Unified structural optimization method using topology optimization and genetic algorithms

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Abstract
This paper presents a new structural design framework that incorporates the concept of topology optimization and genetic algorithms to improve the manufacturability and structural robustness of the optimal structure. The level set function is employed as a topological design variable to obtain clear structural boundaries, and the manufacturability of the structure is mathematically defined based on the manufacturing directions and the fictitious heat fluxes. To gain the manufacturable structure design, the optimization problem is formulated to find both the optimal shape of the structure and the optimal directions of the adjustable manufacturing tools. A level set-based optimization regarding manufacturability has been studied in previous papers, however, due to the influence of the manufacturing directions, the objective value tends to be captured in local optima as the structure becomes more complex. To cope with this issue, we decided to adapt a heuristic based approach, genetic algorithm, to the optimization method. Simultaneously, to reduce the computation time, we applied Design of Experiments for the initial population of the genetic algorithm. The initial population of the manufacturing directions is installed using Latin Hypercube sampling for both a good representation and computational efficiency. To demonstrate the effectiveness of the proposed method, several design examples are provided, and the differences from the optimal solution, derived by a previous gradient-based optimization scheme, are mentioned.

Keywords: Manufacturability, Genetic algorithm, Topology optimization, Geometric constraint, Design optimization

1. Introduction

The topology optimization method, where the design variables are defined by the physical properties of each finite element, has higher design freedom compared to other design methods such as optimization on dimensions or shape optimization, can provide an innovative design candidate at the conceptual design stage, and has been successfully applied in various structural design problems. Especially, with the introduction of the level set method, topology optimization became a more practical design technique for solving numerical instability problems caused by the gray-scale elements (Bendsoe, 1989), (Allaire et al., 2004), (Sethian and Wiegmann, 2000), (Wang et al., 2003). However, the optimal configuration of the structure obtained through topology optimization often contains complex shapes which are unable to be manufactured by conventional manufacturing techniques such as molding, casting, and milling. Recent additive manufacturing techniques can be applied to fabricate such complicated shapes, nevertheless, they cause excessive manufacturing costs and are not yet suitable for mass production processes. Thus, the manufacturability of the optimal design should be regarded from the problem formulation in the topology optimization process to ensure both the cost effectiveness and the performance.

To consider the manufacturability in the optimization, the evaluation process should be mathematically established. The evaluation process is based on a typical framework of Design for Manufacturing (DFM) by Boothroyd (Boothroyd et al., 2001) that is taken into account in the product design phase. DFM offers a quantitative criterion for the manufacturability evaluation of products and vouchers for cost efficiency of the production, and has been successfully applied in product design (Joshi and Ravi, 2010), (Armillotta et al., 2016), (Selvaraj et al., 2009), (Gupta and Nau, 1995). However,
the manufacturing evaluation approaches in DFM, based on intuition of the engineers and lumped parameters, cannot be directly applied to the topology optimization process where the structural configuration is implicitly defined by the topological variables. To deal with this issue, several papers have proposed a new problem formulation that incorporates a quantitative manufacturability evaluation technique(Sato et al., 2017) and an updating scheme for the topological design variable regarding the manufacturing constraint(Xia et al., 2010). They utilized pre-defined parting directions of the molds to define and minimize the total area of non-manufacturable domain, such as undercuts and interior voids, and to restrict the propagation of the structural boundaries to gain manufacturability during topology optimization process. These methods provided an appropriate structural shape to the casting process, however, the structural performance of the optimal design may be severely constrained by the prefixed molding directions. Thus, Hur et al.(Hur et al., 2019) introduced a concurrent optimization by treating the manufacturing directions as design variables and verified the performance improvement of the optimal structure with respect to the optimal molding direction. However, since the manufacturing directions have a dominant effect on calculating topological design sensitivity, a local optima problem can be occurred with the gradient-based optimization for the directional variable.

Thus, this paper presents a new integrated topology optimization framework where a meta-heuristic algorithm is partially used for optimization of the manufacturing directions. The optimization problem is formulated to minimize both the mean compliance of the structure and the non-manufacturable area, which is calculated based on the fictitious flux, the manufacturing direction, and the shadow domains where the heat flux is prevented(Sato et al., 2017). In the beginning of the optimization process, the initial set of the manufacturing directions is determined by Latin Hypercube(LH) sampling. The structural shape, represented by a level set distribution, is updated using a topological derivative of the multi-objective function. The genetic algorithm is then applied to find an optimal set for the manufacturing directions to overcome the limitations of the adjoint design sensitivity and gradient descent algorithm. Several meta-heuristic approaches are considered to deal with this limitations, such as Particle Swarm Optimization(PSO)(Kennedy and Eberhart, 1995) or Differential Evolution(DE)(Price, 2013). However, we applied genetic algorithm as a representative way of metaheuristic optimization method to maximize merit on elitism and mutation. Two different types of design variables, the level set function and the manufacturing directions, are simultaneously optimized in each iteration to settle in a global optimum. A three-dimensional cantilever design example is provided to verify the effect of the proposed optimization framework and enhanced performance of the optimal design.

