Stress intensity factor of a crack normal to the interface between a homogeneous and a functionally graded piezoelectric layers under an electric load

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Abstract
In this paper, the fracture problem of a functionally graded piezoelectric material strip (FGPM strip) containing a crack perpendicular to the interface between the FGPM strip and a homogeneous layer under an electric load is considered. Material properties are assumed to be exponentially dependent on the distance from the interface. The superposition technique is used to solve the governing equations. The stresses induced by the electric load in the un-cracked laminate are calculated, and the obtained normal stress is used as the crack surface tractions with opposite sign to formulate the mixed boundary value problem. By using the Fourier transforms, the electro-mechanical fracture problem is reduced to a singular integral equation, which is solved numerically. The stress intensity factors of the internal crack and the edge crack are computed and presented for the various values of the nonhomogeneous and geometric parameters.

Keywords : Functionally graded piezoelectric material, Fracture mechanics, Elasticity, Normal crack, Stress intensity factor, Integral transform, Electric load

1. Introduction

Recently, piezoelectric composites have been used in a wide variety of applications including vibration control and actuators. These systems can be achieved by incorporating a thin piezoelectric layer into a structural system, and several kinds of piezoelectric actuators have been designed. Hall et al. fabricated a mono-morph actuator made from semi-conductive piezoelectric ceramics (Hall et al., 2005). Therefore, the elucidation of fracture behavior of piezoelectric systems such as actuators is important.

On the other hand, the concept of the well-known functionally graded materials (FGMs) can be extended to the piezoelectric material to improve its reliability (Wu et al., 1996). Since then the thermoelectromechanical fracture of the functionally graded piezoelectric materials (FGPMs) has received much attention, and many investigators have reported interesting results. For example, fracture of FGPMs with two-dimensional crack has been studied under thermal load (Wang and Noda, 2001), anti-plane mechanical and in-plane electric loads (Li and Weng, 2002; Wang and Zhang, 2004), and in-plane mechanical and electric impact loads (Chen et al., 2003). The present author also studied mode III crack problem in an FGPM strip with elastic surface layers (Ueda 2003), in-plane crack problems under static electromechanical loadings (Ueda 2005a; Ueda 2006a), electromechanical impact problems in a cracked FGPM (Ueda 2005b; Ueda 2006b; Ueda 2007a), and thermally induced fracture problems of an FGPM strip (Ueda 2004; Ueda 2007b). Moreover, it is also important to investigate the fracture behavior of a mono-morph actuator made from FGPMs, and some interesting results induced by a thermal load have been reported for the normal crack (Ueda et al., 2018, Mabuchi and Ueda, 2019).

In this paper, we focused on the fracture problem induced by an electric load of monomorph actuators. The problem of the normal crack in the FGPM strip bonded to a homogeneous elastic layer is analyzed. Material properties of the FGPM strip are exponentially dependent on the distance from the interface between the FGPM strip and the
homogeneous elastic layer. The superposition technique is used to solve the governing equations. The stresses induced by an electric load in an un-cracked laminate are calculated, and the obtained normal stress is used as the crack surface tractions with opposite sign to formulate the mixed boundary value problem (Ueda et al., 2018). By using the Fourier transform technique (Sneddon and Lowengrub, 1969), the electro-mechanical problem is reduced to a singular integral equation, which are solved numerically (Erdogan et al., 1972). The stress intensity factors of the internal crack and the edge crack are computed and presented for the various values of the nonhomogeneous and geometric parameters.

2. Formulation of the problem

Consider an FGPM strip of thickness \( h_1 \) containing a finite crack bonded to an elastic layer of thickness \( h_2 \) with the rectangular Cartesian coordinate system \((x, y, z)\) as shown in Fig. 1. The crack of length \( 2c \) is located along \( z \)-axis from \( a \) to \( b \) \((b-a=2c, \ 0 < a < b \leq h_1)\). The FGPM strip is poled in the \( z \)-direction and is in the plane strain conditions perpendicular to the \( y \)-axis. It is assumed that the electric potential \( \Phi(z) \) along the boundaries \( z=0 \) and \( h_1 \) is \( \Phi(0)=0 \) and \( \Phi(h_1)=\Phi_0 \), respectively. In the following, the subscripts \( x, y, z \) will be used to refer to the direction of coordinates.

