Advance Publication by J-STAGE

Mechanical Engineering Journal

DOI:10.1299/mej.19-00462

Received date : 26 September, 2019

Accepted date : 28 October, 2020

J-STAGE Advance Publication date : 4 November, 2020

Effect of flexible plates and micro-orifices on acoustic filter response for frequency-specific earplugs

Akira SANADA* and Hideki TAKASO**

*Industrial Technology Center of OKAYAMA Prefecture
5301 Haga, Kita-ku, Okayama-shi, Okayama 701-1296, Japan
E-mail: akira_sanada@pref.okayama.lg.jp

**ae The Sounds Company
2-6-6 Mainosato, Koga-shi, Fukuoka 811-3114, Japan

Abstract

In recent years, the use of earplugs with analog acoustic band pass filters has gained prominence. These earplugs can protect the ears of factory workers, who work in noisy environments, and simultaneously allow for necessary sounds such as colleagues' voices, danger alarms, and equipment-failure sounds to be heard. However, certain doubts still exist regarding the ability of these earplugs to protect the ears of the workers, particularly for low frequency ranges because of the relatively wide passing frequency ranges of sounds. From this standpoint, there exists a requirement for earplugs that can precisely control the passing amount and range of sounds in different working environments. Thus, we investigated an earplug with a simple lightweight structure, involving a filter consisting of a micro-orifice and flexible elastic plate. First, the frequency response of the proposed earplug was theoretically modeled using the transfer matrix method for each component. Second, the validity of the model was established experimentally, as confirmed by the results. Moreover, the proposed earplugs were confirmed to provide sufficient insulation against noise in a low frequency range and simultaneously allow for efficient passage of sounds in the range of 1–2 kHz. Further fine tuning can be expected by changing the detailed properties of the components.

Keywords: Earplug, Acoustic filter, Flexible plate, Micro-orifice, Transfer matrix method, Helmholtz resonance

1. Introduction

With the growing awareness of labor safety, hearing protection for factory workers has become increasingly important. However, there are dilemmas over hearing protection. By using earplugs, factory workers can not only protect their ears to prevent hearing loss and tinnitus but also reduce their fatigue and psychological stress that may occur due to a prolonged exposure to high-level noise that is prevalent in their working environments. Conversely, using earplugs have their negative sides too and have the potential to disturb the safe and efficient working conditions. For instance, earplugs may block the abnormal noise of an equipment, which is one of the stopping indicators in manufacturing lines. The earplugs also block the voices of coworkers and interrupt efficient communication among workers. Moreover, the earplugs block the danger-alarming sounds that should be detected quickly to facilitate immediate evacuation during danger. Due to these problems workers hesitate in wearing earplugs although they are aware of the importance of the hearing protection that earplugs provide. As a consequence, rate of earplug usage has not reached a satisfactory level in Japanese factories.

As a solution to these problems, many types of earplugs with analog acoustic band pass or low-pass filters have been developed and widely distributed in the hearing protection market. Casali (2010) reviewed the various kinds of hearing protection devices developed recently. One of the devices he reviewed is the canal cap-style passive earplugs with muffler tubes (Zwislocki, 1998) or ducts that are closed at their upper ends and open at their ear-canal ends. He reports that the earplugs with the muffler tubes take the advantage of the quarter wave resonance principle that

improves the attenuation in a low-frequency range and allows speech and other useful sounds to pass. However, according to his report, in the case of only one muffler, the attenuation benefit of the quarter-wave resonance is limited to a narrow frequency range. In addition, no empirical, scientific evaluations of these earplugs are available, although some manufacturers show anecdotal evidence. In his review, passive adjustable attenuation earplugs are also introduced. With these types of earplugs, users have some choices over the amount of attenuation and prevent overprotection, especially in moderate noise environments. The configurations of these types vary, for instance, by setting an obstructing valve in the tunnel/vent through the earplug, by inserting the available filters/dampers into the tunnel/vent, etc. However, with all these configurations, the users are supposed to have choices for the attenuation spectra along with easy and appropriate ear protection in their specific situations that allows them to retain their necessary sounds. Although he mentions the complicated process of standard testing for these earplugs because of the user-specific settings, the detailed mechanism behind the acoustic controls are not discussed.

Apart from the earplugs investigated by Casali, filtered earplugs with a micro-orifice structure are widely used commercially in hearing protection devices. These types of earplugs typically exhibit strong attenuation, particularly to the noise in the high frequency range, and allow for noise in low frequency range to pass (Sanada and Takaso, 2016). However, there are still some doubts regarding whether these earplugs can provide sufficient hearing protection since they allow a wide range of sound frequencies to pass, particularly for the low-frequency range. To have an adequate level of hearing protection, the passing range needs to be narrowed.

In this paper, an earplug is proposed that not only allows efficient passing of sound for a narrow frequency range but also sufficiently insulates the noise for a wide frequency range. The proposed earplug has relatively simple structures consisting of a flexible elastic plate, a small attached weight, and micro-orifice along with internal tube cavities, and it is supposed to function as an analog acoustic filter that only allows sounds of selected frequency ranges to pass. The steep band-edge property of the filter was achieved on the basis of a synergetic combination of the principles of flexible plate resonance and Helmholtz resonance through the micro-orifice and the cavity. To the best of our knowledge, earplugs operating on the basis of such a synergetic effect have not been reported previously. A detailed investigation of the mechanism of bandpass filtering and attenuation with these configurations is expected to reveal the potential of analog acoustic filtering to foster hearing protection in the working environment of factories.

