Advance Publication by J-STAGE

Mechanical Engineering Journal

DOI:10.1299/mej.20-00226

Received date : 30 April, 2020

Accepted date : 20 October, 2020

J-STAGE Advance Publication date : 27 October, 2020

Level-set based topology optimization considering milling directions via fictitious physical model

Doe YOUNG HUR*, Yuki SATO*, Takayuki YAMADA*, Kazuhiro IZUI* and Shinji NISHIWAKI*

* Department of Mechanical Engineering and Science, Graduate School of Engineering,
Kyoto University Kyotodaigaku-katsura C3, Nishikyo-ku, Kyoto 615-8540, Japan
E-mail: doe.hur.88s@st.kyoto-u.ac.jp

Abstract

Topology optimization is the most flexible structural optimization method that allows the topological modification as well as the shape changes. Topology optimization techniques have recently been utilized in engineering applications regarding multi-physics or multi-disciplinary optimization. The manufacturing evaluation is an important factor for the practical application to the field of industry. For example, conventional manufacturing processes such as molding and casting, require fulfillment of specific geometric conditions due to the path of the tools. To obtain the optimal structures satisfying such conditions, a new level set-based topology optimization method, based on the convection-diffusion equations with fictitious fluxes, are utilized to impose the geometric constraints regarding manufacturability, has been developed. In this method, handling geometric constraints for the manufacturability depends on the fictitious heat fluxes. The propagation of the fictitious fluxes distinguishes the non-manufacturable domain. However, in previous approaches, the manufacturing directions are regarded as design parameters where the values are fixed before the optimization procedure. As the non-manufacturable domain is dominated by the path of the manufacturing tools, the appropriate selection of these parameter values is necessary to obtain the optimal design with the best manufacturability constraints. Thus, these parameters should be appropriately set during the optimization process. In this method, we develop a new topology optimization method where the manufacturing directions are also considered as design variables. A sensitivity analysis based on the adjoint equation is introduced to obtain optimized manufacturing directions. Several engineering applications are introduced and optimized through the new method and the previous methods to confirm the availability of the proposed method.

Keywords : Manufacturability, Topology optimization, Geometric constraint, Design optimization

1. Introduction

Structural optimization can be realized through several classified optimization procedures, known as sizing, shape, and topology. Through the parameter settings as the design variables, the size of the design is optimized in the sizing optimization. With the evolution of the boundaries, the shape is optimized in the shape optimization (Pingen, 2008), (Marck et al., 2013). Topology optimization is the most flexible method among them since it can allow the topological changes as well as shape changes. The basic concept of topology optimization is the replacement of the optimization problem by the material distribution problem. Typical topology optimization methods include homogenized design methods (Bendsoe and Kikuchi, 1988), density approach, such as the Solid Isotropic Material with Penalization (SIMP) method (Bendsøe and Sigmund, 1999), and level-set based approach (Allaire et al., 2004). In the level-set based approach, the level-set function (Sethian and Wiegmann, 2000), (Wang et al., 2003) is introduced for the boundary expression of a target structure. The level set method generates a clear interface between two materials. In addition, it also provides smooth transition between two materials for the gradient calculation. Unlike the density method, the level set based method defines the interface as zero using the level set function, which is usually calculated by the Hamilton-Jacobi equation with the sensitivities. However, since the update scheme of this method is the boundary movements of the target design domain, the topological

changes to the introduction of the holes essentially do not occur. Yamada et al. (Yamada et al., 2010) proposed a different type of the level-set based approach where the conventional topology optimization concept and the phase field model are introduced. This method allows any topological changes in addition to the clear boundary expressions using the piecewise constant type of the level set function.

