Limitation of estimation method of surface texture parameters by inverse analysis of hammering test data with disturbance

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Abstract
The surface texture formed by the micro asperities on the surface of the solid members induces the stiffness reduction of the structure constructed by screws and bolts. In recent years, the fixing by screws and bolts is one of the primary fixing methods to construct multi-material structures, and the fixing state such as the interfacial stiffness at the clamped part is required to be evaluated for the proper fixing. The interfacial stiffness affects the total stiffness of the structure and is dominated by the surface texture and the clamping force. Especially, the standard deviation of the asperity peak heights and the homogenized elastic modulus collectively named as the surface texture parameters mainly determine the interfacial stiffness. This study has developed an estimation method of the surface texture parameters by the inverse analysis of the hammering test data, and numerically investigated the basic behavior and the estimation error of the inverse analysis for the establishment of the methodology to find the limitation of the proposed method against the disturbance in the hammering test data. In the investigation, a theoretical limitation of the inverse analysis has been derived by the linear approximation, and a numerical simulation which is the same approach as the Monte Carlo simulation has been performed by using a bolted joint model consisting of two plates joined by the eight sets of the bolt and nut. From the results of the numerical simulation, the followings were summarized: 1) The recalculation error of the natural frequency subjected to the clamping force is as large as the disturbance in the hammering test data. 2) The sensitivity of the homogenized elastic modulus is much smaller than that of the standard deviation of the asperity peak heights in the inverse analysis. 3) The theoretical limitation of the linear approximation can qualitatively predict the estimation errors of the surface texture parameters.

Keywords: Inverse problem, Finite element method, Homogenization method, Surface texture, Natural frequency, Fixing, Tribology, Surface roughness, Interfacial stiffness, Multi-material structures

1. Introduction
Surface texture, or surface topography is formed by quite small asperities on surface of solid. The surface texture dominates the wear characteristics and the interfacial stiffness at the interface between the solid members constructing mechanical structures. Many researchers have discussed the contact mechanics and the mathematical models that describe the wear characteristics and the interfacial stiffness (Greenwood and Williamson, 1966; Greenwood and Tripp, 1970; Olofsson, 1995; Björklund, 1997; Olofsson and Hagman, 1997; Hagman and Olofsson, 1998). Especially regarding the interfacial stiffness, the surface texture at the clamped part affects the total stiffness of the structure constructed by screws and bolts. Since the interfacial stiffness is lower than the elastic modulus of the base material, the natural frequency of the structure with bolted joints is generally lower than that of the structure without bolted joints.

In recent years, the fixing by screws and bolts is one of the primary fixing methods to construct multi-material structures, and the fixing state such as the interfacial stiffness at the clamped part is required to be evaluated for the proper fixing (Catalanotti et al., 2011; Anami, 2016; Lambiase et al., 2016; Hirsch et al., 2017). One of the simple techniques is the hammering test in which the loosened screw or bolt is detected by the change of the vibration after hammering the
target. Similar techniques have been developed, and most of the techniques seem to be based on the comparison with the measurement data and the calculation data of some reference states (Nichols et al., 2006; He and Zhu, 2011; Zhu and He, 2013; Hammami et al., 2016).

On the contrary, some studies have clarified the mechanism that the surface texture and the clamping force dominate the interfacial stiffness (Ito et al., 1977; Koizumi et al., 1978; Ono et al., 1991; Yamamoto et al., 2008). For that matter, the surface texture and the clamping force determine the real contact area of the interface. The large real contact area given by the smooth surface and the high clamping force generates the high interfacial stiffness. In this connection, one of the authors of this paper developed an estimation method of the natural frequency of the structure with bolted joints based on the Hertz-Mindlin theory (Kishimoto and Endo, 2007). In the estimation method, the natural frequency is determined by the compressive stress on the interface at the clamped part and the surface texture parameters; the standard deviation of the asperity peak heights and the homogenized elastic modulus. The homogenized elastic modulus is the product of the standard deviation of the asperity peak heights, the number of the asperity peaks per unit area, the mean radius of the curvature at the asperity peaks and the elastic modulus of the base material. Once the surface texture parameters are obtained, the relationship between the clamping force and the natural frequency of the arbitrary structure can be estimated.

The surface texture parameters are usually obtained by the 3D surface measurement in advance. Some of the conventional approaches to obtain the surface texture parameters are the curve fittings applied on the surface texture (Nayak, 1971; Sakaguchi et al., 1999; Noro et al., 2006). However, the 3D measurement with a spatial resolution of a micron is required in the measurement of the surface texture, and the measurable area is limited. Our previous research (Kishimoto, et al., 2017) focused on the surface texture parameters (the standard deviation of the asperity peak heights and the homogenized elastic modulus) that determine the interfacial stiffness, and proposed the estimation method of the surface texture parameters by the inverse analysis of the dynamic characteristic of the specimen in the hammering test. The specimen consisted of two plates joined by bolts, nuts and washers. The proposed method evaluates the surface texture of the interface of the plates in terms of the Hertz-Mindlin theory by using the natural frequency of the specimen and the clamping force of the bolts and nuts. Thus, the proposed method has an advantage that the mean values of the surface texture parameters can be obtained regardless of the size of the target surface. The previous research clarified that the proposed method could be workable in the actual measurements and the estimation accuracy of the proposed method was comparable to that of the 3D surface texture measurement.

On the other hand, the estimation accuracy of the surface texture parameters decreases with the decrease of the measurement accuracy of the clamping force and the natural frequency. The limitation of the proposed method such as the allowable amount of the disturbance in the hammering test data has been little investigated, but it is necessary to guarantee the estimation result against the disturbance. In the field of inverse problems, the theoretical limitation of the estimation accuracy is determined by the condition number of the coefficient matrix (The Japan Society of Mechanical Engineers ed., 1991; Kubo, 1992). This theorem is limited to the linear inverse problem in which the relationship between the unknown values and the measured values is linear. The limitation of the estimation accuracy in the non-linear inverse problem such as the proposed estimation method is still concerned.