First, to verify the frequency-selective attenuation mechanism of the proposed earplugs, the frequency response was theoretically modeled via the transfer matrix method. The matrix for each composing element of the earplug was separately derived. Then, through multiplication of all these separate matrices, the attenuation of the entire configuration was calculated. The viscosity and thermal effects were also considered because they influenced the acoustic response through the micro-orifice and the tube cavities with small diameters. Second, the calculations were performed on the basis of the derived theoretical model to determine the expected attenuation levels for the different earplug configurations. The acoustic response changes due to parameters such as the size of the micro-orifice and the mass of a small attached weight were closely examined to note the extent of attenuation as well as the frequency ranges of the attenuated and passing sounds. Finally, the theoretical models were verified by the experimental results, and the validity and feasibility of the proposed earplugs were also established. Specifically, it is clarified that the proposed earplugs can not only sufficiently attenuate noise in a wide frequency range but also allow efficient passage for selected narrow-frequency ranges of sounds (1–2 kHz for this study that are important for efficient communications).

2. Theory

2.1 Structure and theoretical model of the proposed earplug

Figure 1(a) shows the schematic view of the proposed earplug configurations used in this study. As shown in the figure, the earplug consists of a flexible elastic plate, Cavity 1, a micro-orifice, and Cavity 2 in a chronological order, when viewed from the outside where the sounds come from. Figure 1(b) describes the model for calculation of the proposed earplug configuration. On arrival at the earplug, the sounds cause vibrations in the flexible plate, and the plate radiates the sound to Cavity 1. After the radiation, the sound goes through the micro-orifice, Cavity 2, and the ear canal (Cavity 3) and finally reaches the eardrum. In the analytical model, only the wave propagation in the ear canal is considered as the acoustical characteristic of the occluded ear. The eardrum impedance is assumed to be infinite in the model. This is because our main interest in this study was the basic filter-response mechanism of the proposed earplug. The influence of the eardrum impedance will be investigated in future works.

In the model, the plate was expected to function as an insulator of the incoming sound and simultaneously as a

bandpass filter. By controlling the resonance of the flexible plate with a small weight attachment at the center of the plate surface, the proposed earplug was expected to have sufficient attenuation for most of the frequency ranges and simultaneously allowed only the targeted frequency range to pass.

The analytical model was constructed as a one-dimensional model, since the wavelength of sound was assumed to be larger than the size of the earplug. In this calculation, for simplicity, the cross-section of the earplug was assumed to be circular anywhere along the earplug, and the inside wall of the sound passage within the earplug was assumed to be rigid. Additionally, only the linear responses were considered in the model because the non-linear responses were assumed to occur at higher sound-pressure levels than that considered in our calculations as the consideration of linear responses was sufficient for our purpose.

Equation 1 shows p_i and v_i , which are the sound pressure and particle velocity on the right side of the i -th element of the earplug, respectively, whereas p_{i-1} and v_{i-1} are the sound pressure and particle velocity on the left side, respectively. The relationship between both sides is expressed using the transfer matrix as follows (Munjal, 1987):

$$\begin{bmatrix} p_{i-1} \\ v_{i-1} \end{bmatrix} = \mathbf{T} \begin{bmatrix} p_i \\ v_i \end{bmatrix}, \quad (1)$$

where $\mathbf{T}$ denotes the transfer matrix with 2×2 factors. To estimate the insulating and band-pass effects of the proposed earplug, the transfer matrices of each constituent element of the earplug were first derived. Then, these derived transfer matrices were multiplied to obtain the transfer matrix of the total effects. The relationship between the sound fields at the incident surface and the eardrum surface is expressed as

$$\begin{bmatrix} p_{in0} \\ \tilde{v}_0 \end{bmatrix} = \mathbf{T}_p \mathbf{T}_{D1} \mathbf{T}_a \mathbf{T}_{D2} \mathbf{T}_t \mathbf{T}_{D3} \begin{bmatrix} p_{ear} \\ v_{ear} \end{bmatrix}, \quad (2)$$

where the sound pressure on the outer surface of the flexible plate due to the incident sound and the average particle velocity on the surface are denoted as p_{in0} and $\tilde{v}_0$, respectively, whereas those on the surface of the eardrum are denoted as p_{ear} and v_{ear} , respectively. The matrices of Cavity 1, Cavity 2, and Cavity 3 are denoted by $\mathbf{T}_{D1}$, $\mathbf{T}_{D2}$, and $\mathbf{T}_{D3}$, respectively, whereas the matrices of the flexible plate, micro-orifice, and cross-sectional area change are denoted by $\mathbf{T}_p$, $\mathbf{T}_a$, and $\mathbf{T}_t$, respectively.

Fig. 1 Schematic view of the proposed earplug. The earplug consists of a flexible elastic plate, Cavity 1, a micro-orifice, and Cavity 2 in a chronological order when viewed from the outside. The earplug is installed with the seal material into the ear canal without any gap. Cavity 3 is the surrounded space between the tip of the earplug and the eardrum in the ear canal. The analytical model is constructed as a one-dimensional model because the wavelength of sound is assumed to be sufficiently larger than the size of the earplug. The cross-section of the model is circular along the whole configuration.

Since the eardrum impedance was assumed to be infinite in this study, the particle velocity v_{ear} on the eardrum was considered to be zero. From Eq. (2), the following equation is derived.

$$\begin{bmatrix} p_{ear} \\ 0 \end{bmatrix} = (\mathbf{T}_p \mathbf{T}_{D1} \mathbf{T}_a \mathbf{T}_{D2} \mathbf{T}_t \mathbf{T}_{D3})^{-1} \begin{bmatrix} p_{in0} \\ \tilde{v}_0 \end{bmatrix} \quad (3)$$

The attenuation effect of the entire earplug configuration is defined as the ratio of the sound pressure at the eardrum to the incident sound pressure at the outer side of the earplug.

$$\tau = \frac{p_{ear}}{p_{in0}}, \quad (4)$$

which is calculated from Eq. (3). Finally, the attenuation level is defined as follows:

$$R [\text{dB}] = 20 \log(|1 / \tau|). \quad (5)$$

In the subsequent sections, the acoustic matrix of each composing element is explained.