Manufacturing requirements regarding the cost and availability for mass production are critical for engineering applications. Traditional manufacturing methods, such as molding and casting, have been widely used. In contrast, recent manufacturing methods, such as the additive manufacturing, have a higher degree of freedom. However, most industries are still using the traditional manufacturing methods that have strict constraints which the conventional topology optimization rarely satisfies the core requirements. Thus, a topology optimization method considering manufacturing constraints is necessary for industrial adaptation. The influence of these constraints towards the design is critical. If the optimum result does not fulfill the constraints strictly, the performance of the model does not satisfy its objective nor the manufacturability. Thus, it is important to define the evaluation method for the manufacturability.

Design freedom of the manufacturable parts is highly restricted by the directions of the design tools. This limitation leads to huge geometrical change on the final design of the products. Manufacturing methods like molding and casting, for example, need void spaces for the molds to be removed while the physical requirements are also satisfied (Xia et al., 2010). In the traditional design processes, these constraints were satisfied through the intuition of the designers or manufacturing process managers. Failure to consider this would cause large cost increment or non-manufacturable parts. In addition, the traditional topology optimization process focuses on its performance as a major task and considers the manufacturability next, or vice versa.

The Design for Manufacturing (DFM) process (Boothroyd et al., 2001), which builds manufacturable design via CAE (Joshi and Ravi, 2010), (Armiliotta et al., 2016), only focuses on the evaluation of the manufacturing cost via simplification or standardization of the design (Selvaraj et al., 2009). Merging with the DFM method, the solid isotropic material with penalization (SIMP) method (Bendsøe and Sigmund, 1999) is popular in the topology optimization field. Generally, the design engineers interpret and modify the design process to obtain manufacturability (Bharanidaran and Ramesh, 2017). Thus, the intuition or the experience of the designer dominates the result. It is important to adapt Multi-disciplinary Design Optimization regarding manufacturability as an objective, or as a constraint, to make a break-point in the early design process. Regarding the physical performance and the manufacturability, or more, simultaneously, enables the process to achieve all the characteristics. In this research, an optimum which satisfies manufacturability constraints and minimizes or maximizes the physical objectives is derived. To deal with this physical characteristics concerning manufacturability, several optimization methods have been introduced, such as imposing geometrical constraints in shape and topology optimization (Liu and Ma, 2016), or the topology optimization based on the length scale constraints which has been proposed for use in the solid isotropic material with penalization (SIMP) schemes (Poulsen, 2003), (Guest et al., 2004), (Zhou et al., 2015). To satisfy the manufacturability and the topological optimum simultaneously, the level set-based topology optimization considering geometric constraints related to the manufacturability is proposed by Sato et al. (Sato et al., 2017). In this research, fictitious physical phenomena, fictitious fluxes in this research, are applied to the numerical analysis for evaluation of the manufacturability. To obtain optimal configurations for certain devices that provide high performance while ensuring manufacturability using molding techniques, the evaluation of the molding manufacturability incorporates with a topology optimization procedure. However, in Sato's approach, the manufacturing directions, same as propagation of the fictitious fluxes which represents parting direction of molding and casting, are fixed constraints although these directions have a great influence on the optimum shape and the objectives of the topology optimization. See the validation of this phenomenon, which has done on a numerical example in sec.3.2. Thus, an optimization process considering the manufacturing directions for milling is necessary to obtain the optimum results incorporating the manufacturability appropriately.

Therefore, we construct a topology optimization method that provides the optimal manufacturing directions regarding manufacturability using milling techniques in this research. Here, two independent manufacturing directions, which are milling directions, are designated as design variables, which are optimized through the sensitivity analysis. An optimization process incorporated with the manufacturability evaluation method for the optimal configuration with high performance and ensured manufacturability is introduced.

2. Methodologies and Mathematical Background

2.1. Manufacturability Evaluation

The conventional topology optimization method may generate voids or undercut geometries inside the optimal configurations as shown in Fig.1. To obtain a manufacturable structure, the design process should contain a stage that deduces the manufacturability. The level-set based approach considering manufacturability is proposed by Sato et al.,(Sato et al., 2017). This approach utilizes the numerical model that implements nonmanufacturable regions via fictitious heat flux from the outline boundaries, which propagates with its initial direction only.