2. Genetic Algorithm-based Topology Optimization Regarding Manufacturability

In this paper, two different types of design variables, a topological design variable and manufacturing directions, are employed to represent the structural design and to evaluate the manufacturability. Since the manufacturability directions greatly affect the optimal design of the structure, it is important to choose the initial values of these directional variables and a proper optimizer that can prevent converging in a local minimum. Thus, the LH sampling(Iman and Conover, 1980),(McKay et al., 2000) and genetic algorithm(Holland et al., 1992) are employed to the initial population and the obtained global optimum of the manufacturing directions, respectively. Since the genetic algorithm does not require information about the design sensitivities of the objective functions and can cope with both continuous and discrete variables, it has been applied to the topology optimization by Chapman(Chapman et al., 1994), Rahami(Rahami et al., 2008), and Hajela(Hajela and Lee, 1995) for avoiding the initial-dependency and achieving the global optimum. However, in the topology optimization where the number of design variables is the same as the number of finite elements or their nodes, it is a time consuming process to update the topological variable by using the genetic algorithm. Thus, the conventional topology optimization method is still used in the proposed optimization process to obtain the structural configuration, and the genetic algorithm is only applied to find the optimal manufacturing directions, which are used as the input analysis condition for the topology optimization. The detailed problem formulations and optimization process are defined in the following sections.

2.1. Level Set-Based Topology Optimization

To represent the structural boundary in the design domain, a level set function ($\phi$), which is based on a piecewise constant function at each finite element node, is used as the topological design variable(Wang et al., 2003),(Yamada et al., 2010). Based on the zero iso-boundary of the level set function, positive and negative level set regions represent the material and non-material domains, respectively. The maximum and minimum values of the level set function are restricted to 1 and -1 for applying the regularization scheme(Allaire et al., 2002),(Allaire et al., 2004),(Challis, 2010) to
functions in the design domain for the fictitious heat flux model are defined as follows:

\[ \text{undercut and the interior void. A demonstration of the manufacturability can be seen in Fig.2.} \]

The governing equations of the fictitious heat fluxes can be regarded as non-manufacturable domains for the casting and the injection molding methods such as where a fictitious heat flux model is applied. In this fictitious physical model, the fictitious heat fluxes propagate from volumes. The manufacturability evaluation of this research is based on the previous research by Sato et al.(Sato et al., 2017) to 1 for material domains and shadow regions with sufficiently large value. The non-dimensional second order diffusion tensor matrix \( A_i \) and the fictitious heat flux vector \( V_i \) propagates in the \( i \)-th manufacturing direction, defined as follows:

\[ \text{subject to } \quad G[x_f] = \int_D g(x, x_f) d\Omega - G_{\max} \leq 0 \]

where \( G \) is the density function for the design constraint and \( G_{\max} \) is the upper limit. \( f_{\text{domain}} \) and \( f_{\text{boundary}} \) are the objective functions in the design domain \( D \) and on the boundaries \( \Gamma \), respectively. Based on level set function \( \phi \), \( \chi_f \) is the characteristic function for distinguishing the material and non-material domain clearly and defined as a smoothed Heaviside function with respect to the value of the level set function. \( x \) is a nodal coordinate of the finite element.

To solve the constrained optimization problem, a total Lagrangian \( F \) is defined as the following equation.

\[ \inf \chi_f \quad F[\chi_f] = \int_D f_{\text{domain}}(x, \chi_f) d\Omega + \int_\Gamma f_{\text{boundary}}(x, \chi_f) d\Gamma \]

subject to \( G[\chi_f] = \int_D g(x, \chi_f) d\Omega - G_{\max} \leq 0 \)

To consider the manufacturability of the structure in the topology optimization process, it is needed to detect an inner void and undercut, which are inappropriate to manufacture, and define a mathematical formulation for determining those.

\[ \frac{\partial \phi}{\partial t} = -K(\bar{F}' - \tau \nabla^2 \phi) \quad \text{in} \quad D \]

\[ n \cdot \nabla \phi = 0 \quad \text{on} \quad \partial D \]

2.2. Manufacturability Evaluation

To solve the constrained optimization problem, a total Lagrangian \( F \) is defined as the following equation.