The present study discusses a methodology to find the limitation of the proposed method to estimate the surface texture parameters by the inverse analysis of the hammering test data with the disturbance. A theoretical limitation of the inverse analysis has been derived by the linear approximation, and a numerical simulation which is the same approach as the Monte Carlo simulation has been performed by using a bolted joint model consisting of two plates joined by the eight sets of the bolt and nut. The theoretical limitation provides the maximum estimation error without repeated computations such as the Monte Carlo simulation even in the case the configuration of the analysis object is changed. In the calculation process of the theoretical limitation, the major factor of the estimation error is also clarified.

In the numerical simulation, the model is assumed to consist of two steel plates, or one aluminum alloy plate and one steel plate. The latter case is a kind of the multi-material structures. The correct values of the surface texture parameters of the interface of the plates was given in advance, and the direct analysis was applied to obtain the simulation data of the natural frequency subjected to the clamping force of the bolts and nuts. Then the inverse analysis was applied to the simulation data with artificial disturbances. Based on the results of the numerical simulation and the comparison with the theoretical limitation, the estimation error of the proposed method and the effectiveness of the theoretical limitation of the linear approximation have been investigated.
2. Estimation method of surface texture parameters

2.1. Outline

Figure 1 shows the outline of the proposed estimation method. The specimen is hammered and the natural frequency of the specimen is obtained by the acceleration pickup and the Fourier analysis. The finite element (FE) model of the specimen is also prepared by using the interfacial element, and the calculated value of the natural frequency is obtained by the stress analysis and the modal analysis in the finite element method (FEM). The elastic modulus of the interfacial element depends on the surface texture parameters (the standard deviation of the asperity peak heights $\sigma$ and the homogenized elastic modulus $C$).

![Fig. 1 Outline of the proposed method.](image)

The inverse analysis is applied to obtain the most likelihood set of the surface texture parameters ($\sigma$, $C$) that minimizes the residual between the hammering test result and the FEM result. In the inverse analysis, the sample data set of the natural frequency and the clamping force of the bolted joint is necessary. Once the surface texture parameters are obtained, the relationship between the clamping force and the natural frequency of the arbitrary structure in which the interface has the same surface texture can be estimated by the recalculation of the direct analysis. Moreover, in the structure clamped by more than one bolt and nut, the natural frequency can be estimated in advance even if the clamping forces of the bolts and nuts are different each other.

The previous research (Kishimoto and Endo, 2007) has reported that the natural frequency of the monolithic specimen was almost constant with respect to the tightening torque of the bolts and nuts. Therefore, the influence of the other interfaces such as the interface between the bolt and the washer can be ignored in the case that the bolts, the nuts and the washers are small in comparison with the specimen size.

2.2. Interfacial element

Figure 2 illustrates the contact model of nominally flat surfaces and the interfacial element based on the contact model. As shown in the left of Fig. 2, the piece 1 and the piece 2 are joined by the compressive stress $p_z$. $z$-axis is defined along the normal direction of the interface of the pieces. $x$-axis and $y$-axis are defined to be normal to each other as shown in Fig. 2. The contact model describes the contacts of the spherical asperities shown in the middle of Fig. 2. The total height of the asperities is $L$ and it is equal to the thickness of the interfacial element. The variable parameter $d$ is the gap between the mean planes of the surface topography formed on the surface of the pieces. The interfacial element shown in the right of Fig. 2 simulates the elasticity of the asperities contact. The elastic modulus of the interfacial element $E_z$, $G_{yz}$ and $G_{zx}$ are derived by the following process, and the elastic modulus on the other directions is set to zero.

Assuming that the asperity peak heights obeys the Gaussian distribution and the contact of asperities is the Hertzian contact, the relationship between the nominal compressive stress $p_z$ and the gap $d$ in the static equilibrium state is described by using the homogenized elastic modulus $C$ as follows (Greenwood and Tripp, 1970).

$$p_z = CF_1^2(d)$$

(1)

where

$$F_m(d) = \frac{1}{\sqrt{2\pi}} \int_0^\infty z^n \exp \left[ -\frac{1}{2} \left( z + \frac{d}{\sigma} \right)^2 \right] dz$$

(2)
σ = \sqrt{\sigma_1^2 + \sigma_2^2} \tag{3}

Using the subscript \( i \) as the identifier for the piece \( i (= 1, 2) \), \( \sigma_i \) is the standard deviation of the asperity peak heights of the piece \( i \). The homogenized elastic modulus \( C \) is described as follows.

\[
C = \frac{64\pi\sigma^2\eta_1\eta_2}{15} \left( \frac{1}{\beta_1^2} + \frac{1}{\beta_2^2} \right) \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1} \tag{4}
\]

where \( \eta \) is the number of the asperity peaks per unit area, and \( \beta \) is the mean radius of the curvature at the asperity peaks. \( E_i \) and \( \nu_i \) are the Young’s modulus and the Poisson’s ratio of the base material of the piece \( i \), respectively. In Eq. (3) and Eq. (4), the standard deviation \( \sigma \) and the homogenized elastic modulus \( C \) depend on the surface topography of the interface and the material constants of the pieces, and are independent of the stress state. Therefore, the parameters \( \sigma \) and \( C \) are assumed to be constant and collectively named as the surface texture parameters.

The interfacial stiffness under the compressive stress \( p_z \) is approximated by using the interfacial element whose thickness is \( L \). Assuming that the small displacement occurs in the vibration of the specimen, the \( z \)-direction Young’s modulus of the interfacial element is described as follows.

\[
E_z = \begin{cases} 
\frac{\partial p_z}{\partial d} L = \frac{F_z(d)}{F_z^2(d)} + \frac{d}{\sigma} \frac{p_z L}{\sigma} & (p_z \geq 0) \\
0 & (p_z < 0) 
\end{cases} \tag{5}
\]

In Eq. (5), the interface is assumed to have a gap and the interfacial stiffness is zero if the compressive stress is negative.

