Fast quasi-implicit NOSB peridynamic simulation based on FIRE algorithm

Yoshinori SHIIHARA***, Shoki TANAKA*** and Nobuhiro YOSHIKAWA**

*School of Engineering, Toyota Technological Institute
2-12-1 Hisakata, Tenpaku-ku, Nagoya, 468-8511, Japan
E-mail: shiihara@toyota-ti.ac.jp
**Institute of Industrial Science, The University of Tokyo
4-6-1 Komaba, Meguro-ku, Tokyo, 153-8505, Japan
***School of Engineering, The University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8654, Japan

Abstract
The peridynamics is a particle methods considered superior to the finite element method in describing fracture phenomena since fracture can be simply expressed as bond breaking between the peridynamics particles. The Non-Ordinary State-Based (NOSB) peridynamics is a variant of the particle method which allows us to implement constitutive laws as in the finite element method. To maintain its computational accuracy, parameters used in the peridynamic simulations needs to be optimized through parameter searches, hence the peridynamics requires a fast quasi-implicit algorithm which allows to directly compare the results to the one obtained by the finite element method. The FIRE algorithm is such an algorithm well examined through molecular dynamics studies and is easy to implement existing explicit algorithms. The objective of this study is to describe the details of the implementation of the FIRE method to the peridynamic elastic and elasto-plastic computational codes and the effectiveness through the comparison with another quasi-static algorithm, the energy based relaxation method. The computational results show the effectiveness of the algorithm and some examples of the parameter search in simple problems such as elastic and elasto-plastic deformation analyses and a stress analysis near a crack.

Keywords: Peridynamics, Quasi-implicit algorithm, Elasto-plasticity, Finite element method, Continuum mechanics

1. Introduction
The peridynamics (Silling 2000) is a kind of the particle method mainly developed to treat failure and fracture problems in continuum mechanics, considered superior to the Finite Element Method (FEM) in describing fracture phenomena. The peridynamics can simply express fracture as bond breaking between the peridynamics particles while this is not the case in the FEM which requires special treatments such as the cohesive element (e.g., Muñoz et al. 2006) to deal with the discontinuities of stress and strain emerging after fracture. Because of this advantage, the peridynamics has been examined in various applications such as fracture in concrete (Gerstle et al. 2007), glass (Kilic & Madenci 2009), complex (Hu et al. 2015), and polycrystalline materials (Meo et al. 2016); crack initiation, propagation and branching in brittle and ductile materials (Bobaru & Hu 2012)(Madenci & S. Oterkus 2016)(Zhou et al. 2016)(Panchadhara et al. 2016); high-speed impact and explosion problems (Xu et al. 2008)(Diyaroglu et al. 2016), etc.

The main disadvantage of the peridynamics is its computational cost. As also shown in this paper, it is well known that the method requires large number of degrees of freedom to obtain an accurate outcome comparable to that in the FEM. Hence, searching a parameter set providing a good balance between the accuracy and cost is particularly important in the peridynamic simulation. For this purpose, the peridynamic result should be compared with the one obtained by the FEM or the theory of elasticity through an implicit static analysis because such comparison requires with an equilibrium state. Since many peridynamic applications in past studies were done as explicit dynamic simulations to treat dynamic
phenomena such as impact problems, a fast quasi implicit algorithm easily implemented to existing peridynamic explicit codes is desirable since finding an optimum parameter set may require many trial calculations.

A quasi-implicit algorithm is basically not an efficient way because the algorithms can find a mechanically stable equilibrium after a vast number of explicit time integration steps. The FIRE algorithm proposed in Bitzek et al. 2006 was originally developed for molecular dynamics as a way to find such an equilibrium configuration in atomic systems. They showed that the FIRE algorithm has more excellent performance compared to the conjugate gradient method and comparable performance to the BFGS method as a quasi-Newton method. Such an algorithm seems suitable to be implemented to the quasi implicit peridynamic simulation. There is an example where the FIRE algorithm has been employed to compute stress intensity factors using the peridynamics (Panchadhara et al. 2016), however, the details of the algorithm and its effectiveness has not been yet discussed especially through the comparison with the other quasi implicit algorithm applied to the past peridynamic simulations such as the dynamic relaxation method (e.g., Namadchi & Alamatian 2016) or the Energy Based Relaxation (EBR) method (Kondo et al. 2007).