2.2 Flexible circular plate

The matrix of the flexible circular plate is considered in this section. Figure 2 shows the theoretical model of the circular flexible plate. The plate with radius R_p and thickness h was attached to the outer side of the earplug. The plate was assumed to be embedded in an infinite baffle, and a fixed boundary condition was formed at the edge of the plate.

The plate was supposed to be excited by a normal incident sound wave and simultaneously receive the reaction force from the air surrounding the plate. The reaction forces were assumed to be caused by the sound pressure resulting from the vibration of the plate itself. The sound pressure at an arbitrary position $\mathbf{r}$ on the outer surface of the earplug was expressed as $p_0(\mathbf{r})$ whereas the sound pressure on the inner plate surface was p_1 . When the plate moved in the w -axis positive direction, the pressure at the left side of the plate was assumed to be $-p_v(\mathbf{r})$. Therefore, $p_0(\mathbf{r})$ can be expressed as $p_0(\mathbf{r}) = p_{in0} - p_v(\mathbf{r})$. Hence, the equation of the flexible plate motion is obtained as follows (Sanada and Tanaka, 2012):

$$\begin{aligned} D' \nabla^4 w(\mathbf{r}, t) + \rho_s h \frac{\partial^2 w(\mathbf{r}, t)}{\partial t^2} &= p_0(\mathbf{r}) - p_1 \\ &= p_{in0} - p_1 - p_v(\mathbf{r}) \end{aligned}, \quad (6)$$

where the displacement of the flexible plate is denoted as $w(\mathbf{r}, t)$, ρ_s is the density of the plate, and $D' = D(1 + j\eta)$, where D is the plate bending rigidity, and η is the loss factor. Incidentally, when the normal incident sound entered the earplug, p_{in0} was assumed to be uniform at any location on the plate surface. On the other hand, p_v was assumed to depend on $\mathbf{r}$ because the manner of sound radiation from the plate is typically not uniform on the outer surface of the plate whereas p_1 did not depend on $\mathbf{r}$ because the sound field inside the earplug was considered to be one-dimensional.

The vibration velocity of the plate is expressed in the following equation under the assumption that the structural vibration can be described by a summation of N modes.

$$\dot{w}(\mathbf{r}) = \mathbf{\Psi}^T(\mathbf{r}) \dot{\xi}, \quad (7)$$

where the transpose of the matrix is denoted as the superscript T; the modal function vector and the modal velocity

vector are represented by $\Psi(\mathbf{r})$ and $\dot{\xi}$, respectively. These are defined as

$$\Psi^T(\mathbf{r}) = [\psi_1(\mathbf{r}) \quad \psi_2(\mathbf{r}) \quad \cdots \psi_i(\mathbf{r}) \cdots \psi_N(\mathbf{r})], \text{ and} \quad (8)$$

$$\dot{\xi}^T = [\dot{\xi}_1 \quad \dot{\xi}_2 \quad \cdots \dot{\xi}_i \cdots \dot{\xi}_N], \quad (9)$$

where $\psi_i(\mathbf{r})$ and $\dot{\xi}_i$ are the modal function and modal velocity amplitude of the i -th order mode, respectively. The modal function of a thin circular plate clamped at its edge is expressed by the following equation.

$$\psi_i = A_i \left\{ J_{m_i} \left(k_{m_i n_i} R_p \right) I_{m_i} \left(k_{m_i n_i} r \right) - I_{m_i} \left(k_{m_i n_i} R_p \right) J_{m_i} \left(k_{m_i n_i} r \right) \right\} \cos m_i \theta, \quad (10)$$

where m_i and n_i are the modal indices of the i -th mode. J_{m_i} and I_{m_i} are the m_i -th order Bessel function and modified Bessel function of the first kind, respectively. A_i is a constant. r and θ are the polar coordinates of which the origin is the center of the plate. $k_{m_i n_i} R_p$ is λ , which satisfies the following equation that is derived from the clamped boundary condition.

$$J_{m_i}(\lambda) I_{m_i+1}(\lambda) + I_{m_i}(\lambda) J_{m_i+1}(\lambda) = 0 \quad (11)$$

When the sound normally comes in, because the vibration modes with nodal lines in the circumferential direction are not excited, only the modes with $m_i = 0$ are excited. Hence, the first mode and the modes with the nodal lines in the radial directions should be considered. These modes are expressed as

$$\psi_i = J_0(\lambda_i) I_0\left(\frac{\lambda_i}{R_p} r\right) - I_0(\lambda_i) J_0\left(\frac{\lambda_i}{R_p} r\right), \quad (12)$$

where λ_i is the solution of the following equation.

Fig. 2 Analytical model of a flexible circular plate. The plate is assumed to be embedded in an infinite baffle. The entering sound wave is incident perpendicular to the plate. The plate is excited by the normal incident sound wave, and the plate simultaneously receives the reaction force from the air surrounding the plate.

$$J_0(\lambda)I_1(\lambda) + I_0(\lambda)J_1(\lambda) = 0 \quad (13)$$

Under the assumption of harmonic excitation, Eq. (7) is substituted into Eq. (6). Then, Eq. (6) is multiplied by Ψ from the left and integrated over the area S of the plate. Finally, the following equation is obtained.

