Influence of substrate twisting on Young's modulus measured by four-point bending test for thermal barrier coatings

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Abstract

The influence of specimen twisting on Young's modulus of thermal barrier coatings (TBCs) was studied using four-point bending tests based on ISO 19477 and three-dimensional finite element analysis (3D FEM). Twisted substrate specimens were four-point bent using a jig with or without pin rotation to investigate the influence of pin contact conditions. The Young's modulus was calculated by strain gauge and maximum-displacement methods. The results showed that load-strain and load-maximum displacement curves of the twisted substrate specimens showed linear relationships for fixed and rotating pin conditions. The decrease in the Young's modulus with increasing twisting angle was not observed for the strain gauge method but was for the maximum-displacement method. However, the maximum decrease of around 5% for the maximum-displacement method was improved by pin rotation. 3D FEM results showed that the Young's modulus obtained by the FEM quantitatively agreed well with the experimental value. Analyzed strain distribution clarified the insensitivity to specimen twisting for the strain gauge method because of the small difference in the strain at the center of the specimen where the strain gauge was attached. The decrease in the Young's modulus of the twisted specimen for the maximum-displacement method corresponded to the inhomogeneous curvature distribution of the specimen. The influence of the specimen twisting of TBCs on Young's modulus was evaluated by 3D FEM. The same tendency as that for the substrate specimen was observed, but the amount of decrease became large. A decrease around 5 times larger than that for the substrate specimen was observed with a twisting angle of 0.058 by the maximum-displacement method under fixed pin conditions. However, the decrease was improved by pin rotation. The analysis result showed that the strain gauge method had sufficient accuracy to evaluate the Young's modulus of twisted TBC specimens up to a twisting angle of 0.058.

Key words: Thermal barrier coatings, Young's modulus, Twisting, Four point bending test, Finite element analysis

1. Introduction

Thermal Barrier coatings (TBCs), consisting of an yttria-stabilized zirconia (YSZ) layer (top coat) and a metallic
layer (bond coat), are widely applied to gas turbine blades in power generation facilities to improve thermal efficiency (Goward, 1998; Nicholls, 2003; Padture et al., 2002; Zhu and Miller, 2000). Young's modulus of the TBCs is an important parameter in designing gas turbine blades. However, accurate measurement of the Young's modulus is generally difficult because the Young's modulus of the TBCs is generally lower and the thickness is much smaller than those of the substrates. Therefore, it is important to develop accurate measurement methods.

The Young's modulus is measured using a variety of test methods including impulse excitation (Tillmann et al., 2013), nano-indentation (Basu et al., 1999; Guo and Kagawa, 2006; Jang and Matsubara, 2005; Thompson and Clyne, 2001; Tillmann et al., 2013; Wallace and Hlavsky, 1998), and bending methods (Schwingel et al., 1998; Thompson and Clyne, 2001; Tillmann et al., 2013). The nano-indentation method has the advantage of easy specimen preparation, but the obtained Young's modulus tends to be larger than the actual value owing to material densification (Tillmann et al., 2013). The impulse excitation method has the merit of simple testing apparatus, but the material density is required to obtain the Young's modulus (Tillmann et al., 2013).

The bending method evaluates the Young's modulus using the theory of composite beams, and this method is widely used to evaluate Young's modulus of variety of materials. Waki et al. (2014) proposed an accurate evaluation method of the Young's modulus of TBCs using a four-point bending test, where the Young's modulus was evaluated from the gradient of load-strain or load-displacement curves and the parameters influencing the accuracy of the Young's modulus was discussed both theoretically and experimentally. The four-point bending method is issued as an ISO standard (ISO standard, 2016).

For the four-point bending method, the measurement error becomes large with decreasing relative thickness of the top coat \( h_t/(h_t+h_b) \), where \( h_s, h_b \) and \( h_t \) are the thickness of substrate, bond coat, and top coat, respectively. When the +5% measurement error is introduced at a relative thickness of 0.1, the measurement error of the Young's modulus becomes larger than 30% (Waki et al., 2014). Hence, the thicknesses of the substrate, bond coat, and top coat are defined in the ISO standard as \( 1.5 \text{ mm} \leq h_s \leq 3.0 \text{ mm}, h_s/20 \leq h_b, 0.1 \text{ mm} \leq h_b, 0.20 \text{ mm} \leq h_t \) and \((h_t+h_b)/10 \leq h_s\), to guarantee the accuracy of the data.