The objective of this study is to describe the details of the implementation of the FIRE method to the peridynamic elastic and elasto-plastic computational codes and to demonstrate the effectiveness through the comparison with another quasi-static algorithm, the EBR method. While several formulations have been proposed for the peridynamics, here we showed an implementation for the Non-Ordinary State-Based (NOSB) formulation (Silling et al. 2007)(Warren et al. 2009). This is because the original formulation called bond-based peridynamics was not able to straightforwardly deal with constitutive laws which has been carefully developed through the past history of the FEM. In this paper, we first describe the formulation of the NOSB peridynamics to show the parameters in peridynamics influencing the accuracy. Then we describe the details of the FIRE algorithm and the flowchart for its implementation to elastic and elasto-plastic simulations. The rest are devoted to the computational results showing the effectiveness of the algorithm and some fundamental examples of the parameter search in several simple problems such as elastic and elast-plastic deformation analyses and a stress analysis near a crack in an elastic body.

2. Computational methods

2.1 Peridynamics

2.1.1 Formulation

In the NOSB peridynamics, an individual concept called "state" has been utilized for the description of physical quantities, i.e., stress, strain, force, etc. While the concept is useful to simply describe equations in peridynamics because it represents a physical quantity of all the particles by a single vector, it may look unaccustomed to most researchers, especially, familiar with the continuum mechanics. Hence, in this paper, we described the formulation in a conventional manner without the state.

The NOSB peridynamic formulation was based on the total Lagrangian description. The biggest difference of the NOSB peridynamics from the standard one (bond-based peridynamics) lies in its formulation which expresses the deformation via the deformation gradient tensor. While the tensor in the continuum mechanics is basically defined as a derivative of the displacement with respect to the material coordinates, the derivative cannot be computed in the peridynamics because the displacement as well as the other quantities are defined only on particle positions in the peridynamic formulations. Thereby the deformation gradient for particle $i$ is given as an approximated form,

$$
F_i = \left[ \sum_{j=1}^{n} (x_j - x_i) \otimes (X_j - X_i) V_{ij} \right] \left[ \sum_{j=1}^{n} (X_j - X_i) \otimes (X_j - X_i) V_{ij} \right]^{-1},
$$

where the symbol $\otimes$ indicates a tensor product; and $X_i$ and $x_i$ are the position of particle $i$ in the material and spatial coordinates, respectively. As shown in this equation, the deformation gradient on the particle $i$ is determined from the information on the other particles $j$, which locates close to the particle $i$ within a distance called influence radius, $\delta$. As in Fig. 1, each particle occupies some volume of the material. $V_{ij}$ in Eq. (1) is the volume of $j$ inside the influence radius of the particle $i$. This approximation is given so that the approximated deformation gradient can represent the relative displacements between the particles $i$ and $j$ accurately as possible and is also employed in the molecular dynamics
Simulations (Gullett et al. 2008).

Fig. 1 Schematic of interaction between particles $i$ and $j$. Only particles within the horizon $\delta$ from particle $i$ can interact. The definition of $V_{ij}$ is also shown in this figure.

The equation of motion in the NOSB peridynamics is obtained from the minimization of the Lagrangian:

$$\rho_i \dot{u}_i = \sum_j (f_{ij} - f_{ji})V_{ij} + b_i, \quad (2)$$

where $\rho_i$, $u_i = x_i - X_i$, and $b_i$ are the density of the material, the displacement and the body force on the particle $i$, respectively. The first term in the right hand of Eq. (2) is the inner force acting on the particle $i$, which consists of the force $f_{ij}$ on $i$ from $j$ and reaction $f_{ji}$ as the force on $j$ from $i$. As discussed in the introductory part, the force can be estimated based on the constitutive law needed to describe the mechanical behavior of the material. The procedure is as follows. First, infinitesimal change in a deformation gradient tensor, $\boldsymbol{F}' \rightarrow \boldsymbol{F}'$, is considered under infinitesimal change of relative current positions $y_{\gamma \eta} = x_{\gamma} - x_{\eta}$ of the particles, $\Delta y_{\gamma \eta}$:

$$\boldsymbol{F}' - \boldsymbol{F} = \sum_j (\Delta y_{\gamma \eta} \otimes (X_j - X_i))K^{-1}. \quad (3)$$