![Normal crack in an FGPM strip bonded to a homogeneous elastic layer under an electric load.](image)

The material property parameters are taken to vary continuously along the \( z \)-direction inside the FGPM strip. The material properties of the FGPM strip, such as the elastic stiffness constants \( c_{ii}(z) \), the piezoelectric constants \( e_{ii}(z) \), the dielectric constants \( \varepsilon_{ii}(z) \) \((i=1)\) are one-dimensionally dependent as

\[
(c_{ii}, e_{ii}, \varepsilon_{ii}) = (c_{i0}, e_{i0}, \varepsilon_{i0}) \exp(\beta z) \quad (i = 1)
\]

where \( \beta \) is positive or negative constant, and the subscript 0 indicates the properties at the interface plane \( z=0 \). From Eq. (1), the non-dimensional parameter \( \beta h_1 \) is denoted by

\[
\beta h_1 = \ln \left( \frac{c_{i1}}{c_{i0}} \right) = \ln \left( \frac{e_{i1}}{e_{i0}} \right) = \ln \left( \frac{\varepsilon_{i1}}{\varepsilon_{i0}} \right)
\]

and \( \beta h_1 \) indicates the changing rate of the material properties. The material properties of the homogeneous elastic layer are the elastic stiffness constants \( c_{ii} \) \((i=2)\), and the subscript \( i=1,2 \) denotes the physical quantities of the FGPM strip and the homogeneous elastic layer, respectively.

The crack problem may be solved by using the superposition technique. In the problem considered here, since the electric field is one-dimensional and straight crack does not obstruct the electric filed in this arrangement, determination of the electric potential distribution and the resulting stress would be quite straightforward and the related crack problem would be one of mode I. We suppose that the crack is opened under the action of the same distribution of the internal pressure \( \sigma_n(z) \), where \( \sigma_n(z) \) is the stress induced by the electric load.
3. Electro-mechanical fields induced by the electric load in the un-cracked strip

We consider the electro-mechanical fields due to the electric load. The stress and electric displacement components can be found from following constitutive equations:

\[
\begin{align*}
\sigma_{x1}^E(z) &= c_{111}^E \sigma_{x1}^E(z) + c_{311}^E e_{x11}^E(z) - e_{311}^E E_{z1}^E(z) \\
\sigma_{x2}^E(z) &= c_{333}^E \sigma_{x2}^E(z) + c_{331}^E e_{x12}^E(z) - e_{331}^E E_{z2}^E(z) \\
\sigma_{x3}^E(z) &= c_{441}^E \sigma_{x1}^E(z) - e_{351}^E E_{z1}^E(z) \\
D_{x1}^E(z) &= c_{131}^E \sigma_{x1}^E(z) - e_{111}^E E_{z1}^E(z) \\
D_{x2}^E(z) &= e_{111}^E \sigma_{x1}^E(z) + c_{311}^E e_{x13}^E(z) + e_{311}^E E_{z1}^E(z) \\
\sigma_{x1}^E(z) &= c_{112}^E \sigma_{x1}^E(z) + c_{132}^E e_{x13}^E(z) \\
\sigma_{x2}^E(z) &= c_{332}^E \sigma_{x1}^E(z) + c_{332}^E e_{x12}^E(z) \\
\sigma_{x3}^E(z) &= c_{442}^E \sigma_{x1}^E(z)
\end{align*}
\]

where \(\sigma_{x1}^E(z), \sigma_{x2}^E(z), \sigma_{x3}^E(z)\) \((i = 1, 2)\) are the stress components, \(D_{x1}^E(z), D_{x2}^E(z)\) are the electric displacement components. And \(e_{x1}^E(z), e_{x2}^E(z), e_{x3}^E(z)\) \((i = 1, 2)\) are the strain components, \(E_{x1}^E(z), E_{x2}^E(z)\) are the electric field components. The superscript \(E\) denotes the electrically induced quantities.