$$\frac{1}{j\omega} \mathbf{K}_p \dot{\xi} + j\omega \mathbf{M}_p \dot{\xi} = \mathbf{F}_{in0} - \mathbf{F}_1 - \mathbf{F}_v, \quad (14)$$

where $\mathbf{K}_p$ and $\mathbf{M}_p$ are the $N \times N$ diagonal matrices owing to the orthogonality of the vibration modes. The diagonal elements of these matrices are

$$K_{pi} = \int_S D' \psi_i(\mathbf{r}) \nabla^4 \psi_i dS, \quad \text{and} \quad (15)$$

$$M_{pi} = \int_S \rho_s h \psi_i^2(\mathbf{r}) dS, \quad (16)$$

where K_{pi} and M_{pi} denote the modal rigidity and the modal mass of the i -th mode, respectively. $\mathbf{F}_{in0}$, $\mathbf{F}_1$, and $\mathbf{F}_v$ are

$$\mathbf{F}_{in0} = \int_S \Psi(\mathbf{r}) p_{in0} dS = \begin{bmatrix} F_{in01} & F_{in02} & \cdots & F_{in0i} & \cdots & F_{in0N} \end{bmatrix}^T, \quad (17)$$

$$\mathbf{F}_1 = \int_S \Psi(\mathbf{r}) p_1 dS = \begin{bmatrix} F_{11} & F_{12} & \cdots & F_{1i} & \cdots & F_{1N} \end{bmatrix}^T, \quad \text{and} \quad (18)$$

$$\mathbf{F}_v = \int_S \Psi(\mathbf{r}) p_v(\mathbf{r}) dS = \begin{bmatrix} F_{v1} & F_{v2} & \cdots & F_{vi} & \cdots & F_{vN} \end{bmatrix}^T. \quad (19)$$

F_{in0i} , F_{1i} , and F_{vi} are the i -th modal excitation forces caused by the incident sound, the reaction force of the air in Cavity 1, and the reaction force of the air outside, respectively.

In this case, Eqs. (17) and (18) are

$$\mathbf{F}_{in0} = \mathbf{G}^T p_{in0}, \quad \text{and} \quad (20)$$

$$\mathbf{F}_1 = \mathbf{G}^T p_1, \quad (21)$$

where

$$\mathbf{G} = \int_S \Psi^T(\mathbf{r}) dS. \quad (22)$$

Here, for simplicity, the radiation from a circular rigid piston was employed as the approximation for the radiation from an elastic flexible plate vibrating with an average velocity $\tilde{w}$. If the radiation impedance is Z_v , the sound pressure generated by the vibration of the plate can be expressed as follows.

$$\begin{aligned} p_v &= Z_v \tilde{w} \\ &= Z_v \frac{\int_S \dot{w}(\mathbf{r}) dS}{S} \\ &= \frac{Z_v}{S} \mathbf{G} \dot{\xi} \end{aligned} \quad (23)$$

When the radiation impedance Z_v is used in the case of Fig. 2, $-p_v$ is considered to be the average pressure over the outer surface of the plate and does not change with $\mathbf{r}$. The radiation impedance of the circular piston with radius R_p is expressed as follows (Blackstock, 2000):

$$Z_v = \rho_0 C \left[1 - \frac{J_1(2kR_p)}{kR_p} + j \frac{K_1(2kR_p)}{kR_p} \right], \quad (24)$$

where ρ_0 and C are the density of air and the speed of sound, respectively. k denotes the wave number, and K_1 is the Struve function. In the case when the wavelength is sufficiently larger than the size of the plate, the approximation is calculated using Eq. (24), since $kR_p \ll 2$.

$$Z_v = \rho_0 C \left[\frac{(kR_p)^2}{2} + j \frac{8kR_p}{3\pi} \right] \quad (25)$$

Therefore, substituting Eq. (23) in Eq. (19), $\mathbf{F}_v$ is expressed as

$$\begin{aligned} \mathbf{F}_v &= \frac{Z_v}{S} \int_S \boldsymbol{\Psi}(\mathbf{r}) dS \dot{\boldsymbol{\xi}} \\ &= \frac{Z_v}{S} \mathbf{G}^T \mathbf{G} \dot{\boldsymbol{\xi}} \end{aligned} \quad (26)$$

Substituting Eqs. (20), (21), and (26) in Eq. (14), the following equation is obtained:

$$\dot{\boldsymbol{\xi}} = \left(\frac{1}{j\omega} \mathbf{K}_p + j\omega \mathbf{M}_p + \frac{Z_v}{S} \mathbf{G}^T \mathbf{G} \right)^{-1} (\mathbf{G}^T p_{in0} - \mathbf{G}^T p_1). \quad (27)$$

Hence, the average vibration velocity is derived as

$$\tilde{\dot{\mathbf{w}}} = \frac{1}{S} \mathbf{G} \left(\frac{1}{j\omega} \mathbf{K}_p + j\omega \mathbf{M}_p + \frac{Z_v}{S} \mathbf{G}^T \mathbf{G} \right)^{-1} (\mathbf{G}^T p_{in0} - \mathbf{G}^T p_1). \quad (28)$$

This equation is transformed to

$$p_{in0} = \frac{S}{\mathbf{G} \left(\frac{1}{j\omega} \mathbf{K}_p + j\omega \mathbf{M}_p + \frac{Z_v}{S} \mathbf{G}^T \mathbf{G} \right)^{-1} \mathbf{G}^T} \tilde{\dot{\mathbf{w}}} + p_1. \quad (29)$$

Since the particle velocities ($\tilde{v}_0$ and v_1) at both sides of the plate surfaces are assumed to be identical to the average vibration velocity of the flexible plate $\tilde{\dot{\mathbf{w}}}$, the transfer matrix of the flexible plate is obtained as

$$\begin{bmatrix} p_{in0} \\ \tilde{v}_0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{S}{\mathbf{G} \left(\frac{1}{j\omega} \mathbf{K}_p + j\omega \mathbf{M}_p + \frac{Z_v}{S} \mathbf{G}^T \mathbf{G} \right)^{-1} \mathbf{G}^T} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \mathbf{T}_p \begin{bmatrix} p_1 \\ v_1 \end{bmatrix}. \quad (30)$$

The possibility existed to utilize the resonance of the flexible plate as a filter to allow passage of the targeted frequency range of sound. Through tuning of the eigenfrequency of the plate, the passing frequency range of the filter could be changed. The eigenfrequency of the plate is expressed as follows:

$$\omega_i = \sqrt{\frac{K_{pi}}{M_{pi}}} = \left(\frac{\lambda_i}{R_p} \right)^2 \sqrt{\frac{D}{\rho_s h}}. \quad (31)$$

This equation indicates that the eigenfrequency is determined by the radius R_p , Young's modulus E , poison ratio ν , density ρ_s , and the thickness h of the plate (in the case that the plate is flat, $D = Eh^3 / (12(1 - \nu^2))$), and the eigenfrequency could be controlled by changing any of these properties. However, careful consideration is needed for changing these properties, otherwise, the changes could complicate the tuning. For instance, when the Young's modulus or the size of the plate is changed, not only the eigenfrequency of the plate but also the attenuation effect in the low-frequency range is affected.