Each component of Eq. (3) can be represented as follows after some modification:

$$F'_{\gamma \eta} = F_{\gamma \eta} + \sum_j \sum_k \delta_{\gamma \eta} (X_{kj} - X_{ki})K^{-1}_{\gamma \eta}V_{ij}$$

$$= F_{\gamma \eta} + \sum_j \sum_k \delta_{\gamma \eta} (X_{kj} - X_{ki})K^{-1}_{\gamma \eta}V_{ij}, \quad (4)$$

where the summation for $k$ and $l$ run over the directions, $x$, $y$, and $z$. $F_{\gamma \eta}$ is the $\gamma \eta$ component of the approximated deformation gradient tensor of particle $i$. In Eq. (4), Dirac's delta is used to exclude $\Delta y_{\gamma \eta}$ from the summation for $j$ and $k$ for the formulation described later. Under the change $\Delta y_{\gamma \eta}$, the change of strain energy $W$ of the system concerned becomes

$$W' - W = \frac{\partial W}{\partial F'_{\gamma \eta}} (F'_{\gamma \eta} - F_{\gamma \eta})$$

$$= P_{\gamma \eta} (F'_{\gamma \eta} - F_{\gamma \eta}), \quad (5)$$

where and $P_{\gamma \eta}$ is the 1st Piola-Kirchhoff stress tensor given as the partial derivative of the strain energy with respect to
the deformation gradient tensor. The stress tensor can be obtained from the deformation gradient tensor through constitutive laws to which the material considered obeys as done in the finite element method. Its practical expression can be found elsewhere (Belytschko et al. 2006). Substituting Eq. (4) to Eq. (5), we have

\[ W' - W = P \sum_{i,j} \left( \sum_{i,j} \delta_{ij} (X_{ij} - X_{ij}) K_{ij} \nu_i \nu_j \right) \Delta y_{ij} . \] (6)

Assumption that the energy change is given by the work induced by the displacement,

\[ W' - W = \sum_j f_j \Delta y_j . \] (7)

leads to the following relationship between the force density and the stress, \( f_j = PK^{-1}(X_j - X_i) \). Substituting this relationship to Eq. (2), the following equation of motion in NOSB peridynamics is eventually obtained:

\[ \rho \ddot{u}_i = \sum_j \left( P \kappa^{-1}_{ij} + P \kappa^{-1}_{ji} \right) (X_j - X_i) \nu_j + b_i . \] (8)

The right-hand side of Eq. (8) apparently corresponds to the force acting on the particle \( i \), i.e., \( f_i \).

2.2 FIRE method as quasi implicit method

2.2.1 Algorithm

To perform an explicit dynamic simulation based on Eq. (8), time integration algorithms such as the Verlet method have been employed. Here, we describe the implementation of the FIRE algorithm on the NOSB peridynamics. Since the FIRE algorithm is the quasi implicit method having a similar program structure to the explicit algorithm, it is easy to implement the FIRE algorithm to the explicit peridynamic simulation. The following is the procedure of the FIRE (Bitzek et al. 2006):

1. Calculate \( y_i(t) \), \( v_i(t) \), and \( f_i(t) \) for each particle. Check the convergence.
2. Calculate \( p = \sum f_i(t) \cdot v_i(t) \).
3. Correct the velocity as \( v_i(t) \rightarrow (1 - \alpha) v_i(t) + \alpha \hat{f}(t) | v_i(t) | \).
4. If \( p > 0 \) and time step \( N \) satisfies \( N_{\text{min}} \) after \( p \leq 0 \), then \( \Delta t \rightarrow \min \left( f_{\text{inc}} \Delta t, \Delta t_{\max} \right) \) and \( \alpha \rightarrow f_{\text{inc}} \).
5. If \( p \leq 0 \), then \( \Delta t \rightarrow f_{\text{dec}} \Delta t \) and \( v_i(t) \rightarrow 0 \) for all the particles.
6. Go to next step.

\( \hat{f} \) is the unit vector of \( f \), \( v_i \) is the velocity vector of the particle \( i \). In this procedure, \( \Delta t_{\max}, N_{\text{min}}, f_{\text{inc}}, f_{\text{dec}}, \Delta t_{\text{dec}}, \text{ and } \alpha_{\text{start}} \) as the initial value of \( \alpha \) are the parameters used in this calculation. \( \Delta t_{\max} \) should be small so as to ensure the stability of the calculation and, conventionally, is set to the Courant time step multiplied by some coefficient between 0 and 1. We empirically found that the other parameters gave little effect to its computational stability of the peridynamics.