Similarly, Björklund (1997) derived the relationship between the shear stress and the displacement tangential to the interface under the compressive stress \( p_z \) based on the Mindlin’s theory (Mindlin, 1949; Mindlin et al., 1952). Assuming that the stiffness in the tangential direction to the interface is isotropic, the shear modulus is described as follows.

\[
G_{yz} = G_{zx} = 4 \left( 1 - \frac{\nu_1^2}{E_1} + 1 - \frac{\nu_2^2}{E_2} \right) \left( 2 - \frac{\nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \right)^{-1} E_z \tag{6}
\]

where \( G_{yz} \) is the shear modulus of the interfacial element at the \( y \)-surface in the \( z \)-direction, and \( G_{zx} \) is the shear modulus of the interfacial element at the \( x \)-surface in the \( z \)-direction. \( G_i (i = 1, 2) \) is the shear modulus of the base material of the piece \( i \).

### 2.3. Inverse analysis

Focusing on Eq. (1) and Eq. (5), the Young’s modulus \( E_z \) depends on the compressive stress \( p_z \) and the surface texture parameters \( (\sigma, C) \) by solving Eq. (1) for the gap \( d \). The compressive stress \( p_z \) can be calculated by the stress analysis using the FEM if the clamping force of the bolted joint is given. Although the elastic modulus of the interfacial element \( (E_z, G_{yz}, G_{zx}) \) is undetermined in the stress analysis, the interfacial element is approximated to be the isotropic material whose Young’s modulus \( E \) and the Poisson’s ratio \( \nu \) are given as follows.

\[
E = \left( \frac{1}{E_1} + \frac{1}{E_2} \right)^{-1} \quad \text{and} \quad \nu = \frac{\nu_1 + \nu_2}{2} \tag{7}
\]
On the other hand, the natural frequency of the specimen depends on only the Young’s modulus $E_z$ in the FEM because of Eq. (6). Therefore, the problem setting is that the parameters $\sigma$ and $C$ are estimated from the data set of the clamping force and the natural frequency.

Using the subscript $j$ as the identifier for the data set $j (=1, 2, \ldots, n)$, where $n$ is the total number of the data set, the natural frequency of the $j$-th data set in the hammering test is denoted by $f_j^{\text{EXP}}$ and the natural frequency of the $j$-th data set in the FEM is denoted by $f_j^{\text{FEM}}(\sigma, C)$. First, the solution space $(\sigma, C)$ is meshed by the 2D finite elements, and the natural frequency at each node $f_j^{\text{FEM}}(\sigma_k, C_k)$ is calculated. The subscript $k$ is the identifier for the $k$-th node of the meshed solution space. Using the superscript $(l)$ as the identifier for the $l$-th element of the meshed solution space, the natural frequency $f_j^{\text{FEM}}(\sigma, C)$ at the arbitrary $(\sigma, C)$ in the $l$-th element is interpolated with the 2D piecewise polynomial interpolation which is the well known interpolation method used in the FEM as follows.

$$f_j^{\text{FEM}}(\sigma, C) = \sum_k N_k^0(\xi^0, \eta^0) \times f_j^{\text{FEM}}(\sigma_k, C_k) \quad (k \in l \text{-th element}) \quad (8)$$

where $N_k^0(\xi^0, \eta^0)$ is the shape function of the $l$-th element. $\xi^0$ and $\eta^0$ are the local coordinates in the element. If the finite element for the solution space is isoparametric,

$$\sigma = \sum_k N_k^0(\xi^0, \eta^0) \times \sigma_k \quad \text{and} \quad C = \sum_k N_k^0(\xi^0, \eta^0) \times C_k \quad (k \in l \text{-th element}) \quad (9)$$

Equation (9) can be solved for the local coordinates $(\xi^0, \eta^0)$ by the Newton-Raphson method.

Then the coordinate $(\sigma, C)$ that minimizes the following residual sum of square $\Pi(\sigma, C)$ is searched by the downhill simplex method (Nelder and Mead, 1965).

$$\Pi(\sigma, C) = \begin{cases} \sum_{j=1}^{n} (f_j^{\text{EXP}} - f_j^{\text{FEM}}(\sigma, C))^2 & (\text{inside meshed solution space}) \\ \text{penalty} & (\text{outside meshed solution space}) \end{cases} \quad (10)$$

where $f_j^{\text{FEM}}(\sigma, C)$ is calculated by using Eq. (8). penalty is a large value for the penalty in the case of that the $(\sigma, C)$ is located outside the meshed solution space in the process of the downhill simplex method. Since the residual sum of square $\Pi(\sigma, C)$ at each node can be calculated before the operation of the downhill simplex method, the coordinate of the node $(\sigma_k, C_k)$ that minimizes the residual sum of square $\Pi(\sigma_k, C_k)$ is set to the initial.

2.4. Theoretical limitation of the inverse analysis by linear approximation

The exact value of the natural frequency of the $j$-th data set for the true values of the surface texture parameters ($\sigma^{\text{TRUE}}, C^{\text{TRUE}}$) is denoted by $f_j^{\text{EXA}}$, and the vectors $[f]$ and $[g]$ are defined as follows.

$$[f] = \begin{bmatrix} f_1^{\text{EXA}} \\ f_2^{\text{EXA}} \\ \vdots \\ f_n^{\text{EXA}} \end{bmatrix} \quad \text{and} \quad [g] = \begin{bmatrix} \log_{10}(\sigma^{\text{TRUE}}) \\ \log_{10}(C^{\text{TRUE}}) \end{bmatrix} \quad (11)$$

where the components of the vector $[g]$ are defined to be the log of the surface texture parameters because the surface texture parameters must be positive values. The relationship between the natural frequency and the surface texture parameters are linearly approximated as follows.

$$[f] \approx [A][g] + [b] \quad (12)$$

where $[A]$ is the coefficient matrix and $[b]$ is the intercept vector. The components of the matrix $[A]$ and the vector $[b]$ are invariable. The least squares solution of Eq. (12) is described as follows.

$$[g] = ([A]^T[A])^{-1}[A]^T([f] - [b]) \quad (13)$$

where the superscript $T$ denotes matrix transposition. If the disturbance of the natural frequency $[\Delta f]$ and the estimation error of the surface texture parameters $[\Delta g]$ are induced, Eq. (13) is

$$[g] + [\Delta g] = ([A]^T[A])^{-1}[A]^T([f] + [\Delta f] - [b]) \quad (14)$$
and the difference of Eq. (13) and Eq. (14) is as follows.