To solve this possible tuning problem, the attachment of a small weight (m_s) on the surface of the plate was considered as a simple process for tuning the eigenfrequency without affecting the attenuation effect in the low-frequency range. The analytical model was formed under the assumption that the attached weight on the plate did not affect the rigidity of the plate. In addition, the shape of the plate vibration mode was assumed to be unaffected since the attached weight was sufficiently less than the mass of the plate. The modal mass of the plate with a small weight is obtained in the following equation:

$$M'_{pi} = \int_S (\rho_s h \psi_i^2(\mathbf{r}) + m_s \delta(\mathbf{r} - \mathbf{r}_{ms}) \psi_i^2(\mathbf{r})) dS, \quad (32)$$

where δ is the Dirac delta function and $\mathbf{r}_{ms}$ is the location where the weight is attached. When the eigenfrequency of the plate was calculated for a small weight being attached, M'_{pi} was used instead of M_{pi} , as shown in Eq. (31).

2.3 Cylindrical tube cavity with a small radius

The cylindrical tube in Fig. 3 is considered as the cavity that facilitates the sound passage within the earplug. When the wavelength is sufficiently longer than the diameter of the cylindrical tube, the sound is assumed to propagate only in one dimension. In this case, the relationship between the sound pressure and the particle velocity at both sides of the cylindrical tube can be expressed as follows (Munjaj, 1987):

$$\begin{bmatrix} p_{i-1} \\ v_{i-1} \end{bmatrix} = \begin{bmatrix} \cos kl_d & j Z_0 \sin kl_d \\ j \frac{\sin kl_d}{Z_0} & \cos kl_d \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} = \mathbf{T}_D \begin{bmatrix} p_i \\ v_i \end{bmatrix}, \quad (33)$$

where Z_0 is the characteristic impedance of air, and l_d is the distance between both the sides.

When the diameter of the cylindrical tube is substantially small, the sound propagation in the tube could be affected by the viscous friction effect as well as the thermal effect. Viscous friction occurs at the inner surface of the tube due to the viscosity of the air. The thermal effect results from the thermal conduction between the air and the inside wall of the tube. These effects are observed as the damped propagation and the reduction in the speed of sound and can be expressed using the equivalent density ρ_e and the bulk modulus K_e (Allard and Atalla, 2009) as complex numbers. ρ_e is defined from the equation of motion per a unit mass for the air in the tube as follows:

$$\rho_e = \frac{\rho_0}{1 - \frac{2}{k_s R_d} \frac{J_1(k_s R_d)}{J_0(k_s R_d)}}, \quad (34)$$

where $k_s = (-j\omega\rho_0/\mu)^{1/2}$, μ is the viscosity of the air, and R_d is the radius of the tube.

The bulk modulus K_e can be expressed as follows:

$$K_e = \frac{\gamma P_0}{1 + (\gamma - 1) \frac{2}{Bk_s R_d} \frac{J_1(Bk_s R_d)}{J_0(Bk_s R_d)}}, \quad (35)$$

where γ denotes the ratio of specific heats ($= C_p / C_v = 1.4$, where C_p and C_v are the specific heats at constant pressure and constant volume, respectively). P_0 and B denote the atmospheric pressure and the square root of the Prandtl number B^2 , respectively.

Therefore, the wave number inside the tube is obtained from Eqs. (34) and (35) as follows:

$$k_d = \sqrt{\rho_e / K_e} \omega. \quad (36)$$

If k_d is used instead of k in Eq. (33), the transfer matrix of the cylindrical tube with a small radius is obtained.

Fig. 3 Analytical model of a cylindrical tube with a small radius. Viscosity effects and thermal effects inside the tube are considered in the model.

2.4 Micro-orifice

Figure 4 shows the analytical model of a micro-orifice. The difference between the sound pressures at both sides of the orifice is expressed in a linear manner as follows, under the assumption that the pressure gradient is uniform along the x -axis in the orifice (Melling, 1973).

$$p_i - p_{i-1} = z_a \tilde{v}_a, \quad (37)$$

where $\tilde{v}_a$ is the average particle velocity at the cross-section of the orifice, and z_a is the specific acoustic impedance of the orifice.

$$z_a = \frac{j\omega\rho_0 l_a}{1 - \frac{2}{k_s R_a} \frac{J_1(k_s R_a)}{J_0(k_s R_a)}}, \quad (38)$$

where R_a and l_a are the radius and the depth of the orifice, respectively. The real and imaginary parts of z_a express a resistance factor and a reactance factor, respectively.

Fig. 4 Analytical model of a micro-orifice. The cross-section is assumed to be circular. The end correction lengths on both sides of the orifice are considered in the model.