2.2.2 Implementation in peridynamics

In this subsection, we discuss the implementation of the FIRE algorithm to the peridynamics. The flowchart is shown in Fig. 2. For the updating process of the velocity and the displacement, a time integration method such as the Verlet method is used. As easily understood from this figure, the procedure is almost the same as the one of the explicit algorithm except for the modification process of the velocity before the convergence checking part. In this flowchart, the implementation of the Energy Based Relaxation (EBR) method to the peridynamics is also described as the counterpart of the FIRE. They are similar in the point that they can be easily implemented by small modification on the explicit algorithm. In the EBR, the velocity is modified as \( v \rightarrow kv \) where \( k \) is the correction parameter. The correction process is taken when the following strain energy increases during the time step:

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where \(S\) and \(E\) are the 2nd Piola-Kirchhoff stress and Green-Lagrange strain tensors, respectively. The Green-Lagrange strain tensor is calculated from the deformation gradient tensor as done in the finite element method:

\[
E = \frac{1}{2} \left( F^T \cdot F - I \right)
\]

The strain energy density is given as a scalar product of \(S\) and \(E\). The two algorithms are compared in the next section to demonstrate the effectiveness of the FIRE.

\[
W = \frac{1}{2} \int S : E \, dV
\]

The flowchart for elasto-plastic materials, as an example of nonlinear materials, is also shown in Fig. 2, where positions, boundary condition such as forced displacement, and the quantities like strain and stress are updated incrementally.
according to the framework of the nonlinear FEM. Practically, we obtain the strain increment \( \text{d}\mathbf{E} \) from Eq. (10) as the strain difference between the current and previous iteration step shown in Fig. 2(b). Then the stress increment \( \text{d}\mathbf{S} \) can be obtained through the constitutive law as follows:

\[
\text{d}\mathbf{S} = C^{EP}\text{d}\mathbf{E},
\]

where \( C^{EP} \) is the 4-rank tensor representing the elasto-plastic constitutive law. The details of such matrices can be seen elsewhere (Belytschko et al. 2006). Using this increment, the stress is updated. This updated stress is used to check if the stress state satisfies yield condition or not (in this study, the von Mises yield criterion was used). Then this 2nd Piola-Kirchhoff stress tensor is converted to 1st Piola-Kirchhoff stress tensor using the deformation gradient tensor to calculate the force in Eq. (8). Numerical examples on an elasto-plastic simulation will be shown in the following chapter.

3. Result

3.1 Models and settings

To demonstrate the effectiveness of the FIRE algorithm and show several numerical examples performed via the algorithm, here we considered simple problems: (i) elastic deformation of an elastic rectangular box, (ii) elasto-plastic deformation of a sheared bar, and (iii) stress concentration near a zero-thickness crack inside an elastic body. All the calculation were performed as implicit analyses based on the FIRE.

Firstly, the results for the item (i) will be discussed with the model in Fig. 3 to examine the performance of the FIRE and to show some examples of parameter searches. In this model, the positions for the particles at a side along the longitudinal axis (\( x \) in the figure) were completely fixed and the other side are forced to move in order to give tension, compression, and shear deformations to the structure. As shown in the figure, we gave these boundary conditions to a certain amount of volume because in the peridynamics there are no concept corresponding to the surface as in the FEM. Macek and Silling suggested that such volume should have a thickness more than the horizon size so that the prescribed condition are appropriately ensured in the constrained area (Madenci & E. Oterkus 2016). In Fig. 3, the part "fixed particles" is completely fixed and the part "particles to be moved" can only move in the direction of its prescribed motion. Then inhomogeneous stress and strain field can appear inside the rectangular box due to the effect of the Poisson's ratio. The FEM was utilized to check the accuracy in this analysis. For this purpose, well-converged FEM results are required. Using this finite element results, the accuracy of the peridynamics was estimated by considering the following L^2 relative error norm in the displacement vectors of all the particles:

\[
e = \sqrt{\frac{\sum_{i=1}^{N} \left| \mathbf{u}_{i}^{PD} - \mathbf{u}_{i} \right|^{2} V_{i}}{\sum_{i=1}^{N} \left| \mathbf{u}_{i} \right| V_{i}}},
\]

where \( V_{i} \) and \( \mathbf{u}_{i}^{PD} \) are the volume and the deformation vector of \( i \) th particle, respectively. We obtained \( \mathbf{u}_{i} \) at the position of \( i \) th particle by interpolating the well-converged finite element solution obtained by the commercial finite element software, ANSYS®.