The first assumptions are based on the casting and the injection molding techniques. There are two types that cannot be generated through these methods, which are the undercut geometry and the interior void.

Fig. 1 Examples of Void and Undercut

A structure without voids and undercuts can be manufactured by molding method. An example of the structure which can be manufactured via molding method can be demonstrated as Fig.2.

Fig. 2 An Example of Molding Part

The evaluation of the manufacturability is easy for simple structures, however, for complex structures, as shown

in Fig.3, it is hard to figure out. In traditional design methods, there is a modification procedure for the manufacturing constraints. However, as the number of these feedback processes are limited, the modification range of the design is also limited. A manufacturing evaluation method suggested by Sato, et al.(Sato et al., 2017), is based on the fictitious physical models. For example, for two fictitious physical fluxes generated from the top and bottom of the design domain, respectively, the material domain, the interior void and the undercut regions remain unaffected. In this research, the fictitious physical models are implemented by advection-diffusion equations via designated advection directions. In addition, the evaluation is focused on the availability of milling method to secure clear definition of manufacturability.

Fig. 3 Examples of a Complex Structure(Sato et al., 2017)

2.2. Optimization for manufacturability

The basic formulation of the optimization problem is shown in eq.(1), and the objectives of the optimization are minimization of the mean compliance and manufacturing failure. The constraint, which is shown in eq.(2), is designated to the volume or the area of the material domain.

Objective:

Minimize Mean Compliance

Minimize Volume of Voids or Undercut Regions

(1)

Constraints:

$$[\text{Volume of Solid}] - [\text{Maximum Volume}] \leq 0 \quad \text{in } D$$

(2)

Mathematical formulations of the optimization using weighted sum method for manufacturability can be expressed as,

Objective:

$$\inf_{\chi_\phi} F = \int_{\Gamma_t} t \cdot u_1 d\Gamma + \hat{\gamma} F_M^m$$

(3)

Constraints:

$$G_1 = \int_D \chi_\phi d\Omega - V_{\max} \leq 0$$

(4)

$$E_1 = \int_{\Gamma_i} t \cdot \tilde{u} d\Gamma - \int_D \varepsilon(u) : C_{\chi_\phi} : \varepsilon(\tilde{u}) d\Omega = 0 \quad (5)$$

$$E_i^{FPM} = \int_D \beta \chi_\phi (1 - \psi_i) \tilde{\psi}_i d\Omega - \int_D L_i^2 \nabla \tilde{\psi}_i \cdot A_i \nabla \psi_i d\Omega - \int_D \tilde{\psi}_i L_i V_i \cdot \nabla \psi_i d\Omega = 0 \quad (6)$$

The variable for the manufacturing directions for milling, denoted as by θ_i and ϕ_i , are based on the heat advection concept, and included in the matrices $\mathbf{A}$, the non-dimensional second order diffusion tensor which is anisotropic and relies on the parting directions, and $\mathbf{V}$, the advection vector, respectively. These matrices depends on the manufacturing directions which will be introduced in sec.2.5. The i th order state variable, which represents the existence of the fictitious flux, is designated as ψ_i , the parameter for the virtual heat generation as β , and the outward unit vector for the level-set function, which is normal to the boundary, is represented as n . A non-manufacturable region and a material domain are evaluated as shade regions. A volume constraint is expressed as V_{max} in Eq.(4), which means the maximum allowance volume, or mass, of the material domain. As shown in Eq.(3), the objective of this optimization is minimization of the mean compliance and manufacturing failure. The mean compliance field is represented as u_i , the elastic tensor as $C(\chi_\phi)$, and the strain tensor as ε , where $\varepsilon(u_i) = 1/2(\nabla u_i + \nabla u_i^T)$. The weighting coefficient for the molding constraint is designated as χ and the weak form test functions as $\tilde{u}$ and $\tilde{\psi}_i$. The weak form equilibrium equation for the mean compliance field and the fictitious physical models of Eqs.(8) and (9), which will be introduced in section 2.3, are indicated as E_1 and E_i^{FPM} in Eqs.(5) and (6) (Sato et al., 2017). The manufacturing objective functional F_M^m is defined as,