Although the specific acoustic impedance of air in the orifice is expressed in Eq. (38), the effect of the radiation resistance at the end of the orifice should be considered. This effect was considered as the end-correction length and was added to the depth of the orifice l_a . In this study, the correction length of an aperture, which was derived by Ingard (1953), was used. The end correction length at one side of the orifice in the resistance factor of z_a is as follows:

$$\Delta_{resistance} = R_a. \quad (39)$$

In the case of sound radiation from the circular aperture to the tube cavity of the circular cross-section, the end-correction length in the reactance factor is expressed as follows:

$$\Delta_{reactance} = 0.48(s)^{1/2} \left(1 - 1.25 \frac{R_a}{R_d} \right), \quad (40)$$

where s denotes the cross-sectional area of the orifice. In consideration of the end correction lengths at both the sides, the resistance factor of z_a is obtained from Eqs. (38) and (39).

$$\text{Re}(z_a) = \text{Re} \left[\frac{j\omega\rho_0(l_a + 2R_a)}{1 - \frac{2}{k_s R_a} \frac{J_1(k_s R_a)}{J_0(k_s R_a)}} \right] \quad (41)$$

The reactance factor is obtained from Eqs. (38) and (40) that is as follows:

$$\text{Im}(z_a) = \text{Im} \left[\frac{j\omega\rho_0 l_a}{1 - \frac{2}{k_s R_a} \frac{J_1(k_s R_a)}{J_0(k_s R_a)}} + j\omega\rho_0 \left\{ 0.48(s)^{1/2} \left(1 - 1.25 \frac{R_a}{R_{di-1}} \right) + 0.48(s)^{1/2} \left(1 - 1.25 \frac{R_a}{R_{di}} \right) \right\} \right], \quad (42)$$

where R_{di} and R_{di-1} denote the radii of the cavity at x_i and x_{i-1} , respectively. From the continuity of the volume velocity at x_i and x_{i-1} , the following equations are obtained.

$$s\tilde{v}_a = S_i v_i, \text{ and} \quad (43)$$

$$s\tilde{v}_a = S_{i-1}v_{i-1}, \quad (44)$$

where S_i and S_{i-1} denote the cross-sectional area of the cavity at x_i and x_{i-1} , respectively. From these equations, we get

$$v_{i-1} = (S_i / S_{i-1})v_i. \quad (45)$$

From Eqs. (37) and (43), we get

$$p_{i-1} = p_i - z_a \frac{S_i}{s} v_i. \quad (46)$$

Then, the transfer matrix of the micro-orifice is derived from Eqs. (45) and (46) as follows:

$$\begin{bmatrix} p_{i-1} \\ v_{i-1} \end{bmatrix} = \begin{bmatrix} 1 & -z_a \frac{S_i}{s} \\ 0 & \frac{S_i}{S_{i-1}} \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} = \mathbf{T}_a \begin{bmatrix} p_i \\ v_i \end{bmatrix}. \quad (47)$$

2.5 Change of cross-sectional area

As shown in Fig. 5, at the boundary with an abrupt cross-sectional area change, the particle velocity is discontinuous while the pressure is continuous. Therefore, the transfer matrix at this boundary was needed to describe the velocity change.

Since the volume velocity is preserved at the boundary, the following equation is obtained:

$$S_i v_i = S_{i-1} v_{i-1}. \quad (48)$$

The transfer matrix of the cross-sectional area change is obtained as follows:

$$\begin{bmatrix} p_{i-1} \\ v_{i-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{S_i}{S_{i-1}} \end{bmatrix} \begin{bmatrix} p_i \\ v_i \end{bmatrix} = \mathbf{T}_t \begin{bmatrix} p_i \\ v_i \end{bmatrix}. \quad (49)$$

Fig. 5 Analytical model of a cross-sectional area change. At the boundary, the particle velocity is discontinuous while the pressure is continuous.

3. Calculation and design of filter configurations

3.1 Basic structure of earplugs for calculation

To verify whether the expected properties were obtained for the proposed earplug configurations, the attenuation levels were calculated using the constructed theoretical model. The dimensions of the basic structures of the earplug were determined, considering the actual use, estimated insulation, and practically available experimental settings. The length of the earplug was set at 23.0 mm. The aluminum flexible plate 10.0 mm in diameter and 0.1 mm in thickness was used and was fixed at its edge. The depth of the orifice was set at 1.0 mm. The other parameters used in the calculation are listed in Table 1. These parameters, except the loss factor, were referenced from the literature (Allard and Atalla, 2009 and National Astronomical Observatory of Japan, 2014). The loss factor of the flexible plate was difficult to evaluate because the value was typically influenced by not only the material damping but also the mounting loss. The value of the loss factor was chosen to be 0.05 so that the calculation results could match the experimental results.

3.2 Calculation from theoretical model for prevalent filtered earplugs

The calculation from the theoretical model was performed for a prevalent filtered earplug. The configuration for this earplug had a micro-orifice structure but without a flexible plate and a small attached weight (i.e., unlike the configuration shown in Fig. 1(b), the prevalent filtered earplug configuration does not contain a flexible plate and weight).

Figure 6 shows the calculation results for the prevalent filtered earplugs. Typically, the frequency response for these types of earplugs was controlled by the orifice radius R_a . Therefore, in our calculation, the cases involving different orifice radius sizes (no orifice, $R_a = 0.5$ mm, $R_a = 0.25$ mm, and $R_a = 0.1$ mm) were compared. A dip in the attenuation was observed below 1 kHz, and this dip tended to shift to lower frequencies as the diameter of the orifice reduced. Based on the observed tendencies, the attenuation dip is believed to result from a phenomenon known as the Helmholtz resonance. The Helmholtz resonance is known to occur when the air in the orifice and cavity functions as a mass and spring, respectively. The depth of attenuation also tended to be shallower as the orifice diameter decreased. This was possibly caused by the resistance at the orifice. When an orifice has a small diameter, its resistance is expected to increase.

Based on these results, the small orifice structure on the earplugs is expected to have the capability to control the dip frequency and the dip depth intentionally. When the orifice diameter was further reduced, the attenuation level increased at high frequencies. Based on these attenuation tendencies, the earplug with a micro-orifice structure is expected to function as a low-pass filter.

3.3 Calculation from theoretical model for proposed frequency-specific earplugs

Figure 7 shows the calculation results for the proposed frequency-specific earplugs. The results for the frequency-specific earplugs were calculated for the case of an orifice radius of 0.5 mm.

