Abstract
The rapid evaluation technique of fatigue limit using infrared thermography has been developed during the past 30 years. In this technique, the fatigue limit is evaluated on the basis of the temperature evolution associated with cyclic loading. However, this technique is still not reliable enough because the temperature evolution has not been verified quantitatively very well. In previous studies, the authors conducted numerical analysis to simulate the temperature evolution. The results of fatigue limit evaluation were compared fairly well with experimental ones. However, the temperature evolution was significantly larger compared to experimental results due to adiabatic assumption. In this paper, heat conduction within the specimen is taken into account in order to conduct more realistic simulation of temperature evolution. The results are compared quantitatively to those obtained in experiment. The fatigue limit obtained from simulation is compared with those determined by Wöhler method and thermography experiment.

Keywords: Fatigue limit, Heat conduction, Infrared thermography, Energy dissipation, Finite element analysis

1. Introduction
Rapid evaluation of fatigue limit has become an interesting research topic from the 19th century because it takes much time and cost in Wöhler method (Vitovec, 1953). This paper focuses particularly on the technique based on temperature evolution when structure is subjected to cyclic loading. Details of this technique were reviewed in Ly et al. (2011). Fatigue limit evaluated using this technique is not reliable as much as the one obtained by using Wöhler method in fact. However, due to the development of infrared thermography and its application for this technique from about 30 years ago (Curti, et al., 1986 and Luong, 1992, 1995, 1998), this technique has become more interesting for many researchers.

Jiang et al. (2001, 2004) and Yang et al. (2001, 2004) analyzed the characterization of the temperature evolution during cyclic loading of Ultimet alloy un-notched specimen by experiment and one-dimensional numerical simulation for some first cycles without fatigue limit evaluation. They are the only group conducted numerical simulation but in simple 1D modeling in order to compare the temperature evolution during some loading cycles. Other related studies have been conducted as follows. Charkaluk and Constantinescu (2006, 2009) focused on high and low cycle fatigue criteria for polycrystalline materials in which the principal concepts are macroscale and mesoscale models especially as well as a shakedown condition. A model using Kröner scheme in coupled thermoplasticity was proposed to predict quantitatively mean temperature measurements which was provided by Boulanger et al. (2004). Ranca et al. (2008) studied the thermal effects associated with the propagation of a fatigue crack in a giga cycle fatigue regime of a high-strength steel. In order to better understand these thermal effects and to make a connection with the initiation and the propagation of the fatigue crack, the fatigue crack is modeled by a circular ring heat source whose radius increases with time. The numerical resolution of the thermal problem allows determination of the time evolution of the temperature fields in the specimen. Gornet et al. (2013) tried to determine fatigue limit of carbon fiber epoxy matrix laminated composites under cyclic
loadings by self-heating measurements using experiment and simulation. It is reported that the proposed self-heating test method combined with tomography and finite element simulations confirms that the fatigue limit of composite materials could be measured in few hours without classical fatigue test campaigns.

The advantage of infrared thermography technique is to provide real-time temperature distribution; therefore critical region of structure in work can be detected. However, the mechanism of this technique has not been clarified quantitatively much.

The authors conducted 3D numerical study for SUS304 stainless steel notched specimens in order to simulate temperature evolution due to cyclic loading (Ly, et al., 2011, 2012). Following issues were found:

1. Plastic energy dissipation is essential to explain temperature evolution. The fatigue limit obtained by numerical study has good agreement with the one determined by Wöhler method except for very sharp notch (mainly due to the fact that the spatial resolution of the infrared thermography is not fine enough) as shown in Fig. 1.

2. Plastic shakedown is necessary to evaluate the fatigue limit from the temperature evolution.

3. 2f-component and mean temperature rise approaches provide similar result in adiabatic condition as shown in Fig. 1. 2f-component approach uses the amplitude of second harmonic component of temperature evolution, while mean temperature rise approach utilizes mean temperature rise on surface of structure to evaluate fatigue limit.

These issues are useful guidelines to apply this technique in industry. However, the result of temperature evolution obtained in the numerical simulation was significantly higher than the one obtained in the experiment. The reason should be adiabatic condition assumption. Therefore, further study need to be conducted to understand the effect of heat transfer on fatigue limit evaluation.

There are some factors which should affect temperature distribution within a structure, such as heat conduction, heat convection and radiation to the surroundings. Among them, heat conduction would be most significant in metallic materials. Therefore, this research aims to explore the effect of heat conduction within the specimen to evaluate fatigue limit by numerical simulation.