$$[\Delta g] = ([A]^T[A])^{-1}A^T[\Delta f]$$

Equation (15) is written for each component as follows.

$$\begin{align*}
\begin{bmatrix} \log_{10}(\sigma^{TRUE} + \Delta \sigma) - \log_{10}(\sigma^{TRUE}) \\ \log_{10}(C^{TRUE} + \Delta C) - \log_{10}(C^{TRUE}) \end{bmatrix} &= \begin{bmatrix} a_{x1}^* & a_{x2}^* & \cdots & a_{x_n}^* \\ a_{c1}^* & a_{c2}^* & \cdots & a_{c_n}^* \end{bmatrix} \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix}
\end{align*}$$

where $\Delta \sigma$ and $\Delta C$ are the estimation errors of the surface texture parameters, and $\Delta f_i$ is the disturbance of the natural frequency. If the maximum amplitude of the disturbance of the natural frequency is merely denoted by $|\Delta f|$,.

$$\begin{align*}
\begin{bmatrix} \log_{10}(1 + \Delta \sigma) - \log_{10}(\sigma^{TRUE}) \\ \log_{10}(1 + \Delta C) - \log_{10}(C^{TRUE}) \end{bmatrix} &\leq \begin{bmatrix} |a_{x1}^*| ||\Delta f| + |a_{x2}^*| ||\Delta f| + \cdots + |a_{x_n}^*| ||\Delta f| \\ |a_{c1}^*| ||\Delta f| + |a_{c2}^*| ||\Delta f| + \cdots + |a_{c_n}^*| ||\Delta f| \end{bmatrix} = \sum_{j=1}^{n} |a_{x_j}^*| ||\Delta f| \end{align*}$$

Therefore,

$$|\Delta \sigma| \leq \left(10^{5\sigma_{x1}^*} |a_{x_j}^*| ||\Delta f| - 1\right) \times \sigma^{TRUE} \quad \text{and} \quad |\Delta C| \leq \left(10^{5\sigma_{c1}^*} |a_{c_j}^*| ||\Delta f| - 1\right) \times C^{TRUE}$$

Because of Eq. (18), the coefficients $a_{x_j}^*$ and $a_{c_j}^*$ provide the enlargement ratio of the estimation errors $|\Delta \sigma|$ and $|\Delta C|$.

3. Analysis condition

3.1. Analysis object

Figure 3 shows the schematic and the FE model of the analysis object. The analysis object simulated the two plates joined by the eight sets of M6 bolt, nut and washer. The constituent material of the base plate was assumed to be steel or aluminum alloy. The constituent materials of the attached plate, the bolts, the nuts and the washers were assumed to be steel. The surface roughness of the interface between the base plate and the attached plate was given by the maximum height roughness of the surface profile $R_z$. The maximum height roughness $R_z$ was set to be $25 \sim 12.5 \mu m$ (usual finishing), $6.3 \sim 3.2 \mu m$ (fine finishing) or $0.8 \mu m$ (mirror finishing).

The FE computation was executed by the self-produced software written in C++. The FE model consisted of the 8-node hexahedral isoparametric elements and the 6-node triangular prism isoparametric elements. The numbers of the nodes and the elements in the FE modeling were 22314 and 18944 in total, respectively. The interfacial elements were deployed at the interface of the base plate and the attached plate. The thickness of the interfacial element $L$ was set to 1 mm. Although this value was not exact in terms of the total height of the asperities on the specimen surfaces, this
approximation was enough for the specimen in this verification according to the previous research (Kishimoto and Endo, 2007). The results in the case when \( L \) was set to the other value are shown in the appendix of this paper. Moreover, the bolt axis in the FE model was also elongated by 1 mm, but this influence could be ignored. The other interfaces and the screw thread of the bolts were also ignored. Specifically, the interfaces in contact with the bolts, the nuts and the washers were formed by the common nodes. The shape of the M6 bolt axis was the 6 mm diameter solid shaft in the FE model.

Table 1 and Table 2 show the details of the parameters set in the analysis object. The six cases shown in Table 2 were executed. The true values of the surface texture parameters \((\sigma_{TRUE}^{TRUE}, C_{TRUE}^{TRUE})\) in this numerical simulation were set by reference to our previous research (Kishimoto et al., 2017) as shown in Table 3. Although the surface texture parameters shown in Table 3 are the values of the plate made of alloy tool steel SKS93, the steel plates and the aluminum alloy plates were assumed to have the same surface texture.

<table>
<thead>
<tr>
<th>Analysis case</th>
<th>Base plate (R_z) [(\mu m)]</th>
<th>Attached plate (R_z) [(\mu m)]</th>
<th>(\sigma_{TRUE}) [(\mu m)]</th>
<th>(C_{TRUE}) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(St-St)-Usual</td>
<td>Steel 25 (\sim) 12.5</td>
<td>Steel 25 (\sim) 12.5</td>
<td>5.600</td>
<td>440.7</td>
</tr>
<tr>
<td>(St-St)-Fine</td>
<td>Steel 6.3 (\sim) 3.2</td>
<td>Steel 6.3 (\sim) 3.2</td>
<td>1.303</td>
<td>55.5</td>
</tr>
<tr>
<td>(St-St)-Mirror</td>
<td>Steel 0.8</td>
<td>Steel 0.8</td>
<td>0.527</td>
<td>3.0</td>
</tr>
<tr>
<td>(Al-St)-Usual</td>
<td>Aluminum alloy 25 (\sim) 12.5</td>
<td>Steel 25 (\sim) 12.5</td>
<td>5.600</td>
<td>220.3</td>
</tr>
<tr>
<td>(Al-St)-Fine</td>
<td>Aluminum alloy 6.3 (\sim) 3.2</td>
<td>Steel 6.3 (\sim) 3.2</td>
<td>1.303</td>
<td>27.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum height roughness, (R_z) [(\mu m)]</th>
<th>Member</th>
<th>Number of asperity peaks per unit area, (\eta_i) [(mm^2)]</th>
<th>Mean radius of curvature at asperity peak, (\beta_i) [(mm)]</th>
<th>Standard deviation of asperity peak heights, (\sigma_i) [(\mu m)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 (\sim) 12.5 (\text{(Usual finishing)})</td>
<td>Base plate</td>
<td>9.35</td>
<td>2.93</td>
<td>4.07</td>
</tr>
<tr>
<td></td>
<td>Attached plate</td>
<td>10.52</td>
<td>2.55</td>
<td>3.01</td>
</tr>
<tr>
<td>6.3 (\sim) 3.2 (\text{(Fine finishing)})</td>
<td>Base plate</td>
<td>16.60</td>
<td>3.27</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Attached plate</td>
<td>18.63</td>
<td>2.88</td>
<td>1.03</td>
</tr>
<tr>
<td>0.8 (\text{(Mirror finishing)})</td>
<td>Base plate</td>
<td>12.06</td>
<td>2.35</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Attached plate</td>
<td>18.75</td>
<td>2.55</td>
<td>0.18</td>
</tr>
</tbody>
</table>