If we now assume that the laminate is infinite (i.e., \(-\infty < x, y < \infty\)), free of surface tractions and electric displacement at \(z = h_1\) and \(z = -h_2\), and subjected to the electric load, it may be seen that

\[
\begin{align*}
\sigma_{x1}^E(z) &= \sigma_{x2}^E(z) = 0 \quad (0 \leq z \leq h_1) \\
\sigma_{x2}^E(z) &= \sigma_{x3}^E(z) = 0 \quad (-h_2 \leq z \leq 0) \\
D_{x1}^E(z) &= 0 \quad (0 \leq z \leq h_1) \\
D_{x2}^E(z) &= D_0 \quad (0 \leq z \leq h_1) \\
E_{x1}^E(z) &= 0 \quad (0 \leq z \leq h_1) \\
\phi(0) &= 0 \\
\phi(h_1) &= \phi_0
\end{align*}
\]

where \(D_0\) denotes the unknown uniform electric displacement, and all non-vanishing field quantities are independent of \(x, y\). By solving Eqs. (3) and (4) with Eqs. (5), it is found that

\[
\begin{align*}
\sigma_{x1}^E(z) &= \bar{c}_1 \exp(\beta z) e_{x1}^E(z) + \bar{d}_0 D_0 \quad (0 \leq z \leq h_1) \\
\sigma_{x2}^E(z) &= \bar{c}_2 e_{x2}^E(z)
\end{align*}
\]

where

\[
\begin{align*}
\bar{c}_1 &= c_{110} - c_{130} (c_{330} e_{330} + e_{310} (c_{310} e_{310} - c_{330} e_{310})) \\
&+ c_{330} (c_{310} e_{330} + e_{310} (c_{310} e_{310} - c_{330} e_{310})) \\
\bar{d}_0 &= c_{130} (c_{330} e_{330} + c_{310} e_{310}), \quad \bar{c}_2 = c_{112} - \frac{c_{132}^2}{c_{332}}
\end{align*}
\]

The compatibility conditions that need to be satisfied become

\[
\frac{d^2 e_{x1}^E(z)}{dz^2} = 0 \quad (i = 1, 2)
\]

giving with the electric loading condition
\[ \varepsilon_{s11}(z) = A^E z + B^E \]
\[ \sigma_{s11}(z) = \tilde{c}_1 \exp(\beta z)(A^E z + B^E) - E_A \left( \tilde{c}_1 \left( \frac{A^E h}{2} + B^E \right) + \varepsilon_0 \frac{\phi_1}{h} \right) \]  
\[ \varepsilon_{s11}(z) = A^E z + B^E \]
\[ \sigma_{s11}(z) = \tilde{c}_2 \left( A^E z + B^E \right) \]  

where \( A^E \) and \( B^E \) are unknown constants to be obtained from boundary conditions for the laminate and

\[ \tilde{c}_3 = \left( c_{130}^E e_{330} - c_{330}^E e_{130} \right)^2 \]
\[ \tilde{c}_4 = \frac{c_{130}^E e_{330} + c_{330}^E e_{130}}{c_{330}} \]
\[ \varepsilon_0 = \frac{\beta h_i}{1 - \exp(-\beta h_i)} \]  

If the laminate is unconstrained along its boundaries, we have

\[ \int_0^h \sigma_{s11}^E(z) dz + \int_0^h \sigma_{s12}^{E'}(z) dz = 0 \]
\[ \int_0^h \sigma_{s11}^{E'}(z) dz + \int_0^h \sigma_{s12}^E(z) dz = 0 \]  

Substituting Eqs. (9) and (10) into Eqs. (12), \( A^E \) and \( B^E \) are obtained as follows

\[ A^E = \frac{\tilde{c}_4 \phi_1}{\tilde{c}_1 h_i h_i} A_0, \quad B^E = \frac{\tilde{c}_4 \phi_0}{\tilde{c}_1 h_i} B_0 \]  

where

\[ A_0 = \frac{2D_{2a} - D_{1b} - E_0}{D_{2b}D_{1a} - D_{2a}D_{1b}} \quad B_0 = \frac{2D_{2a} - D_{1b} - E_a}{D_{2a}D_{1b} - D_{2a}D_{2b}} \]  