\[ \text{subject to } \quad \bar{F}' = 0, \quad \lambda G = 0, \quad \lambda \geq 0, \quad G \leq 0 \]

where \( \lambda \) is the Lagrange multiplier for satisfying the inequality constraint. It is noted that the optimal design can be obtained when the Karush-Kuhn-Tucker (KKT) conditions are satisfied. Thus, the level set function is updated by the derivative of the total Lagrangian \( \bar{F}' \) and the following time evolution equation(Otomori et al., 2012).

where \( K \) is a proportional coefficient, \( n \) is the outward unit vector normal to \( \partial D \), and \( \tau \) is a regularization parameter. According to the level set distribution with the maximum and minimum values, the regularization term \( \tau \nabla^2 \phi \) can exist only near the structural boundaries where the level set function value changes. Thus, the above time evolution equation ensures the smoothness of the level set distribution and helps to make the optimization problem well-posed. It is noted that the zero-Neumann boundary condition is applied on the outer boundaries of the design domain \( \partial D \) during the optimization process.

\[ -L_i^2 \nabla \cdot (A_i \nabla \psi_i) + L_i \nabla \cdot \nabla \psi_i = \beta \chi_f (1 - \psi_i) \quad \text{in} \quad D \]

\[ \psi_i = 0 \quad \text{on} \quad \Gamma_i \]

where \( L_i \) is the length of the design domain along the \( i \)-th direction and \( \beta \) is the parameter for convergence of \( \psi_i \) to 1 for material domains and shadow regions with sufficiently large value. The non-dimensional second order diffusion tensor matrix \( A_i \) and the fictitious heat flux vector \( V_i \) propagates in the \( i \)-th manufacturing direction, defined as follows:

\[ A_i = d_i \otimes d_i \quad , \quad V_i = V_i \otimes d_i \]
where \( V_m \) is the magnitude of the fictitious flux and \( d_i \) is the direction vector, which is defined as follows:

\[
d_i = \begin{pmatrix} \cos \theta_i \\ \sin \theta_i \end{pmatrix} \quad \text{for 2D and} \quad d_i = \begin{pmatrix} \cos \theta_i \cos \phi_i \\ \sin \theta_i \cos \phi_i \\ \sin \phi_i \end{pmatrix} \quad \text{for 3D}
\]  

where \( \theta_i \) and \( \phi_i \) are the inlet directions of the fictitious heat fluxes in 2D and 3D design spaces. An example of the manufacturing direction for milling is shown in Fig.1. The moving direction of the milling tool is indicated as black arrow. Reverse direction is not available for single milling tool that only one manufacturing direction is included in each set of manufacturing variables.

Fig. 1: Example of three-dimensional direction of milling tools

\[
(\cos \theta_i \sin \phi_i, \sin \theta_i \sin \phi_i, \cos \phi_i)
\]

\[
\varphi_i
\]

\[
\theta_i
\]

\[
\psi_i
\]

\[
\chi
\]

\[
\varphi
\]

\[
\psi
\]

\[
\psi_1
\]

\[
\psi_2
\]

\[
\psi_1 \neq 0 \quad \psi_2 \neq 0
\]

\[
\psi_1 = 0 \quad \psi_2 = 0
\]

\[
(1 - \chi) \psi_1 \psi_2 = 0
\]

![Fig. 2: Implementation of manufacturability evaluation via fictitious fluxes](image)

Fig. 2 shows the manufacturability evaluation scheme for a simple 2D structure. Since two different parting dies are needed to manufacture the example model, two heat fluxes \((i = 1, 2)\) are streamed into the design domain. There is a huge gap of heat conductivity between the material and the void domain and, hence, the shadow area where the heat flux cannot pass through is defined according to the inlet direction. \( \psi_i \) in Eq.(6) is defined to have a value only in the material domain \((\chi = 1)\) and the shadow area and \((1 - \chi) \psi_1 \psi_2\) indicates the existence of the inner void and undercut, as illustrated in Fig.2. Thus, the manufacturing difficulty of the structure is evaluated as the following equation.

\[
F_M^m = \int_D (1 - \chi) \prod_{i=1}^m \psi_i d\Omega
\]  

where \( m \) is the number of the manufacturing direction. For manufacturing cases with two manufacturing directions and three manufacturing directions, \( \psi_1 \psi_2 (1 - \chi) \) and \( \psi_1 \psi_2 \psi_3 (1 - \chi) \) represent the non-manufacturable region, respectively.
$F_M^m$ should be minimized during the optimization so that the manufacturing equipment can be smoothly separated from the structure. As shown in Fig.3, molding method as an example manufacturing method, two opponent directions can be pre-determined that converted into manufacturing directions as $\psi_1$ and $\psi_2$. Based on this settings, $F_M^m$ can be calculated that we can evaluate its manufacturability.