### 3.2. Direct analysis

Figure 4 shows the details of the interface in the stress analysis. In the stress analysis to calculate the compressive stress \(p_z\), the element of the bolt axis added for inserting the interfacial element were removed, and the tensile load was applied on the divided surfaces. The tensile load of the bolt axis is equal to the clamping force \(P\). The tensile load per unit area \(p_b\) was calculated as follows.

\[
p_b = \frac{4P}{\pi d_z^2} \tag{19}
\]

where \(d_z\) is the nominal diameter of the bolt (6 mm). The same clamping force was applied on all of the bolts. The clamping force of each bolt was set to be \(P = 0.01 \sim 15\) kN. The maximum value of the clamping force (15 kN) is almost equal to the standard clamping force of the class 12.9 bolt. The class 12.9 means the strength class of the bolt in which the tensile strength is 1200 MPa and the yield stress is 1080 (= 1200 \(\times\) 0.9) MPa in JIS B 1051. After the stress analysis, the modal shape and the natural frequency subjected to the clamping force was obtained by the modal analysis.

### 3.3. Inverse analysis

Figure 5 shows the FE meshed solution space in the inverse analysis. The surface texture parameters \(\log_{10}(\sigma [\mu m])\) and \(\log_{10}(C [\text{MPa}])\) were allocated to the axes. The mesh width was set to 0.5 and the search range was determined as \(-3 \leq \log_{10}(\sigma [\mu m]) \leq 3\) and \(-3 \leq \log_{10}(C [\text{MPa}]) \leq 6\). The natural frequency at each node \(f_j^{FEM}(\varepsilon_i, C_i)\) was rounded
off to the third decimal place in unit of Hertz, taking account of the practical application. penalty in Eq. (10) was set to $10^{12}$. The data set of the clamping force and the natural frequency can be arbitrarily selected in the inverse analysis. This study selected the 1st natural frequencies (the natural frequencies of the 1st vibration mode) when the clamping force $P = 5, 10$ and $15$ kN. The total number of the data set was $n = 3$. In the case (St-St)-Usual, the 1st natural frequencies when the clamping force $P = 3, 4$ and $5$ kN were also selected in order to investigate the behavior of the proposed method when the low class bolts were used.

The disturbance of the hammering test data would appear in both of the natural frequency and the clamping force in the actual situation. However, the disturbance was assumed to appear in only the natural frequency for the simplification of the problem. This assumption also means that the disturbance of the clamping force was included in the disturbance of the natural frequency. The disturbance of the natural frequency was given as the Gaussian distribution whose standard deviation was $\sigma_f$. The value of the disturbance was given as the following geometrical progression.

$$\sigma_f = 10^{i/5}$$

(20)

where the unit of the standard deviation $\sigma_f$ in Eq. (20) is Hertz, and $i$ is integer number. The value of $\sigma_f$ was set to $10^{-4}$ ~ $10^2$ Hz ($i = -20, -19, -18, \ldots, 10$).

The concrete calculation process in this numerical simulation are as follows. The following calculation for each value of the standard deviation $\sigma_f$ was repeated 1000 times.

First, the 1st natural frequencies computed by the direct analysis with the true values of the surface texture parameters $(\sigma^{TRUE}, C^{TRUE})$ and the clamping force $P = 5, 10, 15$ kN (or $P = 3, 4, 5$ kN in the case (St-St)-Usual) were respectively treated as the exact values of the natural frequencies $(f_{EXA}^1, f_{EXA}^2, f_{EXA}^3)$. The exact values $(f_{EXA}^1, f_{EXA}^2, f_{EXA}^3)$ with the disturbance were respectively treated as the simulated measurement values $(f_{EXP}^1, f_{EXP}^2, f_{EXP}^3)$. The disturbance $|\Delta f|$ was calculated by the following equation.

$$|\Delta f| = \frac{1}{3} \sum_{j=1}^{3} |f_{EXP}^j - f_{EXA}^j|$$

(21)

Second, the simulated measurement values $(f_{EXP}^1, f_{EXP}^2, f_{EXP}^3)$ were substituted into Eq. (10), and the estimation results of the surface texture parameters $(\sigma^{EST}, C^{EST})$ were obtained. The estimation errors $|\Delta \sigma|$ and $|\Delta C|$ were calculated by the following equations.

$$|\Delta \sigma| = |\sigma^{EST} - \sigma^{TRUE}|$$

(22)

$$|\Delta C| = |C^{EST} - C^{TRUE}|$$

(23)

Third, the estimated curve by the recalculation of the direct analysis, i.e. the 1st natural frequency for the estimated surface texture parameters $(\sigma^{EST}, C^{EST})$ was computed by the direct analysis. Then the recalculation error was calculated by the following equation.

$$|\Delta f_{re}| = \frac{1}{15 \text{ [kN]} - 0.01 \text{ [kN]}} \int_{0.01 \text{ [kN]}}^{15 \text{ [kN]}} \left| f^{EST} - f^{EXA} \right| dP$$

(24)
where \( f^{EST} \) and \( f^{EXA} \) are the 1st natural frequency for the surface texture parameters \((\sigma^{EST}, C^{EST})\) and \((\sigma^{TRUE}, C^{TRUE})\), respectively. The integral computation in Eq. (24) was based on the trapezoidal integration.