$$F_M^m = \int_D (1 - \chi) \prod_{i=1}^m \psi_i d\Omega \quad (7)$$

2.3. Numerical Formulations

Numerical implementation of the physical model can be described as,

$$-L^2 \text{div}(A \nabla \psi_i) + L V \cdot \nabla \psi_i = \beta \chi (1 - \psi_i) \quad \text{in } D \quad (8)$$

$$\psi_i = 0 \quad \text{on } \Gamma_i \quad (9)$$

The state variable, ψ_i , converges to 1 for the material domain and shadow regions generated by the fictitious heats from Γ_F . This leads the value of $(1 - \chi) \prod_{i=1}^n \psi_i$ to represent either undercuts or void regions for n manufacturing directions. $\psi_1 \psi_2 \psi_3 (1 - \chi)$, for a three-directional manufacturing example, represents the intersection of the shadow regions for a three-directional manufacturing case. This concept might be easily understandable with Fig.4.

Fig. 4 Demonstration of the Fictitious Flux and State Variable

The level-set based topology optimization method, which is used in this research, is based on the level-set function ϕ which represents the structural boundary implicitly via its iso-surfaces (Sato et al., 2017).

$$\begin{cases} 1 \geq \phi(x) > 0 & \text{for } x \in \Omega \setminus \partial\Omega \\ \phi(x) = 0 & \text{for } x \in \partial\Omega \\ 0 > \phi(x) \geq -1 & \text{for } x \in D \setminus \Omega \end{cases} \quad (10)$$

Positive values of ϕ represent the material domain, negative values represent the void domain, and zero represents the structural boundaries. This level-set function ϕ is imposed for the regularization (Allaire et al., 2002), (Allaire et al., 2004), (Challis, 2010). In this research, however, the level set functions are defined in positive values as corresponding to the points in the material domain (Wang et al., 2003), (Yamada et al., 2010). Minimizing an objective function F , for example, under an inequality constraint G , and level set function ϕ can be shown as follows.

$$\inf_{\chi_\phi} F[\chi_\phi] = \int_D f_{domain}(x, \chi_\phi) d\Omega + \int_\Gamma f_{boundary}(x, \chi_\phi) d\Gamma \quad (11)$$

subject to,

$$G[\chi_\phi] = \int_D g(x, \chi_\phi) d\Omega - G_{max} \leq 0 \quad (12)$$

where g is the density function and G_{max} is the upper limit value of the constraint. f_{domain} and $f_{boundary}$ in Eq.(11) represents integrands of the objective functional defined in the design domain Ω and boundaries of the design domain Γ , respectively. In this research, the characteristic function can be defined as follows,

$$\chi_\phi = \begin{cases} 1 & \text{for } \phi \geq 0 \\ 0 & \text{for } \phi < 0 \end{cases} \quad (13)$$

Due to the discontinuous and numerical oscillation issue, the characteristic function χ_ϕ is replaced with a smoothed Heaviside function, H_ϕ , as follows,

$$H_\phi = \begin{cases} 1 & \text{for } \phi > \omega \\ \frac{1}{2} + \frac{\phi}{\omega} \left(\frac{15}{16} - \frac{\phi^2}{\omega^2} \left(\frac{5}{8} - \frac{3}{16} \frac{\phi^2}{\omega^2} \right) \right) & \text{for } -\omega \leq \phi \leq \omega \\ 0 & \text{for } \phi < -\omega \end{cases} \quad (14)$$

where ω represents the width of the transition. In the unconstrained Lagrangian term, the objective and the constraint terms can be re-written as,

$$\inf_{\phi} \bar{F}[\chi_\phi, \phi] = F + \lambda G \quad (15)$$

$$\bar{F}' = 0, \quad \lambda G = 0, \quad \lambda \geq 0, \quad G \leq 0 \quad (16)$$

where $\bar{F}$ and $\bar{F}'$ are the Lagrangian and Lagrangian derivative, respectively, and λ is the Lagrangian multiplier from the Karush-Kuhn-Tucker (KKT) conditions. To solve the KKT conditions in the optimization, a fictitious time evolution term is introduced (Otomori et al., 2012). The time variation of the level set function is proportional to the gradient of Lagrangian $\bar{F}$.

