to as the magnetic stiffness, or K_x . To simulate a hybrid fluid film system with a rotor offset from magnetic center, as shown in Fig. 2, the following mechanical suspension parameters are used: a mass, $m = 0.5$ kg is connected to a spring, $K_m = 3$ kN/mm and damper, $C = 7.75$ N-sec/mm. The nominal air gap is 0.5 mm and the spring (K_m) equilibrium offset is 0.115 mm relative to magnetic center. The magnetic stiffness at the mechanical equilibrium position is $K_x = -1.34$ kN/mm so it is a little more than 1/3 of the mechanical stiffness. The β convergence control parameter was chosen through multiple trials to be $K_\beta = 30 \text{ mm}^{-1} \text{ sec}^{-2}$.

The nominal bias is $I_{b,0}$ is 2.6 amps and the periodic bias is at 5 Hz with amplitude $I_{b,s} = 0.26$ amps. The bias frequency was chosen to satisfy the tradeoff between signal separation and convergence. At 5 Hz, ω_b is well separated from the expected range of the control signal and is the lowest frequency that provides a reasonable convergence rate. The bias perturbation frequency, ω_b , can be understood as a sensor carrier frequency so that the effective bandwidth of the static load estimation process is always less than half of ω_b . Thus, choosing a very low ω_b will slow down the estimation process while choosing a very high ω_b will push this perturbation into the expected spectrum of the external excitations and make it more difficult to distinguish system response to the periodic bias from system response to external excitation. For the present system, 5 Hz appears to be a good compromise.

Figure 2: Mechanical model for the simulation. The springs and dashpots represent a mechanical support adjacent to the magnetic bearing, with which the AMB must cooperate. Notice that the spring equilibrium position due to the spring does not coincide with the magnetic center of the opposed electromagnets.

The results are illustrated in Fig. 3. Control of β first turns on at $t = 1.2$ seconds and it is seen that the value of β rises to a peak at about 3.3 seconds before settling in to a very accurate estimate of the equilibrium value of $\gamma = 0.23$. Although the coil currents are persistent and substantial on both sides, the net magnetic force and associated mechanical force both go to zero as desired.

Figure 3: Simulation results with only bias rebalancing control.

Figure 4: Simulation results with bias rebalancing and active control at 11 Hz. β was initialized at its steady state value.

The second simulation explores the effectiveness of the control signal, $i_p(t)$. In this simulation, β is initialized at a value leading to equilibrium with $i_p = 0$. A sinusoidal signal is then injected so that $i_p(t) = a \sin \omega t$ with ω chosen to be other than the periodic bias frequency of 5 Hz. A typical example is illustrated in Figs. 4 and 5 which illustrate the response at 11 Hz. First, it is clear that the rotor motion tracks the 11 Hz control signal. This is especially evident in the response spectrum presented in Fig. 5 which shows a substantial peak at 11 Hz. Beyond this, it is evident that there is some response

Figure 5: Spectrum of mass displacement with both bias rebalancing and active control at 11 Hz.

at the sum and difference between the target frequency and the carrier: the peaks at 6 Hz and at 16 Hz. Presumably, these are consequences of coupling that arises because the β -adaptation is not perfect and is not able to perfectly maintain $\beta = \gamma$ when the rotor is moving. The β -trace in Fig. 4 shows that β has residual (although small) oscillation in response to this perturbation in i_c and also shows a slight shift in equilibrium position relative to the unperturbed value of 0.23.

The spectrum in Fig. 5 also shows that the displacement has a 1 Hz component related to β . Harmonic fluctuations due to periodic bias control cannot be completely masked because the masking only works perfectly when $\beta = \gamma$. Otherwise, all the other terms in Eq. (9) persist and produce harmonic and subharmonic distortion. We expect sub- and super-harmonics to be present in β as β has to fluctuate as it attempts to track γ . That said, the responses at off-frequencies are quite small relative to the intended response so this coupling may be acceptable in practical use.

5. Experimental Demonstration

To demonstrate this control strategy, a very simple single-DOF magnetic bearing based on a very simple magnetically actuated rocking beam apparatus (Knospe, 2000) is modified by adding a spring between the rocking beam and ground as illustrated in Fig. 6. The goal is to control the AMB in such a way that, regardless of the pre-load position of the spring,

Figure 6: Rocking beam apparatus with electromagnets and preloaded spring.

the AMB will not deflect the spring. This may sound simple, but quick analysis and simple experiments will reveal that, unless it is possible to actually measure the spring deflection (not the same as measuring beam position), it is not possible to control a conventional AMB to ensure that this deflection is actually zero. The feedback control implemented is a simple damping mechanism and the experiments show that the resulting level of damping can be controlled in a predictable manner while ensuring that the static AMB force (indicated by spring strain) stays zero for any spring preload position.