4. Results and discussion

4.1. Direct analysis

Figure 6 shows the example of the deformation shape of the 1st vibration mode of the analysis object. As shown in Fig. 6, the deformation shape of the 1st vibration mode was the plate bending deformation. All of the deformation shape of the 1st vibration mode of the analysis object were almost the same shape.

Figure 7 shows the 1st natural frequency of the analysis object. In Fig. 7, the calculation result when all of the interfaces are ignored (the analysis object is monolithic) is also shown by the dashed black line. This case is named as the case (St-St)-Monolithic. In the cases taking account of the interfacial stiffness, the natural frequency was obviously lower than that in the case (St-St)-Monolithic. Moreover, the natural frequency increased with the clamping force of the bolted joints, and tended to be high when the surface roughness of the interface was low. The value of the natural frequency of the analysis object was from 600 to 800 Hz, approximately.

The natural frequency when the base plate was the aluminum alloy plate was lower than that when the base plate was the steel plate because the Young’s modulus of the aluminum alloy is lower than that of the steel. The gradient of the natural frequency for the clamping force decreased with the increase of the clamping force. The gradient was high when the clamping force was under 1 kN, and gradually became low when the clamping force was over 1 kN in all of the analysis cases. Although the gradient was relatively low, the relationship between the clamping force and the natural frequency was almost linear when the clamping force was over 5 kN.
4.2. Inverse analysis

Figure 8 shows the examples of the 1st natural frequency with the disturbances ($\sigma_f = 1, 10$ and $100$ Hz) and the recalculation results in the case (St-St)-Usual. The estimated surface texture parameters ($\sigma^{EST}$, $C^{EST}$) in Fig. 8 are shown in Table 4. In Fig. 8, the plots show the input data of the inverse analysis ($f^{EXP}_1, f^{EXP}_2, f^{EXP}_3$). The curves show the recalculation results $f^{EST}$. The exact value of the natural frequency $f^{EXA}$ is also shown by the solid blue curve.

Regardless of the condition of the input data, the recalculation results located at the center of the input data because of the minimization of Eq. (10). Thus, when the disturbance was small, the recalculation results agreed with the exact value $f^{EXA}$. Also the estimated surface texture parameters ($\sigma^{EST}$, $C^{EST}$) had good agreement with the true values ($\sigma^{TRUE}$, $C^{TRUE}$) as shown in Table 4. At $\sigma_f = 100$ Hz, the estimated surface texture parameters and the recalculation results disagreed with the correct values because the input data of the inverse analysis had the large disturbance. Therefore, the estimation error increased with the increase of the disturbance in the input data.

Figures 9 and 10 show the 1st natural frequency of the meshed solution space $f^{EM}(\sigma_f, C_k)$. As shown in these figures, the natural frequency was varied in the log scale of the surface texture parameters ($\sigma, C$), and the relationship between the natural frequency and the surface texture parameters was non-linear. The true values of the surface texture parameters ($\sigma^{TRUE}$, $C^{TRUE}$) located at the filled elements. The least squares method was applied on each filled element, and the fitting planes as shown as the dashed lines in Figs 9 and 10 were obtained.

Table 5 shows the parameters of the fitting planes. Using these parameters as the matrix $[A]$ in Eq. (12), the values of $a_{\sigma_f}$ and $a_{C_j}$ ($j = 1, 2, 3$) shown in Table 6 were obtained by Eq. (15). Hereinafter, the name of the analysis case without the clamping force means that the clamping force $P$ of the input data is $5, 10$ and $15$ kN. As shown in Table 5, the gradient of the fitting planes for $\sigma$ was $-70.50$ to $-24.62$, and that for $C$ was $4.10$ to $8.51$. Thus, the standard deviation of the asperity peak heights $\sigma$ seems to be more sensitive for the natural frequency than the homogenized elastic modulus $C$ in that the absolute value of the gradient means the sensitivity. On the other hand, the sum of the absolute values of $a_{\sigma_f}$ was smaller than that of $a_{C_j}$ as shown in Table 6. It can be presumed from Eq. (18) that the

![Figure 8 Examples of the 1st natural frequency with disturbance and recalculation results in the case (St-St)-Usual. The plots show the input data of the inverse analysis. The curves show the recalculation results by the direct analysis based on the estimated surface texture parameters.](image-url)
Fig. 9 The 1st natural frequency of the meshed solution space $f^{FEM}(\sigma, C)$ when the base plate was the steel plate. The natural frequency was varied in the log scale of the surface texture parameters ($\sigma, C$), and their relationship was non-linear. Each fitting plane was obtained by applying the least squares method on the filled element where the true surface texture parameters ($\sigma^{TRUE}, C^{TRUE}$) located.

Fig. 10 The 1st natural frequency of the meshed solution space $f^{FEM}(\sigma, C)$ when the base plate was the aluminum alloy plate. The distribution profile of the natural frequency was same as Fig. 9, except the natural frequency was lower than that when the base plate was the steel plate.

maximum estimation error of the standard deviation of the asperity peak heights $\sigma$ is smaller than that of the homogenized elastic modulus $C$. By the similar comparison in the case (St-St)-Usual, it is inferred that the maximum estimation errors in the estimation by the data set of the clamping force $P = 5, 10, 15$ kN is lower than that in the estimation by the data set of the clamping force $P = 3, 4, 5$ kN. The difference of the sum of $|\alpha^{*}_f|$ and that of $|\alpha^{*}_C|$ were small among all of the analysis cases except the estimation by the data set of the clamping force $P = 3, 4, 5$ kN.