Fig. 3: Example of moldings of manufacturable model

2.3. Simultaneous Optimization Regarding Manufacturability and Mean Compliance

The optimization problem is formulated to minimize the mean compliance and the manufacturing difficulty simultaneously with the constraint regarding the volume of the material domain, as follows:

Objective:

- Minimize Mean Compliance
- Minimize Volume of Voids or Undercut Regions

Constraints:

- Volume of Solid – Maximum Volume $\leq 0$ in $D$

From this concept, the manufacturing objective functional can be mathematically expressed, as follows:

$$\inf_{\chi_0} F = \hat{\gamma} \int_{\Gamma_t} t \cdot u_d d\Gamma + \hat{\gamma} F_M^m$$

subject to

$$G_1 = \frac{\int_{\Omega} \chi_0 d\Omega}{V_D} - V_F \leq 0$$
$$E_1 = \int_{\Omega} t \cdot u d\Omega - \int_{\Omega} \epsilon(u) : \epsilon(\tilde{u}) d\Omega = 0$$
$$E_{iFPM} = \int_{\Omega} \beta \chi_0 (1 - \psi_i) \psi_i d\Omega - \int_{\Omega} L_i^2 \nabla \psi_i \cdot A_i \nabla \psi_i d\Omega - \int_{\Omega} \psi_i L_i V_i \cdot \nabla \psi_i d\Omega = 0$$

where a traction $t$ applied at $\Gamma_t$, the strain tensor $\epsilon(u) = 1/2(\nabla u + \nabla u^T)$, and $\hat{\gamma}$ is the weighting coefficient for making the multi-objective function. $V_D$ is the total volume of the design domain and $V_F$ is the volume fraction constraint for
material usage. \(E_1\) and \(E_{1}^{FPM}\) are weak formulations for the equilibrium equation for the displacement field \((u)\) and the governing equations of the \(i\)-th fictitious heat models, respectively. \(t\) is the traction force on the loading surface \((\Gamma_\ell)\). The test functions for constructing the weak forms are indicated as \(\tilde{u}\) and \(\psi_i\).

2.4. Design Sensitivities using Adjoint Equation

Based on the optimization problem, the Lagrangian is defined for deriving the design sensitivities, as follows:

\[
L = \tilde{h}_1 t \cdot u \text{d} \Gamma + \tilde{h}_2 \int_{\Gamma} (1 - \chi_\phi) \prod_{i=1}^{m} \psi_i \text{d}\Omega + \hat{\rho} \left\{ \int_{\Gamma} \chi_\phi \text{d}\Omega - V_{\text{max}} \right\} + \left\{ \tilde{h}_1 t \cdot \tilde{u} \text{d} \Gamma - \int_{D} \varepsilon(u) : C\chi_\phi : \varepsilon(\tilde{u}) \text{d}\Omega \right\}
\]

\[
+ \tilde{h}_2 \left\{ \int_{\Gamma} L^2 \psi_i \text{d}\Omega - \int_{D} L^2 \varepsilon(\psi_i : A_i \varepsilon) \text{d}\Omega - \int_{D} \psi_i L_i V_i \cdot \nabla \psi_i \text{d}\Omega \right\}
\]

where \(\hat{\rho}\) is the Lagrangian multiplier for the inequality volume constraint.

Using the adjoint variables for the elastic \((p)\) and fictitious heat model \((\xi)\), the stationary conditions for minimizing the mean compliance and the non-manufacturable region are calculated. The stationary conditions require that each Gateaux derivative of the Lagrangian with respect to the state variables \(\chi_\phi\) and \(\xi\) are equal to 0, as follows:

\[
L\langle \chi_\phi, \delta \chi_\phi \rangle = \int_{D} \tilde{h}_2 \left\{ \beta \left( 1 - \xi \right) \xi_{\text{ni}} - \prod_{i=1}^{m} \xi_{\text{ni}} \right\} \delta \chi_\phi + \hat{\rho} \delta \chi_\phi - \varepsilon(p) : C\delta \chi_\phi : \varepsilon(\tilde{\rho}) \right] d\Omega
\]

\[
L < \xi, \delta \xi > = \int_{\Gamma} \tilde{h}_2 \left\{ \left( 1 - \chi_\phi \right) \prod_{i=1}^{m} + \nabla \left( -L^2_i \nabla \xi_i \cdot A_i - \xi_i L_i V_i \right) - \beta \chi_\phi \xi_i \right\} \delta \xi d\Omega
\]