The constants \( D_{ja} \) and \( D_{jb} \) (\( j = 1, 2 \)) are given by

\[ D_{1a} = E_2 - \frac{\tilde{c}_4}{2\tilde{c}_1} E_a - \frac{\tilde{c}_2}{2\tilde{c}_1} h_i^2, \quad D_{1b} = E_2 - \frac{\tilde{c}_4}{\tilde{c}_1} E_a + \frac{\tilde{c}_2}{\tilde{c}_1} h_i^2, \]
\[ D_{2a} = E_2 - \frac{\tilde{c}_4}{4\tilde{c}_1} E_a + \frac{\tilde{c}_3}{3\tilde{c}_1} h_i^2, \quad D_{2b} = E_2 - \frac{\tilde{c}_4}{2\tilde{c}_1} E_a - \frac{\tilde{c}_3}{2\tilde{c}_1} h_i^2 \]  

with

\[ E_1 = \frac{1}{\beta h_i} \left\{ \exp(\beta h_i) - 1 \right\}, \quad E_2 = \frac{1}{(\beta h_i)^2} \left\{ \exp(\beta h_i) (\beta h_i - 1) + 1 \right\} \]
\[ E_3 = \frac{1}{(\beta h_i)^3} \left\{ \exp(\beta h_i) \left( (\beta h_i)^2 - 2\beta h_i + 2 \right) - 2 \right\}, \quad h_0 = \frac{h_i}{h_i} \]  

Therefore the stresses \( \sigma_{s11}^E (z) \) (\( i = 1, 2 \)) are

\[ \sigma_{s11}^E(z) = \left[ \exp(\beta z) \left( \frac{z}{h_i} A_0 + B_0 \right) - \left( \frac{\tilde{c}_4}{\tilde{c}_1} \left( \frac{1}{2} A_0 + B_0 \right) + 1 \right) E_A \right] \frac{\phi_1}{h_i} \]
\[ \sigma_{s11}^{E'}(z) = \frac{\tilde{c}_3}{\tilde{c}_1} \left( \frac{z}{h_i} A_0 + B_0 \right) \frac{\phi_0}{h_i} \]  

In the crack problem under considered, the equal and opposite of the stress \( \sigma_0^E (z) = \sigma_{s11}^E (z) \) (\( a < z < b \)) given
by Eq. (17) will be used as the crack surface traction, and the laminate will be assumed to be under plane-strain conditions. Moreover, the unknown uniform electric displacement $D_0$, the non-zero electric field $E_{cl}^E(z)$ and the electric potential $\phi_1(z)$ are obtained as follows:

$$D_0 = -E_0 \left( \frac{A_0}{2} + B_0 + \frac{\tilde{c}_1}{c_1} \right) \frac{z^2 \phi_0}{h}$$

$$E_{cl}(z) = E_0 \left[ \frac{A_0}{2} \frac{z}{h} + B_0 - E_0 \left( \frac{A_0}{2} + B_0 + \frac{\tilde{c}_1}{c_1} \right) \exp(-\beta z) \right] \frac{\phi_0}{h}$$

$$\phi_1(z) = -\int \frac{E_{cl}(\xi) \, d\xi}{\pi} = -E_0 \left[ \frac{A_0}{4} \frac{z}{h} + B_0 - E_0 \left( \frac{A_0}{2} + B_0 + \frac{\tilde{c}_1}{c_1} \right) \frac{z}{h} \phi_0 \right]$$

where

$$E_{cl} = \frac{c_{33}^{el}}{e_{33}^{el} e_{330} - e_{33}^{el} e_{330}}$$

(19)

4. The crack problem

Referring to Fig. 1, it is assumed that $x = 0$ is a plane of symmetry regarding to geometry and loading conditions. Thus, in analyzing the problem it is sufficient to consider only one-half ($0 \leq x < \infty$) of the FGPM strip and the homogeneous elastic layer. Also, through a proper superposition, the problem is assumed to have been reduced to a perturbation problem in which the crack surface tractions are the only nonzero external loads and the stresses in the layered strip vanish for $x \to \infty$.