Figure 11 shows the disturbance $|\Delta f|$ with respect to the standard deviation $\sigma_f$ and the recalculation error $|\Delta f_r|$ with respect to the disturbance $|\Delta f|$. Figure 11 (a) shows the range of the disturbance $|\Delta f|$, and Fig. 11 (b) shows the maximum of the recalculation error $|\Delta f_r|$ in the 1000 computations at each standard deviation $\sigma_f$. Figure 12 shows the maximums of the estimation errors $|\Delta \sigma|$ and $|\Delta C|$ in the 1000 computations with respect to the disturbance $|\Delta f|$. Figures 11 and 12 show the extracted profiles of the results plotted by the 1000 computations. The curves of the theoretical limitation in Fig. 12 were obtained by Table 6 and Eq. (18).
of Hertz, the recalculation error and the estimation errors were not decreased with the decrease of the disturbance when 0 \leq \Delta f < 0.02 to 3 times the standard deviation \sigma_f. The recalculation error \Delta f was as large as the disturbance \Delta f when the disturbance \Delta f > 0.01 Hz.

As shown in Fig. 11 (a), the disturbance \Delta f increased with the increase of the standard deviation \sigma_f and was given as 0.02 to 3 times the standard deviation \sigma_f. On the other hand, as shown in Fig. 11 (b) and Fig. 12, the maximums of the recalculation error \Delta f and the estimation errors |\Delta \sigma_f|, |\Delta C| were constant when the disturbance \Delta f was small (< 10^{-3} Hz). It was mainly caused by the truncation error of the natural frequency at each node \nu_{j,FEM}(\sigma_k, C_k) that was forming the meshed solution space. Because the number of decimals of the natural frequency \nu_{j,FEM}(\sigma_k, C_k) was 3 in unit of Hertz, the recalculation error and the estimation errors were not decreased with the decrease of the disturbance when |\Delta f| < 10^{-3} Hz.

When |\Delta f| > 10^{-3} Hz, the recalculation error and the estimation errors increased with the increase of the disturbance |\Delta f|. The gradient of the recalculation error |\Delta f_{rec}| for the disturbance |\Delta f| was almost 1 \sim 5 at |\Delta f| = 0.01 \sim 100 Hz. Only in the case (St-St)-Mirror, the recalculation error |\Delta f_{rec}| surged at |\Delta f| = 0.2 Hz. However, it seemed to be caused by the surge of the estimation error |\Delta \sigma_f| at |\Delta f| = 0.2 Hz. In addition, the value of the recalculation error |\Delta f_{rec}|
was about 3 Hz at $|\Delta f| = 0.2 \sim 1$ Hz, and these values were enough small (less than 0.2%) with respect to the natural frequency of the analysis object (600 $\sim$ 800 Hz). Therefore, it is believed that the recalculation error $|\Delta f_{re}|$ is as large as the disturbance $|\Delta f|$ in all of the analysis cases.

The maximum of the estimation error $|\Delta r|$ became constant again at $|\Delta f| = 0.2 \sim 1$ Hz in all of the analysis cases. At this point, the estimation error $|\Delta C|$ reached at $10^6$ MPa which was the upper boundary of the meshed solution space. Although the estimation error $|\Delta r|$ increased again at $|\Delta f| = 1 \sim 100$ Hz, the ratio $|\Delta r| / |\Delta f|$ was lower than before (at $|\Delta f| = 10^{-3} \sim 0.2$ Hz). Thus, the increase of the estimation error $|\Delta r|$ seemed to be suppressed by fixing the homogenized elastic modulus $C$ to be less than $10^6$ MPa. At $|\Delta f| = 100$ Hz, the estimation error $|\Delta r|$ reached at $10^3$ $\mu$m which was the upper boundary of the meshed solution space. Except the region where the increase of the estimation error $|\Delta r|$ was suppressed when $|\Delta f| > 0.2$ Hz, the theoretical limitation of the linear approximation was lower than the maximum estimation errors, but almost agreed with the maximum estimation errors in terms of the increase tendency for the disturbance $|\Delta f|$.

The maximum of the estimation error $|\Delta C|$ was over $100$ MPa at $|\Delta f| = 10^{-3}$ Hz in the case (St-St)-Usual, and became $10^6$ MPa at $|\Delta f| = 1$ Hz in all of the analysis cases. This is caused by the constitutive functions of the interfacial element. Figure 13 shows the function $F_{7/2}/F_{5/2} + d/\sigma$ (Eq. (5)) with respect to the function $F_{5/2}$ (Eq. (1)). As shown in Fig. 13, when the compressive stress $p_2$, the standard deviation of the asperity peak heights $\sigma$ and the thickness of the interfacial element $L$ are constant, the elastic modulus in the vibration state $E_z$ becomes 10 times even if the homogenized elastic modulus in the static equilibrium state $C$ becomes $10^{24}$ times. On the other hand, the elastic modulus $E_z$ is inversely proportional to the standard deviation of the asperity peak heights $\sigma$ by (Eq. (5)). Therefore, the sensitivity of the homogenized elastic modulus $C$ seems to be much smaller than that of the standard deviation of the asperity peak heights $\sigma$ in the inverse analysis. Related to this, the natural frequency subjected to the clamping force seems to be mainly determined by the standard deviation of the asperity peak heights $\sigma$. From the perspective of the estimation of the natural frequency subjected to the arbitrary clamping force, the standard deviation of the asperity peak heights $\sigma$ is the main parameter and the homogenized elastic modulus $C$ works as a fuzzy parameter to tolerate the disturbance. As far as the numerical simulation is concerned, the estimation error $|\Delta r|$ was less than 10 $\mu$m and the recalculation error $|\Delta f_{re}|$ was
less than 15 Hz if the disturbance $|\Delta f|$ was less than 10 Hz, for instance.

In the comparison of the analysis cases, the estimation errors were large if the given surface roughness was large; (St-St)-Usual > (St-St)-Fine > (St-St)-Mirror and (Al-St)-Usual > (Al-St)-Fine > (Al-St)-Mirror. The theoretical limitation mainly depended on the true values ($\sigma^{\text{TRUE}}$, $C^{\text{TRUE}}$) because the difference of the sum of $|a'_{e_j}|$ and that of $|a'_{C_j}|$ were small as shown in Table 6. The theoretical limitation was large when the true values ($\sigma^{\text{TRUE}}$, $C^{\text{TRUE}}$) were large, and agreed with the maximum estimation errors in terms of the order according to the analysis cases. In addition, the material of the base plate had no small influence on the estimation errors and the theoretical limitation though the difference in the theoretical limitation of $|\Delta \sigma|$ was relatively small. As previously presumed, the maximum estimation errors and the theoretical limitation in the estimation by the data set of the clamping force $P = 5, 10, 15$ kN were lower than those in the estimation by the data set of the clamping force $P = 3, 4, 5$ kN in the case (St-St)-Usual. Therefore, it is concluded the theoretical limitation of the linear approximation can qualitatively predict the estimation errors of the surface texture parameters.