$$\frac{\partial \phi}{\partial t} = -K \bar{F}' \quad (17)$$

K in this equation is a proportional coefficient, and by adding a regularization term, it can be normalized.

$$\frac{\partial \phi}{\partial t} = -K(\bar{F}' - \tau \nabla^2 \phi) \quad (18)$$

From this reaction diffusion equation, the diffusive term $\nabla^2 \phi$ ensures the smoothness. The regularization parameter τ decides the diffusivity for the level set function, which prevents the production of non-linear or separated structures (Allaire et al., 2004). From these equations, the boundary conditions can be obtained as,

$$\begin{cases} \frac{\partial \phi}{\partial t} = -K(\bar{F}' - \tau \nabla^2 \phi) & \text{in } D \\ n \cdot \nabla \phi = 0 & \text{on } \partial D \end{cases} \quad (19)$$

The Dirichlet boundary condition, $\phi = 0$, of the design domain is implemented to this boundary condition.

2.4. Sensitivities and Adjoint Equation Lagrangian

The Lagrangian of this level-set based topology optimization, L , is derived for the sensitivity analysis.

$$L = F[v, \zeta_i, \chi_\phi] + \hat{p} G_1[\chi_\phi] + E_1[v, \tilde{v}, \chi_\phi] + \hat{\gamma} \sum_{i=1}^m E_i^{\text{FPM}}[\zeta_i, \tilde{\zeta}_i, \chi_\phi] \quad (20)$$

The Lagrangian multiplier for the inequality volume constraint is indicated as $\hat{p}$, the Lagrangian multipliers for the governing equations are $\tilde{v}$ and $\tilde{\zeta}_i$, and state variables as v and ζ_i . With respect to the Lagrangian multipliers, $\tilde{v}$ and $\tilde{\zeta}_i$, the Gateaux derivative should be equal to 0 for the stationary conditions.

$$\begin{aligned} L = & \int_{\Gamma_t} t \cdot u d\Gamma + \hat{\gamma} \int_D (1 - \chi_\phi) \prod_{i=1}^m \psi_i d\Omega + \hat{p} \left\{ \int_D \chi_\phi d\Omega - V_{max} \right\} \\ & + \left\{ \int_{\Gamma_t} t \cdot \tilde{u} d\Gamma - \int_D \varepsilon(u) : C_{\chi_\phi} : \varepsilon(\tilde{u}) d\Omega \right\} \\ & + \hat{\gamma} \left\{ \int_D \beta \chi_\phi (1 - \psi_i) \tilde{\psi}_i d\Omega - \int_D L_i^2 \nabla \tilde{\psi}_i \cdot A_i \nabla \psi_i d\Omega - \int_D \tilde{\psi}_i L_i V_i \cdot \nabla \psi_i d\Omega \right\} \end{aligned} \quad (21)$$

A self-adjoint equation, as shown above, leads the p to the weak forms of the adjoint equations for the variable ψ_i . The test functions of the weak forms are indicated as $\tilde{p}$ and $\tilde{\xi}$.

$$L \langle \chi_\phi, \delta \chi_\phi \rangle = \int_D \left[\hat{\gamma} \left\{ \beta (1 - \xi_i) \tilde{\xi}_i - \prod_{i=1}^m \xi_i \right\} \delta \chi_\phi + \tilde{p} \delta \chi_\phi - \varepsilon(p) : C_{\delta \chi_\phi} : \varepsilon(\tilde{p}) \right] d\Omega \quad (22)$$