The single-DOF magnetic bearing system used in the experiment is similar to the system simulated in Section 4 in that both examine the response of a mass with a mechanical spring attached, but they are different in that the parameters of the simulation (mass, stiffness, etc.) are different from those of the experiment. As such, the simulation was meant to explore specific aspects of the proposed control approach that are hard to examine in experiment while the experiment stands alone as a physical proof of concept.

Figure 7 shows convergence of the bias balance adaptation. At $t = 0$, the rotor is in its mechanical equilibrium position with the AMB turned off. The AMB is turned on with β initialized to zero and the data show the adaptation process as β approaches 0.141 as well as the recovery of rotor position to its mechanical equilibrium as indicated by the convolution signal going to zero.

Figure 7: Experimental demonstration of bias balance adaptation. The adaptation process only has access to the convolution of the position sensor signal with the reference sine wave so it does not know what the equilibrium value of position or of β should be. Convergence is achieved when the convolution signal goes to zero, indicating no rocker response at the bias frequency of 5 Hz.

Figures 8 through 11 illustrate the impact response of the system with various kinds of AMB control. In each case, the rocker mass is struck with a plastic mallet and the resulting motion recorded. In Fig. 8, the AMB is inactive so the rocker mass and the spring control the dynamics. There is some structural damping, but it is light so the oscillation decays slowly. In Fig. 9, the AMB is active (periodic bias is activated) but its control is configured to do nothing. What is apparent is that the response to the mallet hit is essentially the same as with the AMB turned off. That is, the static position has not changed nor has the damping or natural frequency appreciably changed. In Fig. 10, the AMB is active and its control is designed to generate damping ($K_d s / (\tau s + 1)$). The damping coefficient is set to a moderate level of 0.01 units with $\tau = 0.001$ seconds and the response is seen to be much more heavily damped than without the AMB but to exhibit some oscillation after the impulse. In Fig. 11, the AMB is again active with damping control but here the damping coefficient has been increased by a factor of three to 0.03 units. The response now shows no appreciable oscillation, clearly demonstrating a heavily overdamped behavior. All four responses show an equilibrium position of approximately $-18\mu\text{m}$, although the actual equilibrium is harder to determine in Figures 8 and 9 because the systems are very lightly damped so the position in “steady state” shows slight oscillation due to environmental excitation.

Figure 8: Experimental displacement response to a strike of a plastic mallet. The AMB is not active so damping is quite light.

Figure 9: Experimental displacement response to a strike of a plastic mallet. The AMB is active but damping is set to zero: response is the same as with the AMB inactive.

Figure 10: Experimental displacement response to a strike of a plastic mallet. The AMB is active and damping is moderate (0.01 units).

Figure 11: Experimental displacement response to a strike of a plastic mallet. The AMB is active and damping is high (0.03 units).

6. Convergence

Because the AMB control specifically precludes control action at zero frequency, it is to be expected (and is found to be the case) that the static instability associated with nearly all AMB acts as a limiting factor for this approach. Specifically, the net stiffness which is the sum of the mechanical stiffness and the magnetic open-loop stiffness is expected to need to be positive. The magnetic open-loop stiffness can be found by conventional analysis of (9). In some sense, it must be ensured that the net stiffness of the system is positive throughout the motion of the journal during the convergence of β . This draws attention, at a minimum, in the magnetic open-loop stiffness at the beginning of adaptation where $\beta = 0$ and at the end where $\beta = \gamma$. Noting that $x = g_0\gamma \Rightarrow \gamma = x/g_0$:

$$K_{x,0} \equiv -\left.\frac{df_x}{dx}\right|_{\beta=i_p=0} = -\frac{4\lambda i_b^2(1+3\gamma^2)}{g_0^3(1-\gamma^2)^3} \quad \text{and} \quad K_{x,\gamma} \equiv -\left.\frac{df_x}{dx}\right|_{\beta=\gamma,i_p=0} = -\frac{4\lambda i_b^2}{g_0^3(1-\gamma^2)} \quad (15)$$