The boundary conditions can be written as

$$\sigma_{xx}(0, z) = -\sigma_{xx}^E(z) \quad (a < z < b)$$

$$u_{x1}(0, z) = 0 \quad (0 \leq z \leq a, b \leq z \leq h)$$

(20)

$$\sigma_{xx}(0, z) = 0 \quad (0 \leq z \leq h_1)$$

(21)

$$\sigma_{xx}(0, z) = 0 \quad (-h_2 \leq z \leq 0)$$

(22)

$$\sigma_{xx}(x, h_1) = 0$$

(23)

$$\sigma_{xx}(x, h_2) = 0 \quad (0 \leq x < \infty)$$

$$D_{x1}(x, 0) = D_{x1}(x, 0)$$

(24)

$$D_{x2}(x, 0) = \sigma_{xx2}(x, 0)$$

$$D_{x1}(x, 0) = D_{x1}(x, 0)$$

(25)

where $u_i(x, z), u_i(x, z)$ ($i = 1, 2$) are the displacement components, $\sigma_{xx}(x, z), \sigma_{xx}(x, z), \sigma_{xx}(x, z)$ ($i = 1, 2$) are the stress components, and $D_{x1}(x, z), D_{x2}(x, z)$ are the electric displacement components induced by the crack surface traction.

The singular stresses analysis is the same as the description in the previous paper (Ueda et al., 2018). Then, the expression of the governing equations should be omitted, and only the outline of it may be mentioned here. The stress intensity factors $K_{IA}(t)$ at $z = a$ and $K_{Ie}(t)$ at $z = b$ may be evaluated as
\[ K_{1a} = \begin{cases} \frac{Z^n \exp(\beta a)(\pi c)^{1/2} \Phi(a)}{(a + c)^{1/2}} & (0 < a < b < h : \text{Internal Crack}) \\ \frac{Z^n \exp(\beta a)(2\pi c)^{1/2} \Phi(a)}{(a + c)^{1/2}} & (0 < a < b = h : \text{Edge Crack}) \end{cases} \]  

\[ K_{2a} = \begin{cases} \frac{Z^n \exp(\beta b)(\pi c)^{1/2} \Phi(b)}{(b + c)^{1/2}} & (0 < a < b < h : \text{Internal Crack}) \\ 0 & (0 < a < b = h : \text{Edge Crack}) \end{cases} \]

where the known constant \( Z^n \) is given in (Ueda et al., 2018), and the function \( \Phi(\xi) \) is

\[ G(\xi) = \begin{cases} \frac{c}{(\xi - a)(b - \xi)^{1/2}} \Phi(\xi) & (0 < a < b < h : \text{Internal Crack}) \\ \frac{b - \xi}{(\xi - a)^{1/2}} \Phi(\xi) & (0 < a < b = h : \text{Edge Crack}) \end{cases} \]

In Eq. (28), the function \( G(\xi) \) is the solution of the following singular integral equation obtained from the boundary conditions (20)-(25).

\[ \int_{a}^{b} G(\xi) \left[ \frac{1}{\xi - \zeta} + \sum_{i=1}^{4} M_{i}(\zeta, \xi) \right] d\zeta = \exp(-\beta z) \frac{\pi}{\eta(Z^{n})} \sigma_{\xi y}(z) \quad (a < \zeta < b) \]

where the kernel functions \( M_{i}(\zeta, \xi) (i = 1, 2, 3, 4) \) are also given in (Ueda et al., 2018). The singular integral equation (29) for \( b < h \) is to be solved with the following subsidiary conditions obtained from the second boundary condition (20).

\[ \int_{a}^{b} G(\xi) d\xi = 0 \]

5. Numerical results and discussion

For the purpose of numerical illustrations, the electro-elastic properties of cadmium selenide with the following properties (Ashida and Tauchert, 1998) are used as the properties of the FGPM strip at the plane \( z = 0 \).

\[
\begin{align*}
\epsilon_{110} = 7.41 \times 10^{10} \text{[N/m}^2], & \quad \epsilon_{130} = 3.93 \times 10^{10} \text{[N/m}^2], \\
\epsilon_{330} = 8.36 \times 10^{10} \text{[N/m}^2], & \quad \epsilon_{440} = 1.32 \times 10^{10} \text{[N/m}^2], \\
\epsilon_{330} = -0.16 \text{[C/m}^2], & \quad \epsilon_{440} = 0.347 \text{[C/m}^2], \\
\epsilon_{150} = -0.138 \text{[C/m}^2], & \quad \epsilon_{150} = 0.903 \times 10^{-10} \text{[C/Vm]}, \\
\end{align*}
\]  