The TBCs are frequently both twisted and bent owing to the thermal stress introduced during thermal spraying, and to the best of our knowledge, the influence of the twisting and bending has not been previously studied. Quantitative evaluation of the influence of specimen twisting is important in deciding acceptable twisting angle in the ISO standard. In this study, the influence of the twisting of the substrate specimen on the Young's modulus is clarified by experiment and finite element modelling (FEM), and the influence on the TBCs was analyzed using FEM.

2. Experimental methods

2.1 Four-point bending test

The material used in this study is austenitic stainless steel (JIS:SUS304). Figure 1 shows the shape and dimensions of a specimen, which are based on ISO 19477 (ISO standard, 2016). Specimens with twisting angles of \( \gamma = 0, 0.0019, 0.0038, \) and 0.0058 were cut from cylindrical steel using a wire-cut electrical discharging machine.

![Fig. 1 Shape and dimensions of a specimen. Specimens with a twisting angle of \( \gamma \) were machined using a wire-cut electrical discharging machine.](image)

Figure 2 shows details of the four-point bending test jig. The distance between the pins is 30 mm, which follows ISO 19477 (ISO standard, 2016). To avoid uneven contact of the pins, all pins can rotate along the x-axis. Loading and unloading was performed at a displacement speed of 0.5 mm/min under a maximum load \( P_{max} \) of 100 N. During the
bending test, strain at the back center $\varepsilon$ and maximum displacement $y_m$ of the specimen were measured using a strain gauge (KFG-1-12-D16-16LiM2s, Kyowa Electronics Co. Ltd.) and laser displacement sensor (LS-7000, Keyence Co. Ltd.), respectively. The four-point bending test was repeated three times to obtain accurate data.

The Young's modulus was calculated on the basis of material mechanics theory from load $P$-strain $\varepsilon$ and load $P$-maximum displacement $y_m$ curves using the following equations:

$$E = \frac{3a}{bh^2} \left| \frac{\Delta P}{\Delta \varepsilon} \right|$$  \hspace{1cm} (strain gauge method)  \hspace{1cm} (1)$$

$$E = \frac{23a^3}{4bh^4} \left| \frac{\Delta P}{\Delta y_m} \right|$$  \hspace{1cm} (maximum displacement method)  \hspace{1cm} (2)$$

where $a$ is the distance between pins, and $b$ and $h$ are the width and thickness of the specimen, respectively. $|\Delta P/\Delta \varepsilon|$ and $|\Delta P/\Delta y_m|$ were determined from the gradient of the load-strain and load-displacement curves by the least-squares method.

**2.2 Finite element analysis**

Deformation behavior and strain of the substrate specimen during the four-point bending test were elastically analyzed using 3D FEM (FEM, MARC 2014, MSC software). The model is shown in Fig. 3 (3D elastic deformation, eight-node hexahedral elements, nodes:21489, elements:17082). A four-point bending load was applied to the specimen with a twisting angle of $\gamma$ under the conditions of fixed and rotating pins. The elastic constant and Poisson's ratio of the substrate and pin elements are 198 GPa and 0.30, which are commonly known as those of austenitic stainless steel. Load $P$ was gradually applied to the inner pins until $P$ reached 90 N. The value of the friction coefficient between the pin and specimen was set as 0.

**Fig. 2** Detail of four-point bending test jig. The pins can rotate to avoid uneven contact.

**Fig. 3** Finite element analysis model. 3D modelling of the specimen with twisting angles and pins was performed.
3. Results and discussions

3.1 Four-point-bending test

Figures 4 and 5 show load $P$ - strain $\varepsilon$ and load $P$ - maximum displacement $\gamma_{\text{m}}$ curves obtained from the four-point bending test under the conditions of (a) fixed and (b) rotating pins. Linear relationships are observed for the load-strain and load-maximum displacement curves. For the load-strain curve of the fixed pin (Fig. 4(a)), the gradient slightly decreases with increasing twisting angle. However, the amount of decrease is very small for the rotating pin (Fig. 4(b)). The same trend is observed for the load-maximum displacement curves (Fig. 5).

Fig. 4 Load-strain curves of twisted specimens obtained by strain gauge method under (a) fixed pin and (b) rotating pin conditions. Linear relationships are observed.