\[
+ \tilde{h}_1 \tilde{h}_2 \left\{ - \left( -L^2_i \nabla \xi_i \cdot A_i \nabla \xi_i - \xi_i L_i V_i \right) \right\} \delta \xi d\Gamma
\]

where \(\hat{\rho}\) and \(\bar{\xi}\) are the test functions of the weak forms. The calculated sensitivities are used to update the level set function value by using the time evolutionary equation, as follows:

\[
\frac{\partial \phi}{\partial t} = -K(L\langle \chi_\phi, \delta \chi_\phi \rangle + L\langle \xi, \delta \xi \rangle - \tau \nabla^2 \phi)
\]

Similarly, the design sensitivity for the manufacturing angle can be calculated by a differentiation of the Lagrangian and, by using the following update equation, the simultaneous optimization is performed.

\[
\frac{\partial L}{\partial \theta_i} = - \frac{\partial L}{\partial \theta_i}
\]

\[
\frac{\partial L}{\partial \theta_i} = \int_{\Omega} \tilde{h}_2 \left\{ -L^2_i \nabla \xi_i \cdot \frac{\partial A_i}{\partial \theta_i} \nabla \xi_i - \xi_i L_i \frac{\partial V_i}{\partial \theta_i} \cdot \nabla \xi_i \right\} d\Omega
\]

It is noted that both the level set function and the manufacturing directions are updated by the adjoint design sensitivity and the gradient descent, which are basically able to find a local minimum and are greatly affected by the step size. Especially, the manufacturing directions have a dominant effect on determining the multi-objective function and shape change of the structure, and can easily be driven into a locally optimal design. Thus, a meta-heuristic is needed to achieve the optimal manufacturing directions and a global optimum.

2.5. Genetic Algorithm for Optimizing the Manufacturing Directions

The genetic algorithm is a conventional heuristic based optimization method and has been successfully applied to various design optimization problems(Keane, 2006), based on natural selection and survival of the fittest. The best population, which satisfies the design objectives and the design constraints, are given more chances to survive in the next generation. The genetic algorithm is based on the theory of probability, which is a kind of stochastic algorithm. Thus, dependency on the initial setup can be reduced compare to other non-stochastic algorithms.
In the genetic algorithm, several genetic operators, such as reproduction, crossover, and mutation, are employed. The reproduction operator makes an elite group based on the fitness value that indicates a probability of satisfying the design targets. Since the elite group is selected among the current population, it is important to set the initial population for making the optimization process more efficient. The crossover and mutation are the key operators of the genetic algorithm to prevent getting stuck in a local optimum. The crossover operator selects two different combinations of the design variables, which can be called chromosomes, and creates a next generation by exchanging a part of their combinations. On the other hand, the mutation operator randomly changes a part of the chromosome. In this paper, two different processes, such as a simple and shrink mutation, are performed by changing the sign of the design variable’s value and gradually making it zero. The number of design variables, which are selected for crossover and mutation in a single chromosome, are determined by the mutation rate and crossover fraction. Since the elicited chromosomes survive through the fitness evaluation and the new chromosomes can be searched randomly regardless of the design sensitivity, the genetic algorithm has a merit to achieve a global optimum.

For 3D structural design optimization, each direction requires two angular information, therefore, at least four manufacturing angles are needed to represent two manufacturing tools and define the objective function. Even if a designer can find the desirable initial combination of the manufacturing angles by case studies, it is hard to expect dramatic changes of the angles, which dominate the fictitious heat model and the objective function, and to avoid the initial-dependency when using the gradient-based optimization method. Thus, in this paper, the genetic algorithm is applied to search as many combinations of the manufacturing angles as possible before performing the topology optimization, and find a globally optimal structure and angles.

An example generation can be seen in Fig.4. Through the mutation and the crossover, each chromosome exchanges its genes and the mutation with shrink factor generates new genes.

![Fig. 4: Example of genetic algorithm evolution](image)

### 2.6. Optimization Procedure

The overall process is briefly introduced in Fig. 5. In the first step, the design process starts from Design of Experiment(DoE). The designated number of the experimental points based on LH are spread. In step 2, based on these experimental points, the design variables that are obtained from the DoE become the initial population for the genetic algorithm. In step 3, the normalized experimental points are converted into design variables, the manufacturing directions $\theta_i$ and $\phi_i$ in the mathematical formulation, which was explained in detail in sec.2.2. In step 4, the basic settings for the topology optimization are applied to the design process. In the topology optimization, the manufacturing directions are fixed as design parameters. In step 5 and step 6, structural analyses and sensitivity analysis are conducted. Finally, in step 7, if the objective value is converged, the optimum solution is gained, otherwise, the loop repeats.