The properties of titanium (Ti) with following properties are also used as the properties of the elastic layer.

\[
\begin{align*}
\epsilon_{112} = \epsilon_{332} = \frac{2(1 - \nu)}{1 - 2\nu} \mu, & \quad \epsilon_{112} = \frac{2\nu}{1 - 2\nu} \mu, & \quad \epsilon_{442} = \mu. \\
\end{align*}
\]

where the constants \( \mu \) and \( \nu \) are the modulus of transverse elasticity and Poisson’s ratio, and the values of them are

\[
\mu = 4.26 \times 10^{10} \text{[N/m}^2], \quad \nu = 0.28.
\]

5.1. Electro-mechanical fields by the electric load

First of all, we consider the effects of the nonhomogeneous parameter \( \beta h \) and the thickness ratio \( h_{2}/h_{1} \) on the stress and electric fields induced by the electric load. These electro-mechanical fields can be obtained from Eqs. (17) and (18). Fig. 2(a) shows the distributions of the normalized stress \( \sigma_{\xi y}(z)/\sigma_{0} \) (0 \( \leq z \leq h_{1} \)) and \( \sigma_{\xi y}(z)/\sigma_{0} \)
The effect of the nonhomogeneous parameter $\beta h_1$ on the electric potential distribution $\phi_1(z)$ and the electric field distribution $E_{z1}(z)$ ($0 \le z \le h_1$) for $h_2/h_1 = 1.0$ are also shown in Fig. 2(b). For the case of $\beta h_1 = 0.0$, $\phi_1(z)$ is proportional to $z$ and $E_{z1}(z)$ is constant.

Figure 3 indicates $\sigma_{x1}^{E}(z)/\sigma_0$ ($0 \le z \le h_1$) for $h_2/h_1 = 0.1, 0.2$ and $1.0$ with $\beta h_1 = 1.0$. As $h_2/h_1$ decreases, the absolute value of $\sigma_{x1}^{E}(z)/\sigma_0$ tends to decrease. The stress in the region of $0.0 \le z/h_1 < 0.6$ is compressive and that in the region of $0.7 < z/h_1 \le 1.0$ is tensile. Of course, the opposite is true for the case of $\phi_0 < 0.0$. In this problem, the stresses $\sigma_{x1}^{E}(z)$ ($0 \le z \le h_1$) and $\sigma_{x1}^{E}(z)$ ($-h_2 \le z \le 0$) are statically self-equilibrating. This would mean that, in most cases, as the electric potential $\phi_0$ is cycled above and below the value corresponding the stress free state, the stress $\sigma_{x1}^{E}(z)$ ($0 \le z \le h_1$) near the free surface becomes tensile or compressive and in the interior compressive or tensile.
on the stress $\sigma_{xx}(z)$ ($0 \leq z \leq h_1$) for $\beta h_1 = 1.0$.

### 5.2. Internal crack

Next, we consider the effects of the nonhomogeneous parameter $\beta h_1$, the crack location parameter $(a+b)/2h_1$ and the crack length parameter $c/h_1$ on the stress intensity factors $K_{IA}$ and $K_{IB}$ for the thickness ratio $h_2/h_1 = 1.0$. The stress intensity factors $K_{IA}(t)$ and $K_{IB}(t)$ can be obtained from Eqs. (26), (27) with Eqs. (28), the singular integral equation (29) and the subsidiary condition (30). Figure 4 shows the normalized stress intensity factors $(K_{IA}, K_{IB})/\sigma_0(\pi c)^{1/2}$ versus $(a+b)/2h_1$ for $\beta h_1 = 0.0$, $1.0$ and $-1.0$ with $c/h_1 = 0.1$. The value of $(a+b)/2h_1$ has great influence on $(K_{IA}, K_{IB})/\sigma_0(\pi c)^{1/2}$ for $\beta h_1 = 1.0$, but the values of $(K_{IA}, K_{IB})/\sigma_0(\pi c)^{1/2}$ for $\beta h_1 = -1.0$ are almost uniform. In some cases, the stress intensity factors become negative and the results have no physical meaning, and if both $K_{IA}$ and $K_{IB}$ are negative, the crack would be fully closed. However, a plus or a minus sign of the stress intensity factor depends on the direction of the electric load. Thus we should focus on the absolute values of the stress intensity factors. For the case of $\beta h_1 = 1.0$, the absolute values of $(K_{IA}, K_{IB})/\sigma_0(\pi c)^{1/2}$ of the crack, that is near the interface or the free surface, are very large, and the crack in these cases would propagate under the cyclic electric load.