Fig. 5 Load-strain curves of twisted specimens obtained by the maximum-displacement method under (a) fixed pin and (b) rotating pin conditions. A decrease in the gradient is observed for the fixed pin condition.

Relative Young's moduli $E/E_0$ of the specimens measured by the strain gauge method are shown in Fig. 6, where $E_0$ is the Young's modulus at $\gamma = 0$. The symbols and error bars indicate the average and range between maximum and minimum values of three measurements. The relative Young's modulus slightly decreases with increasing twisting angle. The amount of decrease for fixed pins is slightly larger than that for rotating pins. The strain gauge method is insensitive to both specimen twisting angle and pin fixing condition, as the maximum decrease is within 2%.
In contrast to the strain gauge method, the relative Young's modulus values obtained by the maximum displacement method under fixed pins are sensitive to the twisting angle and the Young's modulus for $\gamma = 0.0058$ is around 5% lower than that for $\gamma = 0$, as shown in Fig. 7. However, the decrease with increasing twisting angle becomes negligible for the rotating pin condition.

3.2 Finite element analysis

3.2.1 Load - strain, load - maximum displacement curves

FEM was performed to investigate the influence of specimen twisting on the Young's modulus. Figure 8 shows an example of deformation of the substrate specimen with a twisting angle $\gamma$ of 0.058 at the cross-section of A in Fig. 3. Even contact between the specimen and pin is not observed for the fixed pin, but is observed for the rotating pin because of pin rotation. The twisting of the specimen along the x-axis for the fixed pin is smaller than that for the rotating pin.

Fig. 8 Deformation of specimen at cross-sectional plane A in Fig. 3. The contact state between the specimen and pin at $P = 90$ N is shown.
The load $P$ - strain $\varepsilon$ and load $P$ - maximum displacement $y_{max}$ curves obtained by FEM under the conditions of (a) fixed and (b) rotating pins are shown in Figs. 9 and 10. Similar to the experimental data, the loads show linear relationship with strain and maximum displacement. For the load-strain curves, the influence of the specimen twisting is almost negligible. However, for the load-maximum displacement curves under the fixed pin condition, the gradient of the curves decreases with increasing specimen twisting angle.

Relative Young's modulus $E/E_0$ of the specimens measured by FEM are shown in Figs. 6 and 7 as solid and broken lines. The FEM result agrees well with experimental values.

3.2.2 Strain and displacement distribution

To discuss the influence of specimen twisting and pin fixing condition, it is important to analyze the strain and displacement distribution of the substrate specimen. Surface strain distributions along the $x$-axis of the twisted substrate specimen between the center of the specimen and inner pin at $P = 90$ N are shown in Fig. 11. Homogeneous strain distribution is obtained around the center of the specimen, but strain concentration occurs as the inner pin is approached. The amount of strain concentration is larger for the fixed pin than that for the rotating pin.
Strain distributions along the $x$-axis of untwisted and twisted substrate specimens under fixed and rotating pin conditions are shown in Fig. 12. Uniform strain distribution is observed for the untwisted specimens, and non-uniform distribution is observed for the twisted specimens. The strain at $z = 0$ is important because the strain gauge is attached here for strain gauge method, and the difference between untwisted and twisted specimen is within 1% for both pin fixing conditions. Hence, the insensitivity to specimen twisting for the strain gauge method (Fig. 6) can be explained by this result.

Fig. 11 Strain distribution of (a) fixed pin and (b) rotating pin conditions. Strain concentration is observed around inner pin.

Fig. 12 Strain distribution of fixed and rotating pin conditions along $x$-axis. Inhomogeneous strain is observed for the twisted specimen.
Curvature, $\kappa$, between the inner pins at $z = -5$ mm, 0 mm, and 5 mm under (a) fixed and (b) rotating pin conditions is shown in Figs. 13(a) and (b). For the fixed pin condition, the curvature changes with increasing and decreasing $y$ at $z = -5$ mm and 5 mm due to the uneven contact between the inner pin and specimen (Fig. 11). However, the concentration is homogenized by the pin rotation. This result corresponds to the improvement of sensitivity to specimen twisting by pin rotation at the maximum displacement method (Fig. 7), because the maximum displacement can be obtained by the integral of curvature along the $y$-direction.