2. Methodology

2.1. Overview

Abaqus is used for numerical simulation. The aim of the present simulation is to investigate the applicability of rapid evaluation technique. Although heat generation due to elastoplastic deformation and heat transfer are coupled in principle, these were analyzed separately because coupled analysis requires huge and complicated computation. The analysis was conducted in the following steps:

1. Elastoplastic analysis was conducted to obtain stress-strain behavior of the specimen,
2. The heat generated by the thermoelastic and thermoplastic effects was computed based on the results of Step 1,
3. Heat transfer analysis was conducted by applying the generated heat obtained in Step 2.

2.2. Analysis of heat generation

Heat generation analysis was conducted in the same manner as in the previous study of Ly, et al. (2011). Thermoeelastic and thermoplastic effects were considered separately first and then total heat generated by these two was obtained as a sum of these two. Besides that electric, magnetic and other factors were neglected.
The heat generated per unit volume due to thermoelastic effect $Q_e$ is obtained as

$$Q_e = -T \alpha \frac{E}{(1 - 2\nu)} \delta \varepsilon,$$

where

$T$: absolute temperature,
$\alpha$: coefficient of thermal expansion,
$E$: Young’s modulus,
$\nu$: Poisson’s ratio,
$\delta \varepsilon$: change in volumetric strain.

The temperature rise is caused by the irreversible plastic deformation. This inelastic effect is obtained with assumption that the irreversible mechanical energy due to the plastic deformation is converted into heat. In consequence, heat generated due to plastic energy dissipation per unit volume per one cycle of loading $Q_p$ can be written as

$$Q_p = \int_{\varepsilon_1}^{\varepsilon_2} \sigma_l d\varepsilon - \int_{\varepsilon_1}^{\varepsilon_2} \sigma_u d\varepsilon,$$

where $\varepsilon_1$ and $\varepsilon_2$ are the minimum and maximum strain of the hysteresis loop, $\sigma_l$ and $\sigma_u$ are the stresses in loading and unloading part of the hysteresis loop, respectively.

Specimen with 2mm-notch is chosen to conduct analysis. This specimen is made of SUS304 stainless steel and its geometry with 3mm thickness is shown in Fig. 2. The specimen is modeled by 7,600 eight-node linear reduced integration elements (C3D8R). Due to symmetry only one-eighth of the specimen is analyzed as shown in Fig. 3(b).

Numerical simulation was conducted for loading amplitude from 6.0 kN to 7.5 kN with 0.5 kN increments. The uniform repeated sinusoidal waveform stress of $\sigma_{\min}/\sigma_{\max} = 0$ in Fig. 3(a) was applied to the model at one end until 900 cycles while the other end was fixed in the loading direction. Herewith, the maximum applied stresses were varied as 133, 144, 156, 167 MPa.

In order to simulate ratcheting, Chaboche model is used. This is a nonlinear hardening material model which provides kinematic and isotropic hardening (Fionn and Petrinic, 2007). It adopts linear back stress evolution rules of which evolution equation is

$$\dot{\chi} = C \frac{1}{\sigma^p} (\sigma - \chi) \dot{\sigma}^p - \gamma \chi \dot{\sigma}^p$$

(a) Sinusoidal waveform of cyclic stress; (b) Stress distribution in the specimen at a certain time and interested element position.

Fig. 3 (a) Sinusoidal waveform of cyclic stress; (b) Stress distribution in the specimen at a certain time and interested element position.
where
\[ x = \frac{C}{\gamma} \left(1 - e^{-\gamma \varepsilon^p}\right), \]  
\[ \sigma^0 = \sigma_{0}^{1} + Q_{\infty} \left(1 - e^{-b \varepsilon^p}\right), \]  

\( \sigma \): current Cauchy stress tensor,
\( C \): initial kinematic hardening modulus,
\( \gamma \): rate at which the kinematic hardening modulus decreases with increasing plastic deformation,
\( \varepsilon^p \): equivalent plastic strain,
\( \sigma_{0}^{1} \): initial yield stress,
\( Q_{\infty} \): saturated value of increase isotropic deformation resistance,
\( b \): speed of saturation.

Material properties of SUS304 shown in Table 1 were taken from handbooks (Databook Fatigue, 1996 and Technical Handbook, 2006) and determined to match experimental data by Kang et al. (2006).