First, the results show that the mass of a small attached weight to the flexible plate had the capability to control the frequency of bandpass filtering in a systematic manner. Figure 7 shows that for masses of 0.05 g, 0.1 g, 0.2 g, and 0.5 g the first eigenfrequency of the plate are calculated to be 2.74 kHz, 1.97 kHz, 1.41 kHz, and 0.90 kHz, respectively. The results for 0.0 g are also shown as the reference, and the first eigenfrequency is calculated to be 10.23 kHz. As shown in Fig. 7, steep dips in attenuation are observed around these calculated eigenfrequencies.

Secondly, the synergistic effect of the plate resonance and Helmholtz resonance could occur. When the mass on the plate was 0.2 g and the orifice radius was 0.5 mm, a notably large sound transmission improvement in the passing ranges was observed at approximately 1.45 kHz on the basis of the calculations. These results could be attributed to the fact that the eigenfrequency of the plate and the resonance frequency of the Helmholtz resonance closely matched at approximately 1.45 kHz. Based on the observed frequency control for bandpass filtering as well as the effective sound transmission in the target ranges, the proposed earplug configuration could function as a highly selective bandpass filter. With these configurations, ranges of sound transmission narrower than those of the prevalent earplugs are expected.

Thirdly, the calculation results show that a flexible plate could maintain a constant high-level attenuation effect. When the flexible plate was attached, a higher level of attenuation was achieved compared to the level when the plate was not attached (Fig. 6). The rigidity of the plate may cause this sufficient and constant attenuation. Below the first eigenfrequency, the plate insulation property is known to be dominantly affected by the plate bending rigidity, and the

rigidity is determined by the choice of the plate material and the thickness of the plate.

Finally, calculation also showed that the flexible plate could have a sufficient insulation property at higher frequencies than that of the attenuation dip. The high levels of attenuation in these high frequencies could be explained by the mass law of the plate (Fahy and Gardonio, 2007), known to be dominant in these ranges.

Figure 8 shows the calculated results for when the orifice radius was changed. As can be seen, the changes in the attenuation level for the proposed earplug when 0.5 g is attached to the flexible plate. The changes in attenuation are

Table 1 Parameters used in the calculations.

flexible plate (aluminum)		air	
Young's modulus (E)	70.3G [Pa]	Ratio of specific heat (γ)	1.4
Poisson's ratio (ν)	0.345	Prandtl number (B^2)	0.71
Density (ρ_s)	2.69E3 [kg/m ³]	Viscosity coefficient (μ)	1.84E-5 [kg/(m·s)]
Loss factor (η)	0.05	Atmospheric pressure (P_0)	1.0132E5 [Pa]
		Air density (ρ_0) (1atm, 15°C)	1.225 [kg/m ³]

Fig. 6 Calculated results of attenuation level for the earplug without the flexible plate and the small attached weight but only with the orifice structure (i.e., for the prevalent filtered earplugs). The results are shown for different orifice radius sizes (no orifice (blue), $R_a = 0.5$ mm (red), $R_a = 0.25$ mm (green), and $R_a = 0.1$ mm (pink)).

Fig. 7 Calculated results of attenuation level for the earplug with a small weight attached flexible plate as well as the orifice structure (i.e., for the proposed highly-selective bandpass-filtered earplugs). The results are shown for different masses of the weight attached on the flexible plate (0.0 g, without any weight (black), 0.05 g (blue), 0.10 g (green), 0.20 g (red), and 0.50 g (orange)). The radius of the orifice is set at $R_a = 0.5$ mm.

Fig. 8 Calculated results of attenuation level of the proposed frequency specific earplug when 0.5 g weight is fixed on the plate. The results are shown for different orifice radius sizes (no orifice (blue), $R_a = 0.5$ mm (red), $R_a = 0.25$ mm (green), and $R_a = 0.1$ mm (pink)).

shown when the orifice radius is changed (no orifice, $R_a = 0.5$ mm, $R_a = 0.25$ mm, and $R_a = 0.1$ mm). As shown in Fig. 6, as the diameter of the orifice decreases, the dip of the attenuation by the Helmholtz resonance shifts to lower frequency ranges and tends to be shallower (i.e., attenuation level increases). In the case of $R_a = 0.5$ mm, at which the resonance frequency of the Helmholtz resonator is expected to be approximately 1.45 kHz, the deepest dip is observed in Fig. 8. These calculation results also suggested that the steep bandpass effect was caused by the synergistic effect of the plate resonance and the Helmholtz resonance.

All these simulation results show that with appropriate configurations, the proposed earplug is expected to achieve high attenuation over a wide frequency range and also demonstrate efficient sound transmission at specific frequencies.

4. Experiment

Experiments were conducted to verify the validity of the proposed theory of acoustic filtering on earplugs and the feasibility of the proposed earplug. Three types of earplug configurations were prepared in the experiments.

The three types of earplug configurations are as follows: (1) the configurations with a small weight attached to the flexible plate and the orifice structure for the proposed highly-selective bandpass-filtered earplugs, as shown in Fig. 1(b), (2) those with a flexible plate but without a small weight for high attenuation earplugs, and (3) those without a flexible plate and a small weight but only with the orifice structure for the prevalent filtered earplugs.

The expected results from the theoretical model were recalculated for the prepared experimental conditions.

4.1 Prototype earplug

The physical appearance of the prototype earplug is shown in Fig. 9. The case body of the earplug was fabricated from aluminum and processed using a lathe turning machine. The dimensions of the prototype earplug were almost the same as those of the analytical model. As can be seen from the figure, the aluminum plate is used as the flexible plate, and its edge is fixed to the case body with an instant adhesive. The orifice structure was 3D printed in resin. The diameters of the orifices were measured using a digital microscope. A small piece of copper was attached to the center of the plate as a small weight to tune the eigenfrequency of the plate, although the piece is not visible in Fig. 9. The mass of the copper piece used in the experiment was 0.153 g.