5. Conclusion

In order to establish the methodology to find the limitation of the proposed method against the disturbance in the hammering test data, the theoretical limitation of the inverse analysis was derived by the linear approximation, and the numerical simulation that was the same approach as the Monte Carlo simulation was performed. The followings were summarized from the results of the numerical simulation.

1. The recalculation error of the natural frequency subjected to the clamping force is as large as the disturbance in the hammering test data.
2. The sensitivity of the homogenized elastic modulus $C$ is much smaller than that of the standard deviation of the asperity peak heights $\sigma$ in the inverse analysis. From the perspective of the estimation of the natural frequency subjected to the arbitrary clamping force, the standard deviation of the asperity peak heights $\sigma$ is the main parameter and the homogenized elastic modulus $C$ works as a fuzzy parameter to tolerate the disturbance.
3. The theoretical limitation of the linear approximation can qualitatively predict the estimation errors of the surface texture parameters.

Acknowledgment

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Appendix

In order to investigate the influence of the thickness of the interfacial element, the additional computations in which the thickness of the interfacial element \( L \) was set to 0.1 mm were executed. The analysis cases were (St-St)-Usual, (St-St)-Fine and (St-St)-Mirror. In the inverse analysis, the data set of the 1st natural frequencies when the clamping force \( P = 5, 10, 15 \) kN were used.

Figure 14 shows the 1st natural frequency of the analysis object obtained by the direct analysis. As shown in Fig. 14, the difference of the natural frequency is small between the thicknesses of the interfacial element \( L = 1 \) mm and \( L = 0.1 \) mm. Figure 15 shows the 1st natural frequency of the meshed solution space \( f_{FEM}^{ij}(\sigma_k, C_k) \) when the thickness of the interfacial element \( L \) was set to 0.1 mm. Table 7 and Table 8 show the parameters of the fitting planes. In comparison with the true surface texture parameters \( (\sigma_{true}^{k}, C_{true}^{k}) \) located.

![Fig. 14](image_url)  
**Fig. 14** The 1st natural frequency (the natural frequency of the 1st vibration mode) of analysis object with respect to clamping force applied per 1 bolt. The difference of the natural frequency is small (the maximum of the difference is 5.19 Hz) between the thicknesses of the interfacial element \( L = 1 \) mm and \( L = 0.1 \) mm.

![Fig. 15](image_url)  
**Fig. 15** The 1st natural frequency of the meshed solution space \( f_{FEM}^{ij}(\sigma_k, C_k) \) when the thickness of the interfacial element \( L \) was set to 0.1 mm. Each fitting plane was obtained by applying the least squares method on the filled element where the true surface texture parameters \( (\sigma_{true}^{k}, C_{true}^{k}) \) located.

<table>
<thead>
<tr>
<th>Analysis case</th>
<th>( P ) [kN]</th>
<th>( [\alpha] ) [Hz/( \mu )m]</th>
<th>( [{\alpha}^*] ) [Hz/MPa]</th>
<th>( b )</th>
<th>( a_{\alpha_j} )</th>
<th>( a_{C_j} )</th>
<th>( a_{\sigma_j} )</th>
<th>( a_{C_j} )</th>
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<tr>
<td>(St-St)-Usual</td>
<td>5</td>
<td>-60.20</td>
<td>5.188</td>
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<td>-0.182</td>
<td>0.061</td>
<td>0.156</td>
<td>0.399</td>
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<td></td>
<td>10</td>
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<td>796.0</td>
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<td>0.740</td>
<td>1.788</td>
<td>4.455</td>
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<td></td>
<td>15</td>
<td>-44.34</td>
<td>4.251</td>
<td>800.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(St-St)-Fine</td>
<td>5</td>
<td>-45.79</td>
<td>5.314</td>
<td>779.6</td>
<td>-0.231</td>
<td>0.068</td>
<td>0.206</td>
<td>0.505</td>
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<tr>
<td></td>
<td>10</td>
<td>-37.94</td>
<td>4.719</td>
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<td>-1.814</td>
<td>0.624</td>
<td>1.745</td>
<td>4.184</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>-34.03</td>
<td>4.409</td>
<td>795.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(St-St)-Mirror</td>
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<td>-39.43</td>
<td>8.354</td>
<td>776.9</td>
<td>-0.222</td>
<td>0.059</td>
<td>0.191</td>
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with Table 5 and Table 6, the differences of the parameters of the fitting planes (the coefficient matrix \([A]\), the intercept vector \([b]\) and the components \(a_{x}^{*}\), \(a_{y}^{*}\)) are small.

Figure 16 shows the maximums of the estimation errors \(|\Delta \sigma|\) and \(|\Delta C|\) in the 1000 computations with respect to the disturbance \(|\Delta f|\). Because of the truncation error of the meshed solution space, the maximums of the estimation errors were slightly increased or decreased by the difference of the thickness of the interfacial element when \(|\Delta f|\) was small (< 0.01 Hz). On the other hand, there is no difference in the maximum estimation errors between the thicknesses of the interfacial element when \(|\Delta f| > 0.01\) Hz. Also there is no difference in the theoretical limitation of the linear approximation between the thicknesses of the interfacial element because the differences of the parameters of the fitting planes are small. Therefore, the thickness of the interfacial element does not affect the limitation of the proposed method as far as the analysis object in this study.

![Fig. 16 Maximums of estimation errors of surface texture parameters in 1000 computations at each standard deviation \(\sigma_f\). Because the theoretical limitations (the dash-dotted curves and the dashed-two dotted curves) overlap each other, they are pointed by the black arrows in the figures. The differences of the maximum estimation errors are small between the thicknesses of the interfacial element \(L\).](image_url)

**References**