The adjoint equation of the minimum mean compliance problem is derived from the Lagrangian (Sato et al., 2017) as,

$$\begin{aligned} L \langle \xi, \delta \xi \rangle = & \int_D \hat{\gamma} \left\{ (1 - \chi_\phi) \prod_{i=1}^m \chi_i + \nabla \left(-L_i^2 \nabla \tilde{\xi}_i \cdot A_i - \tilde{\xi}_i L_i V_i \cdot \right) - \beta \chi_\phi \tilde{\xi}_i \right\} \delta \xi d\Omega \\ & + \int_{\Gamma_t} \hat{\gamma} \left\{ - \left(-L_i^2 \nabla \tilde{\xi}_i \cdot A_i \nabla \xi_i - \tilde{\xi}_i L_i V_i \cdot \right) \right\} \delta \xi d\Gamma \end{aligned} \quad (23)$$

2.5. Optimization scheme regarding the manufacturing directions for milling

The sensitivity of the manufacturing directions can be derived from the Lagrangian Eq.(21)

$$\frac{\partial L}{\partial \theta_i} = \int_D \hat{\gamma} \left\{ -L_i^2 \nabla \tilde{\xi}_i \cdot \frac{\delta A_i}{\delta \theta_i} \nabla \xi_i - \tilde{\xi}_i L_i \frac{\delta V_i}{\delta \theta_i} \cdot \nabla \xi_i \right\} d\Omega \quad (24)$$

where, for a two-dimensional case,

$$A = d \otimes d = \begin{pmatrix} \cos^2 \theta_i & \sin \theta_i \cos \theta_i \\ \sin \theta_i \cos \theta_i & \sin^2 \theta_i \end{pmatrix} \quad \text{and} \quad V = V_m \begin{pmatrix} \cos \theta_i \\ \sin \theta_i \end{pmatrix} \quad (25)$$

And for a three-dimensional case,

$$A = \begin{pmatrix} \cos^2 \theta_i \cos^2 \varphi_i & \cos \theta_i \sin \theta_i \cos^2 \varphi_i & \cos \theta_i \cos \varphi_i \sin \varphi_i \\ \cos \theta_i \sin \theta_i \cos^2 \varphi_i & \sin^2 \theta_i \cos^2 \varphi_i & \sin \theta_i \cos \varphi_i \sin \varphi_i \\ \cos \theta_i \cos \varphi_i \sin \varphi_i & \sin \theta_i \cos \varphi_i \sin \varphi_i & \sin^2 \varphi_i \end{pmatrix} \quad \text{and} \quad V = V_m \begin{pmatrix} \cos \theta_i \cos \varphi_i \\ \sin \theta_i \cos \varphi_i \\ \sin \varphi_i \end{pmatrix} \quad (26)$$

where $d_i = (\cos \theta_i \quad \sin \theta_i)^T$ for two-dimensional cases or $d_i = (\cos \theta_i \sin \varphi_i \quad \sin \theta_i \sin \varphi_i \quad \cos \varphi_i)^T$ for three-dimensional cases. The direction of the i th fictitious physical flux can be represented by θ_i and φ_i . Thus, as shown in Fig.5, for i th direction the values of A_i and V_i change due to the θ_i and φ_i , and V_m is the magnitude parameter of the fictitious flux.

Fig. 5 Demonstrations of the directions

The updating scheme can be simply described as,

$$\theta_{i+1} = \theta_i - \frac{\partial L}{\partial \theta_i} \quad (27)$$

From Eq.(27), the manufacturing directions are updated depending on the sensitivity analysis $\partial L / \partial \theta_i$ which is mathematically utilized in the optimization process.