<table>
<thead>
<tr>
<th>Density</th>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Yield strength</th>
<th>Coefficient of thermal expansion</th>
<th>Specific heat at constant pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>7900 kg/m³</td>
<td>192 GPa</td>
<td>0.33</td>
<td>200 MPa</td>
<td>17.8 x10⁻⁶/K</td>
<td>500 J/kg.K</td>
</tr>
</tbody>
</table>

Table 1 Material properties of SUS304

2.3. Analysis of Heat Conduction

Although the heat generation was obtained by 3D elastoplastic analysis using eight-node linear reduced integration element (C3D8R), it was not possible to conduct 3D heat conduction analysis using the same type element. In addition, it requires much cost to store the heat generation data. Furthermore, the heat conduction in the thickness direction can be assumed to be small enough. Therefore, we conducted the heat conduction analysis using 2D model.

The governing equation of heat conduction in 2D analysis can be summarized as:
\[ \rho C_p \frac{\partial T}{\partial t} - k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = Q_e + Q_p \]  

where \( k = 16.3 \) W/m.K is the thermal conductivity which was searched in handbooks (Databook Fatigue, 1996 and Technical Handbook, 2006).

![Fig. 4 Approach of 2D heat conduction analysis (2mm-notched specimen).](image-url)
The specimen was modeled by 950 four-node, linear-interpolation, heat-transfer element (DC2D4). The mesh used in 2D heat conduction analysis was the same as the surface mesh used in 3D stress-strain analysis, as shown in Fig. 4. The boundary condition at all edges is adiabatic because heat conduction within the specimen is predominant.

The thermal load was specified by element-based distributed heat fluxes. Elastic strain in three coordinate axes \( \varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz} \) and dissipated plastic energy density of 950 elements on the surface of specimen in 3D stress-strain analysis were stored to calculate to the total heat fluxes. These body fluxes (flux per unit volume) together with corresponding time step were assigned to time variation of heat source of corresponding element of 2D heat conduction analysis model.

3. Results

3.1. Stress-strain behaviors

Stress-strain curve at position of notch tip in the specimen in case of 6.0 kN load amplitude is described in Fig. 5. Stress-strain behavior from the beginning to 900 cycles is plotted on the left hand side while five portions: 1 to 5, 16 to 20, 46 to 50, 96 to 100, and 600 to 900 cycles, respectively are plotted on the other side. It is noticed that when the number of cycles increases, the hysteresis loop is gradually shifted to right (ratcheting) and its area per one cycle decreases. After a sufficient number of cyclic loads, about 600 to 900 cycles, the hysteresis loop converges to a single curve when plastic shakedown is achieved.

![Stress-strain response of some cycles in case of 6.0 kN load amplitude.](image)

3.2. Temperature behaviors

Temperature evolution is the combination of the temperature oscillation due to thermoelasticity and the temperature increase due to irreversible plastic energy dissipation which are respectively obtained by

\[
T_e = \frac{Q_e}{\rho C_v},
\]

\[
T_p = \frac{Q_p}{\rho C_p},
\]

where
\( C_v \): specific heat at constant volume,
\( C_p \): specific heat at constant pressure.

The validity of heat conduction analysis was confirmed by a simulation in which frequency and thermal conductivity are set 1 Hz and 0, respectively. This result matches very well with temperature evolution obtained in adiabatic analysis in Ly et al. (2011, 2012) as shown in Fig. 6.

The analysis was continued by applying the experimental condition with thermal conductivity and frequency to be 16.3 W/m.K and 25 Hz, respectively. The spatial resolution of the thermography on the specimen surface was about 0.3 mm square in the experiment. It was found in the previous study (Ly et al., 2012) that the spatial resolution of infrared thermography affects the fatigue limit evaluation significantly. Therefore, temperature evolution was computed for not only the element of interest which is at the notch tip but also for equivalent pixel size (average over the pixel area).

A typical result for the case of 6.0 kN load amplitude is shown in Fig. 7. The left column in this figure illustrates the finite element mesh used in the simulation and the apparent pixel area (0.3 mm square area). The temperature evolutions...
Fig. 6 Validation of temperature evolution (6.0 kN load amplitude).
at element of interest and equivalent pixel size are almost similar. At the first cycle, after the temperature decreases due to thermoelastic effect, it increases rapidly due to the plastic dissipation. Since the applied load has sinusoidal waveform in tension as shown in Fig. 3(a), at the area far away from notch tip, temperature only oscillates around mean value lower than the ambient temperature due to the thermoelastic effect. In consequence, heat is only generated at the vicinity of notch tip. It takes 36 s to complete 900 cycles, which is long enough for heat to diffuse from the vicinity of notch tip to other area of specimen but too short for the specimen to reach thermal equilibrium. Therefore, the temperature at element of interest and also equivalent pixel size keep slightly decreasing. The heat transfer affects severely the mean temperature, which is a consequence of accumulation of heat generated by the plastic energy dissipation. It can be judged that mean temperature rise approach cannot be applied to evaluate the fatigue limit. Note that the mean temperature rise on the surface is usually utilized to evaluate the fatigue limit as proposed by both Curti et al. (1986) and Luong (1992, 1995, 1998).