![Fig. 3. The effect of the thickness ratio $h_2/h_1$ on the stress distribution $\sigma_{xx}(z)$ ($0 \leq z \leq h_1$) for $\beta h_1 = 1.0$.](image)

![Fig. 4. The effects of the crack location parameter $(a+b)/2h_1$ and the nonhomogeneous parameter $\beta h_1$ on the stress intensity factors $K_{IA}$ and $K_{IB}$ for $h_2/h_1 = 1.0$ and $c/h_1 = 0.1$ (internal crack).](image)
\[ c/h_i \to 0.0, \text{ the values of } K_{IA} \text{ and } K_{in} \text{ become to be equal.} \]

Fig. 5. The effects of the crack length parameter \( c/h_i \) and the nonhomogeneous parameter \( \beta h_i \) on the stress intensity factors \( K_{IA} \) and \( K_{in} \) for \( h_2/h_1 = 1.0 \) and \( (a+b)/2h_i = 0.5 \) (internal crack).

Thirdly, we examine the effect of \( h_2/h_1 \) on the stress intensity factors. The normalized stress intensity factors \( (K_{IA}, K_{in})/\sigma_c(\pi c)^{1/2} \) are plotted versus \( (a+b)/2h_i \) in Fig. 6 and versus \( c/h_i \) in Fig. 7 for \( h_2/h_1 = 0.1, 0.2 \) and \( 1.0 \) with \( \beta h_i = 1.0 \). For the case of the internal crack shown in both Figs. 6 and 7, the absolute values of them decrease with decreasing \( h_2/h_1 \), and this phenomenon can be easily understood by the distribution of \( \sigma_{stl}(z)/\sigma_c (0 \leq z \leq h_i) \) shown in Fig. 3. For the cases of \( K_{IA} \) and \( K_{in} \) shown in Fig. 7, because the absolute value of the stress \( \sigma_{stl}(z) \) near the crack tip at \( z = a \) tends to increase with increasing \( c/h_i \) as shown in Fig. 3, the absolute value of \( K_{IA} \) also increases, and the opposite is true for \( K_{in} \).

Fig. 6 The effects of the crack location parameter \( (a+b)/2h_i \) and the thickness ratio \( h_2/h_1 \) on the stress intensity factors \( K_{IA} \) and \( K_{in} \) for \( c/h_i = 0.1 \) and \( \beta h_i = 1.0 \) (internal crack).
F. Fig. 7. The effects of the crack length parameter \( c/h \) and the thickness ratio \( h_2/h_1 \) on the stress intensity factors \( K_{1a} \) and \( K_{m} \) for \( \beta h_1 = 1.0 \) and \( (a+b)/2h_1 = 0.5 \) (internal crack).

5.3. Edge crack

Finally, we study the case of \( b/h_1 = 1.0 \) (edge crack). The normalized stress intensity factor \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) is plotted versus \( a/h_1 \) for \( \beta h_1 = 1.0, -1.0 \) and 0.0 with \( h_2/h_1 = 1.0 \) in Fig. 8. The influence of \( \beta h_1 \) on the stress intensity factor is very large, and the absolute value of \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) decreases with decreasing \( \beta h_1 \). This phenomenon can be explained by the fact that the stress \( \sigma_{aux}(z) \) in the region of \( 0.5 < z/h < 1.0 \) decreases with decreasing \( \beta h_1 \) as shown in Fig. 2(a).