Secondary, an example of the parameter search for an elasto-plastic deformation problem of a sheared bar (Fig. 4 (a)) will be shown. In this model, the upper side of the bar slides in the \( x \)-direction up to 10 mm and the bar itself is sheared. As show in the figure, this large deformation makes most part of this bar yield. The results are compared with the results obtained by a well-converged finite element solution using the commercial finite element software, LS-DYNA®. Finally, the accuracy of the NOSB peridynamics will be examined for stress concentration problem near a zero-thickness crack inside an elastic body, shown in Fig. 4 (b). In this model, a zero-thickness crack is introduced in the elastic plate along the \( z \) direction and the plate is tensiled along the \( x \) direction. Stress distribution near the crack front will be compared between the peridynamic results and the theoretical one obtained from the linear fracture mechanics.
The detailed settings for the peridynamics simulation performed here is as follows. According to (Bitzek et al. 2006), the FIRE parameters, $N_{\text{min}}$, $f_{\text{inc}}$, $f_{\text{dec}}$, and $\alpha_{\text{start}}$ were set to 5, 1.1, 0.5, 0.99, and 0.1, respectively. The iteration was stopped when the relative difference between iteration steps $\varepsilon_{c}$ was less than $10^{-10}$:

$$
\varepsilon_{c} = \frac{\left[ y_{\text{avg}}(t) - y_{\text{avg}}(t - \Delta t) \right]^{2}}{\left| y_{\text{avg}}(t) \right|}.
$$

(11)

The settings for materials considered here are as follows: in the elastic problems, the Young's modulus, Poisson's ratio, and the density of the material were set to 3.31 GPa, 0.4, and 100 kg/m$^3$, respectively. In the elasto-plastic problem, the same settings are considered except that the material yields at 43 MPa and hardening after yielding was set to be linear where the modulus is 2.39 GPa. These material properties are chosen by considering a thermoplastic resin whose crack is problematic especially inside carbon-fiber reinforced plastic materials.

### 3.2 Comparison of FIRE and EBR

The number of the iteration steps required to achieve a convergent result was compared between the FIRE and the
EBR methods. The results are shown in Fig. 5. The algorithms were examined for compression deformation of the box shown in Fig. 3. We gave 0.32 mm compression along \( x \) direction to the box. We observed numerical instability when that deformation was given at once, hence we divided the deformation to multiple steps and gave the deformation equal to 0.1 % of the particle size at each step. In the following elastic calculations shown in this paper, we complied with this procedure. The comparison were made in the cases where the particle sizes are 0.4 mm and 0.8 mm. As noted in Section 2.2.1, the EBR requires the parameter \( k \) while the FIRE does not. Figure 5 shows that except for the case with \( k = 0.1 \), the FIRE always showed faster convergence compared with the EBR: the FIRE is about 100 times higher than the EBR on an average. As seen in the molecular dynamics simulations, the algorithm also showed good adaptivity in the quasi-implicit simulation for the continuum mechanics.

![Graph showing comparison of convergent steps between FIRE and Energy Based Relaxation (EBR) methods with different particle sizes, \( D \).](image)

**Fig. 5** Comparison of convergent steps between FIRE and Energy Based Relaxation (EBR) methods with different particle sizes, \( D \).

### 3.3 Computational examples

#### 3.3.1 Elastic problem

The fast convergence of the FIRE algorithm facilitates parameter searches required to ensure the accuracy in the pridynamics. Here we described some examples of such parameter searches focusing on the horizon radius and the particle size.

The horizon radius influences the computational cost and the accuracy of the peridynamics. The computational cost rises with the increase of the radius because more computational resource is spent in Eqs. (2) and (8) if the number of the particles involved in the horizon radius. On the other hand, the radius can govern the accuracy because the deformation gradient tensor is approximated based on the deformation vector of the particles involved in the horizon radius. Hence, we should search the optimum size for the influential radius to maintain the accuracy and reasonable computational cost.

To show such an example, we performed elastic analyses for the box shown in Fig. 3 with different horizon sizes, 1.5 \( D \), 2.0 \( D \), 2.5 \( D \), and 3.0 \( D \), where \( D \) indicates the particle size and was set to 0.5 mm in this case. The horizon with 1.5 \( D \), nevertheless the for example, 1.5 \( D \) horizon involves 9 particles. The particles on the boundary condition region was made to move 0.32 mm (tension), -0.32 mm (compression) in the \( x \) direction, and -0.2 mm in the \( y \) direction (shear). The results are shown in Fig. 6. The error decreases with the reduction of the horizon size in all the deformation type and the optimum size of the radius was determined as 1.5 \( D \), which, however, involved only neighboring particles. This could be because the interaction between material points in a continuum body is essentially local unlike molecular dynamics where atoms nonlocally interact at long range. While peridynamic particles can interact each other nonlocally, the accuracy in a deformation analysis could be maximized in the case where the particles locally interacts as in a basic continuum body.