Based on the design process, two separated results can be obtained. One result from previous research, the sensitivity analysis based topology optimization shown in the left side of Fig.5, will be introduced for comparison. Another result from the proposed design process containing the genetic algorithm will be introduced. However, the way of applying the genetic algorithm is different in this paper. To gain both efficiency and accuracy, the level set-based method is applied for the structural configuration. Conversely, the genetic algorithm is applied for the optimization in the manufacturing directions. Separation of the method depending on the design variables is caused by its numerical characteristics. As determined from a previous study(Hur et al., 2019), the manufacturing directions, as variables, tend to become stuck in the local optima. Detailed settings for the genetic algorithm is applied for the specific variables. Detail settings for the genetic algorithm will be introduced in sec.3.
As shown in Fig.5, the genetic algorithm in this research is applied to the variables, the manufacturing directions. The initial population is based on the LH design. The LH sampling is focused on the stratified sampling. The LH sampling method (Iman and Conover, 1980), (McKay et al., 2000), which is known as a type of DoE, is used. For the sample space $S$ of input variable $X$, the range of input variables $X_k$ is divided into $N$ strata of equal marginal probability $1/N$ and a sample from each stratum. This leads to that each variable can be represented in a fully stratified manner. Comparison with the random sampling method, which is usually used for the initial population of genetic algorithms, is shown in Fig.6.

The most critical difference between the random sampling and the LH sampling is the number of overlapped points, shown as red-circled points in Fig.6, and this has great influence on small sized populations. In addition, the points with a similar value of a variable, shown as black rectangular regions, are also inefficient groups for the optimization.
3. Numerical Examples

To verify the effectiveness of the proposed method, two simple design problems are introduced in this section. Based on the parameter study by Sato and Hur (Sato et al., 2017), (Hur et al., 2020), the weighting coefficients \( \gamma_1, \gamma_2 \), parameter \( \beta \), regularization parameter \( \tau \), and the manufacturing directions in Eq.(16) are designated as follows,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_1 )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 1 \times 10^3 )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( 1 \times 10^{-4} )</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( \varphi_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>( \pi )</td>
</tr>
<tr>
<td>( \varphi_2 )</td>
<td>0</td>
</tr>
</tbody>
</table>

However, in this paper, a more strict manufacturing condition is applied so that the parameter, \( \beta \) in Eq.(11), for manufacturability evaluation has increased from 1 to \( 1 \times 10^3 \). The weighting coefficient directly affects the adjoint equation and the sensitivity analysis in direct proportion with the manufacturability. This increased weighting coefficient reduced about 9.6% of non-manufacturable domain that cannot be detected due to the small value of \( \psi \) for three-dimensional cases. In this paper, this parameter is set to be 1000 for three-dimensional cases to gain manufacturability with a small margin of error. For the sensitivity based optimization method, the initial setup for two manufacturing directions are \( \theta_1 = 0, \varphi_1 = 0, \theta_2 = \pi, \) and \( \varphi_2 = 0 \). On the other hand, for the proposed optimization method, the genetic algorithm is applied with the condition as follows,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Number of Generations</td>
<td>30</td>
</tr>
<tr>
<td>Number of Population</td>
<td>30</td>
</tr>
<tr>
<td>Mutation Rate</td>
<td>0.5</td>
</tr>
<tr>
<td>Elite Count</td>
<td>3</td>
</tr>
<tr>
<td>Shrink Parameter</td>
<td>1</td>
</tr>
<tr>
<td>Crossover Fraction</td>
<td>0.5</td>
</tr>
</tbody>
</table>

The initial population of the proposed method is set through LH sampling for efficiency.

3.1. Cantilever Beam

For the first engineering application, a cantilever beam studied by Wang et al. (Wang et al., 2003) is selected. As shown in Fig.7, 1000N/m² is applied to one surface on the right side, and four fixed surfaces are located on the opposite side. Based on these surfaces, a rectangular parallelepiped shape is designated as fixed design domain.