These measures of stability are related to the AMB load capacity. Referring to (9), the load capacity f_{sat} may be defined by the force generated by the bearing when the journal is centered (some designers prefer to use a radial offset equal to the touch-down bearing clearance in a direction opposite to the force generating poles) and one pole set is driven to magnetic saturation. Referring to the current that achieves saturation as I_{sat} ,

$$f_{\text{sat}} = \lambda \frac{I_{\text{sat}}^2}{g_0^2} \quad (16)$$

Further, assume that the ratio between the nominal bias and the saturation currents is

$$\eta \equiv \frac{I_{b,0}}{I_{\text{sat}}} : 0 < \eta < 1 \quad (17)$$

and common practice dictates that, typically, $0.3 \leq \eta \leq 0.5$. With this,

$$f_{\text{sat}} = \frac{\lambda I_{b,0}^2}{\eta^2 g_0^2} \quad (18)$$

Given the nonlinear nature of the system with β adaptation active, determining a hard bound on stable limits to λ and x_0 is non-trivial. To investigate this, the system discussed in Section 4 was explored to determine some limits. Using the same mechanical and electrical parameters as previously, λ was varied from 1.0 N-mm²/amp² to 11.6 N-mm²/amp². For each value of λ , the mechanical misalignment x_0 was increased from 0 to the largest value for which the β convergence process could converge. For each combination of λ and x_0 , the control gain K_β in (14) was increased from zero to the highest value for which β converged. In this manner, a stability boundary was determined for the specific mechanical system described in Section 4. The results are presented in Table 1 along with the magnetic open-loop stiffnesses discussed above.

Table 1: Stability boundary at $K_m = 3000$ N/mm

λ N mm ² /amp ²	$\lambda I_{b,0}^2 / 0.5^2 g_0^2$ N	$\lambda I_{b,0}^2 / 0.3^2 g_0^2$ N	$x_{0,\text{max}}$ mm	$K_{\beta,\text{max}}$ mm ⁻¹ sec ⁻²	$K_{x,0}$ N/mm	$K_{x,\gamma}$ N/mm
1	108.2	300.4	0.220	0.11	-652.11	-268.25
2	216.3	600.9	0.197	0.05	-1051.88	-512.14
4	432.6	1201.8	0.145	0.02	-1410.33	-944.73
6	649.0	1802.7	0.076	0.007	-1483.42	-1327.8
8	865.3	2403.6	0.026	0.007	-1757.74	-1753.25
10	1081.6	3004.4	0.009	0.006	-2167.41	-2163.90
11.6	1254.7	3485.2	0.001	0.00021	-2509.37	-2509.33

What is immediately apparent from Table 1 is that the open-loop magnetic stiffness does, as expected, need to be smaller in magnitude than the mechanical / structural stiffness. More importantly, the ideal boundary of $K_x = -K_m$ is only approximately realized for systems with large λ (high load capacity) which can tolerate only a very small mechanical misalignment.

At relatively low values of λ where a larger x_0 is tolerated, the actual limitation relates to the touch-down bearing (modeled in the simulation as a rigid contact at $x = \pm 0.5g_0$ with a coefficient of restitution of 0.5). In these cases, if the initial average magnetic force (with β initialized at 0) is large enough to push the rotor onto the touch-down bearing, then the rotor's motion doesn't include a sinusoidal component at the interrogation frequency. As a consequence, $c(t) = 0$ in (14) and β never evolves. This can be alleviated by initializing β closer to the true value and there are likely heuristic strategies for accomplishing this by detecting that the rotor is in contact with the touchdown bearing ($|c| < \text{some small number}$ and $|x| > 0.95x_{tdb}$ where x_{tdb} is the touch-down bearing clearance) and simply searching β to find any value for which the contact is eliminated.

At relatively high values of λ , a plot of the instantaneous value of K_x throughout the trajectory shows that this value very closely approaches $-K_m$ instantaneously. Figure 12 illustrates a typical trajectory for $\lambda = 10$ N mm²/amp² and $x_0 = 0.009$ mm. The persistent sinusoidal variation in K_x after convergence of β is due entirely to the sinusoidal biasing term.

Doubling K_m results in a stability boundary essentially identical to that of Table 1 but with the values of λ doubled, as might be expected. Figure 13 plots the limiting value of mechanical offset for stability *versus* the scaled ratio of λ/K_m , illustrating that the dependence is essentially the same for $K_m = 3000$ N/mm and $K_m = 6000$ N/mm. This result is

Figure 12: Trajectories of β and K_x for $\lambda = 6$ and $x_0 = 0.055$. Note that K_x dips briefly below $-3000 = -K_m$.

not a proof of the boundary scaling with K_m but is strongly suggestive of such a conclusion for some range of physical parameters.