4.2 Experimental setup

Figure 10 shows a schematic view of the experimental setup. To evaluate the extent of attenuation of the filtered earplugs, a simple straight-pipe coupler was used as a pseudo artificial ear. The inner diameter of the coupler was 9 mm. At the innermost side of the coupler, a 1/4 inch microphone (B&K 4958) was situated facing outward. The earplug under test was carefully installed at the outer end of the coupler without any gap between the plug and the coupler. A

Fig. 9 Photographs of the prototype earplug. The case body of the earplug is composed of aluminum and processed using a lathe turning machine. The 3D printed micro-orifice is installed inside of the earplug. The aluminum thin plate as the flexible plate is attached to the case body using an instant adhesive.

Fig. 10 Experimental setup. A 1/4 inch microphone is installed in a simple straight-pipe coupler to evaluate the attenuation extent of the earplugs. The loudspeaker for generating white noise is placed at a distance of 1.0 m from the earplug. Another 1/4 inch microphone, which is used as the reference, is placed close to the outer surface of the earplug.

loudspeaker was placed at a distance of 1.0 m from the earplug. To measure the insulation property of the earplug, white noise was presented by the loudspeaker. The reference sound pressure level of the incident sound was measured using another 1/4 inch microphone (B&K 4958) placed close to the outer surface of the earplug. The difference between the measured sound pressure level at the inner coupler microphone and that at the reference microphone was considered as the attenuation level for the earplug under test.

To verify the theoretical model, the resonating behaviors were tested experimentally; the experimental results were compared with the calculated results from the theoretical model, and the effects of the small weights attached to the plate and the orifice were closely examined along with the interactive effect between these.

4.3 Comparison between experimental and calculation results

In Fig. 11 and Fig. 12, the experimental and calculated results are shown when the orifice diameters are 0.98 mm ($R_a = 0.49$ mm) and 0.45 mm ($R_a = 0.225$ mm), respectively. In most of the experimental conditions, the obtained experimental results closely matched the expected results from the calculation. The effect of small weight attached to the flexible plate for bandpass filtering was experimentally confirmed. Moreover, the use of a flexible plate for maintaining a constant high-level attenuation and the use of an orifice for Helmholtz resonance were confirmed. These results implied that the theoretical model could explain the properties of the actual application of the proposed earplug. From a close examination of the results in Fig. 11 and Fig. 12, the possibility of a synergetic effect by plate resonance and the Helmholtz resonance is found. Particularly, if the 0.49 mm orifice radius (Fig. 11) is compared with the 0.225 mm orifice radius (Fig. 12), a more apparent synergetic change in the plate resonance due to Helmholtz resonance is

observed in the former. In this setting, the successful implementation of a high level of attenuation in low frequency ranges and effective sound transmission at approximately 1.5 kHz was observed.

However, the discrepancy between the calculated and experimental results was also observed during the experiments. When the measured values of attenuation were greater than 60 dB, the measured results tended to be smaller than those of the calculated results. These could result from the following two possible technical limitations: (1) insufficient insulation of the experimental setups around the coupler used in this study and (2) the internal noise of the microphone installed in the coupler. These could affect the measurement for insulation property evaluation.

Except for these limitations, the experimental results showed that the proposed earplug configuration exhibited flexible capabilities for sufficient attenuation in low frequency ranges and efficient transmission for the targeted frequency ranges of sounds. These results showed the possibility of practical application for highly selective bandpass-filtered earplugs.

Fig. 11 Experimental and calculation results in attenuation level of the earplug when the orifice diameter is 0.98 mm ($R_a = 0.49$ mm). The red circles are the experimental results for the proposed earplug. The blue squares are the experimental results for the earplugs with a flexible plate but without a small attached weight for high-attenuation earplugs. The green triangles are the experimental results for the earplugs without the flexible plate and the small weight but only with the orifice structure for the prevalent filtered earplugs. The dashed lines indicate the calculated counterparts for these earplugs.

Fig. 12 Experimental and calculation results in attenuation level of the earplug when the orifice diameter is 0.45 mm ($R_a = 0.225$ mm). The red circles are the experimental results for the proposed earplug. The blue squares are the experimental results for the earplugs with a flexible plate but without a small attached weight for high-attenuation earplugs. The green triangles are the experimental results for the earplugs without the flexible plate and the small weight but only with the orifice structure for the prevalent filtered earplugs. The dashed lines indicate the calculated counterparts for these earplugs.

5. Conclusion

In this study, earplugs for effective hearing protection, particularly in factory working environments, were proposed that had the capabilities to insulate noise sufficiently in a wide frequency range and simultaneously allowed the efficient passage of sounds in a specific narrow-frequency range. The suggested earplugs functioned well with simple structures composed of a flexible plate, a micro-orifice, cavities, and a small attached weight.

First, the theoretical model was constructed using the transfer matrix method, for which the matrices for each composing element of the earplug were derived. Second, through calculations from the theoretical model, the possibility of fine control over the resonating behaviors of a flexible thin plate and the Helmholtz resonance was investigated. A careful examination was performed on the application possibilities of the implemented resonance controls in the model to proper and practical acoustic filtering for frequency-specific earplugs. The calculated results showed that plate resonating behaviors and the Helmholtz resonance could be controlled well in appropriate settings, and the configurations were applicable to the actual filtering of the filtered earplugs. By completely utilizing the synergetic effect between the plate resonance and the Helmholtz resonance, the optimal design of the filter configurations could be possible to achieve both the steep bandpass effect and the high insulation performance in other wide-frequency ranges.

Finally, the calculated results were tested experimentally. Confirmatory results were obtained, showing the validity and the feasibility of the proposed earplug. The experimental results indicated that the proposed earplug configurations could attenuate noise sufficiently in a wide frequency range, including low frequencies, and simultaneously, it could efficiently pass sounds in the range of 1–2 kHz.

Further fine tuning can be expected by changing the detailed properties of the components. Although not considered in this study, in practical use, the eardrum impedance can affect the insulation of the earplug. For a more accurate evaluation of the human ear protection, the influence of the eardrum impedance will be investigated in future studies.

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