As shown in Fig. 8, the mean compliance of the optimal design obtained by the proposed method for all volume constraints is reduced compared to that of the sensitivity based case. Especially, for the sensitivity based case with 0.3 as the volume constraint, the mean compliance of the optimal case through the sensitivity based case and the proposed method are 7.12, and 6.61, respectively, that is, approximately 7.2% reduction. In this model, the fixed surfaces and the load surface are located and concentrated on opposite sides. Thus, the structure must endure the vertical force to reduce the mean compliance with supporters in this direction. As shown in Fig. 9 and Table. 3, the manufacturing direction 1 for both the sensitivity based case and the proposed method are similar, however, the manufacturing directions 2 for both cases are quite different from each other. The manufacturing direction 2 of the sensitivity based case is almost vertical to its direction 1. Conversely, the manufacturing direction 2 of the proposed method is relatively similar to direction 1.
The von-Mises stress (Mises, 1913) is shown in Fig. 10. The blue-colored domain indicates structures with small pressure which are less effective to minimize the objective. Clearly, the GA based model showed less blue-colored domain which means that each part of the structure is efficient. In addition, the stress is uniformly distributed all around the
Table 3: Optimization results of the manufacturing directions of the cantilever beam (Volume Constraint: 0.3)

<table>
<thead>
<tr>
<th>Case</th>
<th>$\theta_1$</th>
<th>$\phi_1$</th>
<th>$\theta_2$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity Based</td>
<td>218°</td>
<td>280°</td>
<td>142°</td>
<td>93°</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>217°</td>
<td>210°</td>
<td>296°</td>
<td>259°</td>
</tr>
</tbody>
</table>

structure which ensures the effectiveness. Since the fixed surfaces and the load surface are x-z plane symmetric, we expected a symmetric-shaped obtained structure. However, due to the manufacturing constraints, obtained structures from proposed method are not symmetric. Thus, the location and the size of the fixed surfaces can be modified base on these results.

Fig. 10: von-Mises stress contour of three-dimensional cantilever beam (Volume Constraint:0.3, Left: Sensitivity Based, Right: Genetic Algorithm Based)

The generation history of elite members, mean objective values of each generation and the constrains are shown in Fig.11. During the evolution, the structural design of the elite members is developed and the manufacturing constraints are stabilized. Generally, the objective profile shows extreme oscillations in the early stage, however, the LH sampling enabled early stabilization. The best-of-generation profiles showed two phases of evolution. In the first phase, the evolution of the elite member is concentrated on one of the objectives, minimization of the mean compliance. During the second phase, the evolution is focused on another objective and constraint, the manufacturing constraint. It can be also seen in Fig.12, that the shape of the structures become more simpler to satisfy the manufacturing constraint. With more generations, minor improvements can be achieved by means of fine tuning, however, the basic design is expected to remain in similar structure.
3.2. Mechanical Part

Another three-dimensional mechanical part is shown in Fig. 13. The fixed surface is located on the inner-side of one pipe and the load surfaces are located on both sides of the opposite pipe. These forces generate torsion on the part. The center area is designated as the design domain.

Fig. 13: A three-dimensional numerical example (Mechanical Part)

Fig. 14 showed mean compliance of each case. In this graph, comparison between the proposed method and the
sensitivity based method. This result presented that for all volume constraints, the proposed method showed reduced mean compliance.

![Fig. 14: Three-dimensional mechanical part results](image1)

![Fig. 15: Optimization results of three-dimensional mechanical part (Volume Constraint: 0.1, Left: Sensitivity Based, Right: Genetic Algorithm Based, Red Arrows: Moving Direction of Manufacturing Tools)](image2)

![Image of Table 4](image3)

**Table 4: Optimization results of the manufacturing directions of the mechanical part (Volume Constraint: 0.1)**

<table>
<thead>
<tr>
<th>Case</th>
<th>$\theta_1$</th>
<th>$\phi_1$</th>
<th>$\theta_2$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity Based</td>
<td>338°</td>
<td>35°</td>
<td>22°</td>
<td>217°</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>126°</td>
<td>0.00°</td>
<td>0.00°</td>
<td>45°</td>
</tr>
</tbody>
</table>

Similar to the previous cantilever case, a set of manufacturing directions for both methods showed similarity. However, as shown in Table 4, the remaining manufacturing direction of the sensitivity based method is relatively parallel to its direction 1. Conversely, the manufacturing direction 2 of the proposed method is almost vertical to its direction 1. As the mechanical part is based on torsion of the structure, the set of manufacturing directions of the proposed method is more
efficient to build the design. In addition, the outer surface of the structure obtained from the proposed method reached the boundary of the design domain. This secures the structure with larger radius which helps provide robustness against the torsional force.

In addition, as shown in Fig. 16, the sensitivity based case showed more blue colored domain than that of the proposed method. This indicates the uniform distribution of stress is shown in the case of the proposed method.