3.2.3 Analysis of TBCs

Influence of the twisting of TBCs specimen on the Young’s modulus was evaluated using FEM instead of experimentally, because preparation of twisted TBCs without bending is difficult. In general, the specimen is bent and twisted by the residual stress introduced during spraying. Hence, the TBC specimens should be both bent and additionally twisted if TBCs are sprayed on the twisted substrate. The thicknesses of substrate, bond-coat, and top-coat layers of the FEM were 2 mm, 100 $\mu$m, and 200 $\mu$m, respectively. The elastic constants and the Poisson’s ratios of the substrate, bond-coat, top-coat layers were 198 GPa and 0.30, 100 GPa and 0.29, and 30 GPa and 0.31, which are commonly known values of the Young’s modulus and Poisson’s ratio of the substrate, NiCoCrAlY bond-coat, YSZ top-coat, respectively.

The load $P$ - strain $\varepsilon$ and load $P$ - maximum displacement $y_m$ curves obtained by FEM under the conditions of (a) fixed and (b) rotating pins are shown in Figs. 14 and 15. The same tendency as the substrate analysis (Figs. 9 and 10) is observed. Other analysis (strain distribution, displacement distribution) also shows the same tendency as the substrate analysis.

Figure 16(a) and (b) shows the analysis results based on the strain gauge and maximum displacement methods. The Young’s modulus was calculated on the basis of ISO 19477 (ISO standard, 2016). The broken lines are the result for the
substrate (Figs. 6 and 7). A larger decrease in the relative Young's modulus was observed for the TBC specimen than that for the substrate. The lowest value around 0.87 was observed for the maximum-displacement method under fixed pin conditions at a twisting angle of 0.0058, which means a measurement error of 13% is estimated if the Young's modulus is measured using the maximum-displacement method under fixed pin conditions. This value is around 5 times larger than that for the substrate specimen. However, the decrease is remarkably improved by pin rotation. The errors of the rotating pin condition are 1% and 4% for the strain gauge and maximum displacement methods, respectively. The decrease (error) becomes slightly larger for small thickness (100 µm) and small Young's modulus (15 GPa) of the top-coat at a twisting angle of 0.0019 under rotating pin condition, but the amount is around 0.98 and 0.99, respectively.

The error of the strain gauge method under fixed pin conditions (as standardized in ISO 19477) is within 5%. We can note that the measurement method standardized in ISO 19477 has sufficient resilience to specimen twisting, considering the measured twisting angle of the TBCs (YSZ: 0.2 mm in thickness, NiCoClAlY: 0.1 mm, substrate: 2 mm) is less than 1.0×10⁻³.

Fig. 14 Load-strain curves of twisted TBCs obtained by FEM under (a) fixed pin and (b) rotating pin conditions.

Fig. 15 Load-maximum displacement curves of twisted TBCs obtained by FEM analysis under (a) fixed pin and (b) rotating pin conditions.
4. Conclusions

The influence of specimen twisting on Young's modulus of thermal barrier coatings measured by the four-point bending method based on the ISO 19477 standard was studied by experiment and FEM. Twisted specimens were four-point bend tested, where the pin could rotate around the specimen length direction, and the Young's modulus was calculated by strain gauge and maximum displacement methods. In addition, 3D FEM was performed to discuss the influence of specimen twisting. The results are as follows:

1. Load-strain and load-maximum displacement curves of the twisted substrate specimens obtained by four-point bending test showed linear relationships for fixed and rotating pin conditions.

2. A larger decrease in the Young's modulus was obtained for the maximum displacement method than that for the strain gauge method, and the decrease of around 5% at a twisting angle of 0.0058 could be reduced to almost zero by pin rotation.

3. The Young's modulus obtained by FEM agreed well with the experimental value.
4. FEM showed that difference in the strain was small at the center of the specimen where the strain gauge was pasted. Inhomogeneous curvature distribution of the specimen was observed for the twisted specimen under fixed pin condition, but the distribution was improved by the pin rotation, which corresponded to the change observed in the maximum displacement method.

5. The influence of the twisting of the thermal barrier coating specimen evaluated by FEM showed the same tendency as that for substrate, but the amount of decrease became large. A maximum decrease around 13% was observed for the specimen with a twisting angle of 0.058 using the maximum displacement method under fixed pin conditions. However, the decrease was improved to 4% by pin rotation.

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