3. Numerical Examples

In this section, the topology optimization process is applied to several test problems that exhibit features likely to cause difficulty in manufacturing. We compared the performance of the topology optimization regarding manufacturability to two two-dimensional models and two three-dimensional models. Based on the parameter study from Sato et al., (Sato et al., 2017), the weighting coefficient γ and the parameter β are decided as 1 and 1000. The regularization parameter τ is designated as 1×10^{-4} . The assumption for the manufacturing condition is a two-part mold or milling which needs two manufacturing directions, or parting directions, to be decided, denoted as θ_1 and θ_2 for the two-dimensional case and θ_1 , θ_2 , φ_1 and φ_2 for the three-dimensional case in Eq.(27). The initial values of the directions are horizontal directions, that is $\theta_1 = 0$ and $\theta_2 = \pi$ for two-dimensional cases, and $\theta_1 = 0$, $\theta_2 = \pi$, $\varphi_1 = 0$ and $\varphi_2 = 0$ for three-dimensional cases. To verify the efficiency of the optimization on the parting directions, numerical analyses have been done on non-manufacturable models, manufacturable models with two fixed-directions and manufacturable models with optimized directions.

3.1. Two-Dimensional Cantilever Beam

A two-dimensional cantilever beam, a typical engineering application in the topology optimization field, is selected as a study case. As shown in Fig.6, the design domain is colored in gray.

There are two fixed boundaries and one vertical load condition. To gain the design freedom, unlike the typical cantilever beam examples, the upper and the lower boundaries of the design domain exceed that of the fixed edges. A force in the vertical direction (y direction) is applied in the amount of 1000 N/m on the right edge of the design domain. On the opposite side of the design domain, two fixed constraints are located. Young's modulus and Poisson's ratio of the material are designated as 210 GPa and 0.31 , respectively. The upper limit of the allowable volume was set to 25%, 27.5%, 30% and 35% of the volume of the design domain.

Fig. 6 A two-dimensional numerical example: Cantilever Beam

Fig. 7 A two-dimensional numerical example: Cantilever Beam Results

As shown in Fig.7, the results differ for each volume constraint case. For example, improvement of the objective, mean compliance, depends on the manufacturability conditions and is clearly shown in the 0.25 volume constraint. The objective values of the case with 0.25 volume constraint, for example, are $0.70 \times 10^{-7} m$ for the non-manufacturable model, $1.08 \times 10^{-7} m$ for the manufacturable model with fixed-directions and $0.88 \times 10^{-7} m$ for the manufacturable model with the optimized directions.

Fig. 8 Optimization Results of Two-Dimensional Cantilever Beam

(Volume Constraint: 0.275)

(Left: Non-Manufacturable, Center: Manufacturable model with Fixed Directions, Right: Manufacturable Model with Optimized Directions, Black arrow indicates manufacturing direction)

The representative fixtures are as shown in Fig.8. For the fixed directional manufacturing constraint case, the manufacturing directions are designated as 15° and 195° . On the other hand, from the topology optimization based on the sensitivity analysis, the optimized directions for manufacturing came up as 5.9761° and 170.6892° . Tendency between the volume constraint and the advantage through optimization is clearly shown. Furthermore, as the volume constraint become strict, the difference between the fixed direction cases and the optimal direction case becomes larger.

3.2. Two-dimensional Torsion Beam

The two-dimensional torsion beam example is shown in Fig.9.

Fig. 9 A two-dimensional numerical example: Torsion Beam

As shown in the Fig.9, the fixed edges are located in the center of the design domain and the applied repulsive forces, with $1000 N/m^2$, are separated and located on the four corners. The isotropic linearly elastic material is $210 GPa$ of Young's modulus and 0.31 of Poisson's ratio, respectively. The volume constraint, which allocates the allowance of the maximum ratio of the volume or the area relative to the design domain, is designated from 0.4 to 0.5 .