The generation history of elite members, mean objective values of each generation and the constrains are shown in Fig.17. Similar to the Cantilever Beam case, the structural design of the elite members is developed and the manufacturing constraints are stabilized during the evolution. Two phases of evolution are also shown in this case. In the first phase, the evolution of the elite member is concentrated on the mean compliance, and quickly converged. During the second phase, the evolution is focused on the manufacturing constraint. As shown in Fig.18, the structure tends to expand toward the boundary of the design domain which has advantages on the volume constraint and mean compliance.
3.3. Sensitivity Based Optimization Initialized by an Elite Population

In addition to the genetic algorithm based optimization, the sensitivity analysis initialized by an elite population of first generation of the genetic algorithm is added for comparison. In this case, the initial values of the input variables are based on the elite population of the first generation of the genetic algorithm based optimization. This combined method may reduce computation time, and meanwhile, it can acquire similar results from that of the GA based method. However, as shown in Table 5, the mean compliance of each case is between that of sensitivity analysis based optimization and genetic algorithm based optimization.

Table 5: Results from the sensitivity based optimization initialized by the genetic algorithm

<table>
<thead>
<tr>
<th>Case (Volume Constraint)</th>
<th>Mean Compliance</th>
<th>$\theta_1$</th>
<th>$\phi_1$</th>
<th>$\theta_2$</th>
<th>$\phi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever Beam (0.30)</td>
<td>7.0</td>
<td>199°</td>
<td>267°</td>
<td>341°</td>
<td>275°</td>
</tr>
<tr>
<td>Mechanical Part (0.10)</td>
<td>12.6</td>
<td>141°</td>
<td>74°</td>
<td>321°</td>
<td>74°</td>
</tr>
</tbody>
</table>

As shown in Fig. 19, for both cases, the stress distribution has become more equal and the manufacturing directions are similar to that of the proposed method. This represents the effectiveness of the initial value for the sensitivity analysis based optimization. On the other hand, even with the optimized input variables from the genetic algorithm based optimization, the geometric characteristic is similar to that of the sensitivity based cases. Hence, a conclusion that the results from the sensitivity analysis based optimization has a drawback of converging to local optimum values can be drawn.
3.4. Level Set Based Optimization regarding Molding and Casting

Another manufacturing method, molding and casting, has published by Sato et al. (Sato et al., 2017). A huge difference between the manufacturing methods is manufacturing angle as the design variable. For two directional milling method, each manufacturing directions can be independently decided. However, for molding and casting method, one manufacturing direction is depending on another manufacturing direction. Furthermore, for most cases, both directions are decided in perpendicular or parallel to the fixed surfaces for practical reasons. Based on these conditions, I have setup a case regarding molding and casting condition, and compared the results with the designs obtained by the proposed method.

<table>
<thead>
<tr>
<th>Optimization Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (Volume Constraint)</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Cantilever Beam (0.30)</td>
</tr>
<tr>
<td>Mechanical Part (0.10)</td>
</tr>
</tbody>
</table>

As shown in Fig. 20, obtained structures are strictly aligned with the manufacturing directions, parallel to the z-axis in this case, to gain the manufacturability. The results are shown in Table 6 showed relatively similar to that of the sensitivity based method rather than that of the proposed method. Hence, both molding method based and sensitivity based methods can be regarded as local optima and the proposed method has been approached to the global optimum more closely.

Fig. 20: von-Mises stress contour of manufacturable structure regarding molding and casting method (Left: Cantilever Beam, Right: Mechanical Part, Red Arrows: Moving Direction of Manufacturing Tools)
4. Conclusion

In this paper, the level-set based topology optimization regarding manufacturability inevitably contains non-linearity due to the fictitious fluxes. To deal with this characteristic, the proposed method has introduced.
- A heuristic based approach, a genetic algorithm in this paper, was suggested and applied. As a result, the heuristic based approach, which has advantage of non-linearity, showed improved performance.
- Two complex structures were introduced as engineering examples for the verification and successfully proved the effectiveness.
- Comparison with 1st generation of the GA based results underlined the non-linearity of the fictitious flux based method.

Through the fictitious heat, virtual shadow evaluating manufacturability has evaluated. Based on this optimization method, two numerical examples are introduced. For a simple numerical example case, a cantilever beam, maximum reduction of the mean compliance using proposed method was approximately 7.2 % compare to the sensitivity based method. Especially, for a mechanical part case results in more complex shape, which contains torsional forces, maximum reduction of the mean compliance using proposed method was approximately 94.2 % compare to the sensitivity based method. This proved the utility of the proposed design method for both cases, and for a complex design case, the effectiveness can be highlighted. In addition, the residual errors of the computational fluid analysis and sensitivity analysis were handled by weighting coefficients in the adjoint equations.

References

Chapman, C. D., Saitou, K., and Jakiela, M. J. Genetic algorithms as an approach to configuration and topology design, (1994).