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Fig. 6 $L^2$ relative error norm of displacement in peridynamic simulations with different horizon radius.

Fig. 7 Decrease of $L^2$ relative error norm of displacement in elastic simulations with decreases of particle size for (a) tension, (b) compression and (c) shear deformation.

The particles size also governs the computational accuracy in the same way of the element size in the FEM. To check the convergence behavior, we performed peridynamic simulation for tension, compression, and shear deformations of the rectangular box model. The results are shown in Fig. 7. The error decreases with the reduction of...
the particle size except for some cases in shear deformation while the tendency is rather slow compared to the FEM results shown in Fig. 4. With the same number of degree of freedom, the accuracy of the peridynamics was 3 digits worse than that of the FEM. This is because the peridynamics approximates the deformation gradient tensor using the positions of particles inside the horizon radius unlike the FEM. We should treat much number of particles so as to ensure accuracy desired, hence we can say that the FIRE algorithm is particularly effective to determine an appropriate particle size.

3.3.2 Elasto-plastic and crack problems

Here we applied the peridynamics to the elasto-plastic and crack problems described in the subsection 3.1 and examined its accuracy with different parameter settings. The peridynamics has its advantage as a tool to simulate fracture behavior of continuum bodies under large deformation, hence we shortly discuss the following results to examine the adaptability of the FIRE algorithm for these problems.

Fig. 8 shows the results for the elasto-plastic problem, where the distributions of displacement in the $x$ direction and yield region with the different number of particles are compared. The yielding behaviors are similar to that of the FEM shown in Fig. 4. In Fig. 8 (b), the obtained peridynamic results with the different parameter settings are compared with the FEM result with respect to the distribution of displacement in the $x$ direction. As shown in the figure, all the cases considered here exhibit so similar results to the finite element result that their differences are not distinctive. This means that the NOSB peridynamics is accurate enough to reproduce the FEM result even for elasto-plastic bodies under large deformation.

![Fig. 8 Results of peridynamics for elasto-plastic bar under shear deformation: (a) color maps of displacement in $x$](image)
direction and yield region, and (b) distribution of displacement in x direction on central axis of sheared bar.

However, this is not the case in the stress analysis near a crack front shown in Fig. 9, where the stress distribution is plotted along the crack direction as a function of the distance from the crack front. Fig. 9 (a) illustrates how we define the position of the crack front as an example where the horizon size is 1.5 D. As shown here, the crack is modeled as a plane across which no pair bonding between particles run. Then the thickness of this crack is supposed to be zero and the position of the crack front is determined as shown in this figure. The peridynamic results are compared with the theoretical distribution obtained by the linear fracture mechanics, where the theoretical distribution was determined from the corresponding stress intensity factor to this model with an \( r^0 \) term given by fitting to the numerical distribution. Unlike the results in the elasto-plastic sheared bar, the error is large enough to be distinctive especially near the crack front. One of the possible reasons for this error would be that the NOSB peridynamics scheme does not have surface-effect correction (Madenci & Oterkus 2016) unlike the bond-based or ordinary state-based peridynamics. Because particles locating at a surface are surrounded by less number of particles compared to a particle inside the continuum body, its deformation gradient is possibly less accurate especially if we see large stress variation near stress singularities as in this case. This fact may suggest that further development in the NOSB peridynamics is expected to involve the surface-effect correction while this issue is beyond the scope of this study. In any case, the FIRE algorithm is useful because a developer who try to seek and examine such a scheme should perform a large number of trial calculations with different formulations, parameter settings, and models.

Fig. 9 Peridynamic calculations on elastic body with zero-thickness crack: (a) definition of crack front in this simulation and (b) comparison of stress distribution in x direction with linear fracture mechanics.

4. Conclusions

We described the details of the implementation of the FIRE method, as a quasi-implicit algorithm, to the peridynamic elastic and elasto-plastic computational codes and showed several computational results for simple problems such as elastic and elasto-plastic deformation analyses and a stress analysis near a crack. Through the comparison with the energy based relaxation method, we found that the FIRE algorithm is about 100 times faster than the energy based relaxation method to get converged results. We believe that this property is of great importance for parameter searches requiring vast amount of computational cost to ensure the accuracy. Especially we expect that the FIRE algorithm as a fast quasi implicit solver could be more effective in nonlinear problems than in elastic one since one should solve linearized equation expressed in a rate form during a nonlinear material simulation using such a law.

References