Table 1 illustrates that, in order to tolerate a large potential mechanical misalignment x_0 , the control gain K_β needs to be “small.” This raises the question of how small this value is relative to the gain that would be stabilizing for smaller x_0 and what effect this has on rate of convergence. In particular, since it is assumed to be impossible to know x_0 *a priori*, the actual value of K_β can’t be any larger than the value that stabilizes the largest tolerable x_0 . This introduces a level of conservatism and the cost of the conservatism will be time to converge.

To explore this issue, consider the case where $\lambda = 6$. This is a good compromise design for the case presented in Section 4 because the mechanical spring force for that system with a mechanical deflection on the order of the touch-down bearing clearance (0.25 mm) would be 750 N. Choosing a biasing ratio of $\eta = 0.5$ produces an AMB with a load capacity of 649 N, comparable to the mechanical force range. This system was simulated as discussed above and the time to bring β to within two percent of steady state was determined for a range of x_0 from 0.01 mm to the 0.076 mm tolerable limit with K_β ranging from 0.0005 to 0.007.

Figure 13: Limiting mechanical offset, x_0 , versus $\lambda i_b^2/g_0^2/K_m$ for $K_m = 3000$ N/mm and $K_m = 6000$ N/mm.

Figure 14: Settling time versus x_0 for $\lambda = 6$ and K_β ranging from 0.0005 to 0.007 $\text{mm}^{-1}\text{-sec}^{-2}$.

Figure 14 illustrates that, at least for $\lambda = 6$, the settling time is not strongly dependent on initial offset and that there is an optimal value for this particular combination of parameters: K_β gives the lowest settling times for values between 0.002 and 0.003. This is significant in that it implies that tuning the controller to manage the largest potential mechanical misalignments will not come at the expense of poor performance with smaller actual misalignments.

7. Conclusions

The work presented here clearly demonstrates a method to enable an AMB to adapt its bias distribution so as to produce no static force even when the static position of the rotor relative to the AMB magnetic center is unknown. The method exploits adding a low frequency periodic component to the biasing field and detecting rotor response at the same frequency to determine whether or not the AMB is exerting force at the carrier frequency and, by implication, a static force. The relative amplitude of bias in opposing quadrants of the AMB is then adjusted to bring the response at the carrier frequency to zero and this achieves the desired zero static force. Further, it is possible and reasonably simple to invert the periodic actuator gain induced by this periodic bias and the resulting AMB may be used in the same way that a conventionally biased AMB is used with the caveat that the control gain must go to zero at zero frequency.

A parametric study was conducted to identify the capture boundary for the mechanical system described in Section 4 with a mechanical stiffness of 3000 N/mm. The results provide insight to the structural requirements (mechanical stiffness versus magnetic negative stiffness) for the biasing algorithm to work and also to control gain limitations. The bias adaptation is only stable when the mechanical stiffness of the structure is greater than the magnitude of the negative magnetic stiffness of the AMB. Because the bias is periodic, the magnetic stiffness is also periodic, so convergence does not preclude the magnitude of the negative magnetic stiffness from periodically surpassing the mechanical stiffness. The amplitude of the oscillation can be reduced by decreasing the periodic component of the bias, but the trade-off is a weakened control signal and slower convergence. Exploration of the control gain limit with respect to mechanical misalignment (x_0) was achieved through simulation of the mechanical system of Section 4 for a $\lambda - K_m$ pair. The simulation showed that settling time is not strongly dependent on initial offset, which is significant as it implies that tuning the controller to manage the largest potential mechanical misalignments will not result in poor performance with smaller misalignments.

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References

- Dimitri, A. S., Mahfoud, J., and El-Shafei, A., Oil whip elimination using fuzzy logic controller, *Journal of Engineering for Gas Turbines and Power*, Vol. 138, No. 6 (2016), 062502.
- Kasarda, M. E. F., et al., A magnetic damper for first mode vibration reduction in multimass flexible rotors, *ASME J. Eng. Gas Turbines Power*, Vol. 112 (1990), pp. 463–469.
- Knospe, Carl R., Roger L. Fittro, and L. Scott Stephens, Control of a high speed machining spindle via μ -synthesis, in *Proceedings of the 1997 IEEE International Conference on Control Applications*, pp. 912–917.
- Knospe, C.R., The nonlinear control benchmark experiment, in *Proceedings of the American Control Conference*, Vol. 3, 2000, pp. 2134–2138.
- Schweitzer, G. and Maslen, E. (ed.), *Magnetic Bearings Theory, Design, and Application to Rotating Machinery*, Springer, 2009, pp. 79–81.