Fig. 8. The effects of the crack length parameter \( a/h_1 \) and the nonhomogeneous parameter \( \beta h_1 \) on the stress intensity factor \( K_{1a} \) for \( h_2/h_1 = 1.0 \) (edge crack).

The influence of \( h_2/h_1 \) on \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) with \( a/h_1 = 0.9, 0.7, 0.5 \) is indicated in Fig. 9(a) for \( \beta h_1 = 0.0, 1.0 \) and in Fig. 9(b) for \( \beta h_1 = -1.0 \). The value of \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) increases at first, goes through maximum, and then decreases with increasing \( h_2/h_1 \). For the larger value of \( h_2/h_1 \) than the value shown in Eq. (32), \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) tends to increase with decreasing \( h_2/h_1 \), because the rigidity of the laminate reduces with decreasing \( h_2/h_1 \). However the stress intensity factor decreases with decreasing \( h_2/h_1 \) for the smaller value of \( h_2/h_1 \) than the value shown in Eq. (32), because of the reduction of \( \sigma_{aux}(z)/\sigma_0 (0 \leq z \leq h_1) \) shown in Fig. 3. The effect of \( h_2/h_1 \) on \( K_{1a}/\sigma_0(2\pi c)^{1/2} \) for \( \beta h_1 = -1.0 \) is very small.
\[ h_2/h_1 \approx 0.8 \quad (a/h_1 = 0.9, \beta h_1 = 1.0), \quad h_2/h_1 \approx 0.5 \quad (a/h_1 = 0.9, \beta h_1 = 0.0), \]
\[ h_2/h_1 \approx 0.6 \quad (a/h_1 = 0.7, \beta h_1 = 1.0), \quad h_2/h_1 \approx 0.4 \quad (a/h_1 = 0.7, \beta h_1 = 0.0), \]
\[ h_2/h_1 \approx 0.3 \quad (a/h_1 = 0.5, \beta h_1 = 1.0), \quad h_2/h_1 \approx 0.3 \quad (a/h_1 = 0.5, \beta h_1 = 0.0). \]  

(34)

Fig. 9(a). The effects of the thickness ratio \( h_2/h_1 \) and the crack length parameter \( a/h_1 \) on the stress intensity factor \( K_{\text{IA}} \) for \( \beta h_1 = 0.0, 1.0 \) (edge crack).

Fig. 9(b). The effects of the thickness ratio \( h_2/h_1 \) and the crack length parameter \( a/h_1 \) on the stress intensity factor \( K_{\text{IA}} \) for \( \beta h_1 = -1.0 \) (edge crack).

6. Conclusions

The fracture problem of the cracked functionally graded piezoelectric strip bonded to the homogeneous elastic layer by the electric load is studied. The effects of the thickness of the elastic layer, the material nonhomogeneity, the crack location and the crack length on the electro-mechanical fields and the fracture behavior are considered. The following facts can be found from the numerical results.

(1) The electro-mechanical fields depend on the nonhomogeneous parameter \( \beta h_1 \) and the thickness ratio \( h_2/h_1 \). Especially, the stress component \( \sigma_{\text{xx}}(z) \) distribution changes dramatically with the fluctuations of \( \beta h_1 \) and \( h_2/h_1 \).

(2) In some cases, the stress intensity factors under the electric load become negative and the results have no physical meaning. However, when the electric load is combined with the mechanical and thermal loads which induce the positive stress intensity factor, those results can be used effectively. Moreover, because the direction of the electric load is cyclic during the operation of the actuators, we should focus on the absolute values of the stress intensity factors.
(3) The behavior of the normalized stress intensity factors \((K_I, K_{II}) / \sigma_0(\pi c)^{1/2}\) for the internal crack can be understood easily from the stress distribution \(\sigma_{xx}(z) / \sigma_0\). Generally, the decrease of \(\beta h\) is beneficial for reducing the absolute values of the stress intensity factors of the internal crack and the edge crack.

(4) From the fracture mechanical point of view, the reduction of the value of \(h_2 / h_1\) is also beneficial for the case of the internal crack. However, the opposite is true for the edge crack with the relatively thick elastic layer, because the rigidity reduction of the laminate becomes dominant.

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References