As shown in Fig.10, three fixed direction cases are set for a controlled group. The optimization on the directions is based on the sensitivity analysis. For most volume constraint cases, non-manufacturable cases showed smallest deformation and the optimized direction cases followed. Similar to the cantilever beam optimization, for some specific cases,

Fig. 10 A two-dimensional numerical example: Torsion Beam Results

the fixed direction model showed smaller deformation than the optimized direction model. The phenomena are caused by two reasons. First the similarity of the manufacturing directions between the fixed direction and the optimized direction. Another is the limitation of the sensitivity base optimization. Since the nonlinearity of the mean compliance depending on the manufacturing directions is quite high, the results are not the global optima. The figures of the results for the volume constraint 0.45 are introduced in Fig.11. As shown in the figures, the shape of the optimum structure is highly dependent on the manufacturing directions.

Fig. 11 Optimization Results of Two-Dimensional Torsion Beam
(Volume Constraint: 0.45)

(Left to Right: Non-Manufacturable, Fixed Directions: 0°, 15°, 45°, Manufacturable Model
with Optimized Directions)
(Black arrow indicates manufacturing direction)

3.3. Three-dimensional Cantilever Beam

For a three-dimensional numerical example, a cantilever beam, which was studied by Wang et al., (Wang et al., 2003), is selected. As shown in Fig.12, four fixed surfaces are located in one side and a force, magnitude of $1000N/m^2$, is applied to a surface on the opposite side.

As shown from simple control models in Fig.13, and an example shown in Fig.14, both the manufacturable models and the non-manufacturable models showed improved results. Meanwhile, the differences of the objective value between the manufacturable models and non-manufacturable models are infinitesimal relative to that of control models. With satisfying the manufacturing constraints, the manufacturable models achieved similar performance with the non-manufacturable models. These results proved the effectiveness of the optimization process regarding the manufacturability. To clarify the

Fig. 12 A three-dimensional numerical example: Cantilever Beam

effectiveness, another three-dimensional complex numerical example is introduced in the next section.

Fig. 13 A three-dimensional numerical example: Cantilever Beam Results

Fig. 14 Optimization Results of Three-Dimensional Cantilever Beam
(Volume Constraint: 0.45)

(Left: Control Model, Center: Non-Manufacturable Model, Right: Manufacturable Model,
Black arrow indicates manufacturing direction)

3.4. Three-dimensional Mechanical Part

The boundary conditions for another three-dimensional mechanical part are as shown in Fig.15. Two pipe-shaped fixed and load surfaces are located on both sides and the center area is designated as the design domain. To demonstrate torsional phenomenon, two opposite forces are applied to the cross sections of a pipe in vertical directions.

Fig. 15 A three-dimensional numerical example: Mechanical Part

From Fig. 16, similar phenomena occurred as with the three-dimensional cantilever beam. Both optimized models, the manufacturable one and the non-manufacturable one, showed relatively improved results from the simple control model. As shown in Fig. 17, the structure of the non-manufacturable model is extremely complex. In contrast, the structure of the manufacturable model is simple.

Fig. 16 A three-dimensional numerical example: Mechanical Part Results

Fig. 17 Optimization Results of Three-Dimensional Mechanical Part
(Volume Constraint: 0.45)

(Left: Control Model, Center: Non-Manufacturable Model, Right: Manufacturable Model,
Black arrow indicates manufacturing direction)

4. Conclusion

In this paper, we have studied that

- Topology optimization process regarding manufacturability
- Adapted the fictitious flux method
- Applied to two-dimensional and three-dimensional engineering examples

However, this paper is focused on the manufacturing directions for milling. Through the evaluation of the engineering applications, the influence of the manufacturing directions toward the objective, the mean compliance in the research, are verified compare to the fixed direction cases. In two-dimensional applications, the optimization on the manufacturing directions is revealed as an efficient and effective way to obtain the optimized results. In three-dimensional applications, the qualified optimization method is applied to geometrically complex structures to prove its effectiveness toward the control models and the non-manufacturable models. As expected, through the topology optimization process regarding manufacturability, the optimized models achieved both the objectives and the manufacturability with optimized manufacturing directions. However, the existence of local optima depending on the manufacturing directions as the variables has emerged. To treat such a phenomenon, it is necessary to apply an improved optimization method for the process. Research regarding this topic will be followed.

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