Another method to evaluate fatigue limit is 2f-component approach which was firstly suggested by Brémond (1995). The amplitude of second harmonic component of temperature evolution is used. Results in Fig. 8 demonstrate 2f-component amplitudes obtained by applying FFT to every 10 cycles of temperature evolution at the notch tip from be-
beginning to 900 cycles of repeated loading. Hanning window was applied to the data in order to reduce aliasing. They indicate that the 2f-component amplitude under both adiabatic and non-adiabatic conditions converges to a constant at larger number of cycles, namely when plastic shakedown is achieved. The temperature amplitude of 2f-component under non-adiabatic analysis is much smaller than those analyzed under adiabatic condition for every loading amplitude.

<table>
<thead>
<tr>
<th>Loading amplitude (kN)</th>
<th>Adiabatic</th>
<th>Non-adiabatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
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Fig. 8 Temperature amplitude of 2f-component (6.0 kN load amplitude).

A quantitative comparison of 2f-component amplitude at the 600th cycle between experiment (Ly, et. al., 2011) and simulation at the element of interest and equivalent pixel size is shown in Fig. 9. Actually this result is also the same from about 400 to 800 cycles because the temperature amplitudes of 2f-component keep unchanged after the 400th cycle for every loading amplitude as shown in Fig. 8. The results in heat conduction analysis are shifted down compared to corresponding results in adiabatic analysis for both the element of interest and pixel size. This amount of shifting down is larger as applied load amplitude increases. In addition, the results of pixel size were computed as average temperature over all elements covered by the pixel illustrated in Fig. 7(a) (pink square). Thus, they are smaller than those obtained for the element of interest. The most important remark is that temperature results of pixel size in simulation and experiment match very well. Therefore, it can be remarked that the effect of two dimensional heat conduction within the specimen is most significant compared to other heat transfer effects.

Fig. 9 Quantitative comparison of 2f-component temperature amplitude.

3.3. Fatigue Limit Evaluation

A comparison of fatigue limit evaluation by the 2f-component approach between the results obtained in heat conduction and adiabatic analysis are described in Fig. 10 for the interested element and equivalent pixel size, respectively.
Fatigue limit evaluations are similar regardless of heat conduction effect. In other words, heat conduction does not affect significantly on fatigue limit evaluation by the 2f-component approach for 2mm-notched specimen in particular and probably when the stress concentration factor is small enough in general.

![Fatigue limit evaluation graph](image1)

Finally, the fatigue limit evaluated by the 2f-component approach is summarized in Table 2. Even under non-adiabatic condition, the results agree very well with experimental results obtained by Wöhler method and also by thermography which were conducted in Ly et al. (2011).

<table>
<thead>
<tr>
<th></th>
<th>Adiabatic analysis</th>
<th>Non-adiabatic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element of interest</td>
<td>5.7 kN</td>
<td>5.8 kN</td>
</tr>
<tr>
<td>Pixel size</td>
<td>5.9 kN</td>
<td>5.9 kN</td>
</tr>
<tr>
<td>Wöhler experiment</td>
<td>5.7 kN</td>
<td></td>
</tr>
<tr>
<td>Thermography experiment</td>
<td>5.5 kN</td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusions

The numerical analyses conducted in this research together with some experimental results have provided informative knowledge to enhance the reliability of rapid evaluation technique for practical application in industry. The results obtained from 3D stress-strain analysis and 2D heat conduction analyses have been compared quantitatively with temperature of 2f-component together with fatigue limit determined by thermography experiment and Wöhler method. It can be concluded that:

(1) Although only in-plane heat conduction within the specimen is considered in this analysis, numerical and experimental results of 2f-component amplitude are quantitatively comparable.

(2) Although heat conduction causes both mean temperature rise and amplitude of 2f-component change, 2f-component approach used to evaluate fatigue limit is not affected significantly.
3) Mean temperature rise approach should fail to determine fatigue limit in case of stress concentration appearance because mean temperature at notch tip varies depending on the heat conduction phenomena around stress concentration part.

The results obtained in this paper cannot be readily applicable to other notches or materials. However, the applicability of this technique has been already verified for several specimens and materials experimentally in the literature. Further practical experiences are needed to establish this technique as a reliable one.

References


