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Coupling analysis of unsteady aerodynamics and vehicle behavior with road input: Modeling and verification in road tests

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Abstract

This paper discusses that the unsteady aerodynamic forces caused by vehicle motion in road input have a large influence on the vehicle behaviors. In the present coupling analysis, unsteady aerodynamic forces were obtained by using the response functions for vehicle motion derived in our previous study. By attaching a protrusion on the roof which only measures about 3% of the vehicle height, pitch and heave motions are affected significantly by the change of unsteady aerodynamic effects. Due to the coupling of the unsteady aerodynamic forces and vehicle suspension responses, the vehicle behavior depends on the road input frequency. The pitch motion is enhanced in low-frequency road input (<1.6 Hz for a real vehicle) and is suppressed at high frequencies (>1.6 Hz). Meanwhile, the heave motion is suppressed over 1.2 Hz. Such vehicle behaviors were verified in actual running tests, and were consistent to the subjective evaluation of the drivers feeling. Comparing the unsteady aerodynamic coefficients to the equivalent vehicle specification factors, the aerodynamic inertia caused by pitch motion contributes to 39% of the rear vehicle mass inertia effect in pitch motion. This aerodynamic effect is important in designing suspension specifications of vehicles. These results show that the vehicle stability can be affected even by small detail in vehicle shape due to the unsteady aerodynamic forces.

Keywords : Road vehicle, Aerodynamics, Unsteady aerodynamic force, Vehicle dynamics, Coupling analysis, Response function, Motion simulation

1. Introduction

Continuous enhancements have been observed in vehicle performance, with increasing demands for the improvement of running stability by aerodynamic forces. Such improvements are realized by considering every minute change made to the shape of a vehicle, which influences aerodynamic characteristics. However, the understanding of such aerodynamic effects on the running stability cannot be explained appropriately only by analyses based on conventional steady-state aerodynamic forces. This study aims to understand the influence of unsteady aerodynamic forces caused by vehicle motions on the running road input.

Studies on vehicle dynamics affected by unsteady aerodynamic forces due to vehicle motions have been gaining increasing attention in the field of automotive technology, and the necessity of considering unsteady aerodynamic characteristics is being analyzed (e.g., Aschwanden et al., 2006; Passmore et al., 2006; Takeuchi et al., 2009; Pascal et al., 2011; Yamashita et al., 2011; Tsubokura et al., 2012; Nakae et al., 2013; Ikeda et al., 2016; Kuratani et al., 2018; Murayama et al., 2019). Kawakami et al. (2012) and Ogawa et al. (2012) evaluated unsteady aerodynamic forces, such as aerodynamic spring, damping, and mass, and their influence on vehicle motions through their formulation as a modeled function. However, these studies were conducted using basic vehicle models and are not applicable for

examinations of vehicles in actual running conditions. To further improve vehicle performance, motion analyses that evaluate the difference in vehicle behaviors with respect to vehicle shape in road input are necessary in actual running conditions.

Maeda et al. (2020) in their wind tunnel experiments quantitatively measured the differences in unsteady aerodynamic forces according to vehicle shape as aerodynamic response functions. These aerodynamic forces were measured using excitation tests on the scale model, which comprised wheels and a wheelhouse similar to a real vehicle. In the present study, we aimed to clarify the influence of these measured unsteady aerodynamic forces on vehicle behaviors with road inputs. Coupling analyses of unsteady aerodynamic forces and vehicle motions were conducted based on response functions and timeline simulations on pitch and heave motions in the running road input. These analyses were verified by measuring vehicle behavior in an actual running test. Accordingly, the necessity of considering the influence of unsteady aerodynamic forces on vehicle motions and the coupling analysis of aerodynamics with vehicle motions were demonstrated.

Nomenclature

A_f, A_r	: aerodynamic force at the front axle, and at the rear axle
$C_{Amf\theta}, C_{Amr\theta}, C_{Amfz}, C_{Amrz}$	: coefficients of aerodynamic inertia of the axle based on motion with no dimension (These values are quoted from Maeda et al. (2020))
$C_{Acf\theta}, C_{Acr\theta}, C_{Acfz}, C_{Acrz}$	: coefficients of aerodynamic damping of the axle based on motion with no dimension (These values are quoted from Maeda et al. (2020))
$C_{Akf\theta}, C_{Akr\theta}, C_{Akfz}, C_{Akrz}$	: coefficients of aerodynamic spring of the axle based on motion with no dimension (These values are quoted from Maeda et al. (2020))
C_D	: drag coefficient of the vehicle
C_{Lf}, C_{Lr}	: lift coefficients of the front and rear of the vehicle
c_f, c_r	: damping coefficient of suspension at front (for one wheel = 2400 Ns/m), and rear (= 1300 Ns/m)
$c_{f\theta}, c_{fz}, c_{r\theta}, c_{rz}$	: coefficients of the aero-damping at the axle to motion with real dimensions
$c_{r\theta}^*$	: coefficient of reversing the sign convention (only at the rear axle in pitch motion, = $-c_{r\theta}$)
D	: dynamic pressure (= $1/2\rho U^2$)
k_f, k_r	: spring coefficients of suspension at front (for one wheel = 30900 N/m), and rear (= 28400 N/m)
$k_{f\theta}, k_{fz}, k_{r\theta}, k_{rz}$	: coefficients of the aero-spring at the axle to motion with real dimensions
$k_{r\theta}^*$	: coefficient of reversing the sign convention (only at the rear axle in pitch motion, = $-k_{r\theta}$)
L	: length of the vehicle (= 4.228 m)
l_f, l_r	: length from the gravity center to the front axle (= 1.050 m), and to the rear axle (= 1.590 m)
l_{wb}	: length of the wheel base (= 2.640 m)
m	: body mass (= 1376 kg)
m_f, m_r	: body mass of the front axle (= $m \cdot l_r / l_{wb}$ = 847 kg), and of the rear axle (= $m \cdot l_f / l_{wb}$ = 529 kg)
$m_{f\theta}, m_{fz}, m_{r\theta}, m_{rz}$	: coefficients of the aero-inertia at the axle to motion with real dimensions
$m_{r\theta}^*$	: coefficient of reversing the sign convention (only at the rear axle in pitch motion, = $-m_{r\theta}$)
S	: frontal projected area of the vehicle (= 2.208 m ²)
S_f, S_r	: suspension force at the front axle, and at the rear axle
T_{rz}	: first-order delay occurring for aerodynamic forces according to the heave motion with no dimension (only at the rear axle, and these values are quoted from Maeda et al. 2020)
U	: wind velocity
z_b	: body displacement at the wheel-base center (= $(z_{bf} + z_{br})/2$)
z_{bf}, z_{br}	: body displacement at the front axle, and at the rear axle
z_{rd}	: road displacement at the wheel-base center (= $(z_{rdf} + z_{rdr})/2$)
z_{rdf}, z_{rdr}	: road displacement at the front axle, and at the rear axle
z_x	: body height to the road surface at the wheel-base center (= $z_b - z_{rd}$)
z_{xf}, z_{xr}	: body height to the road surface at the front axle, and at the rear axle
θ_b	: pitch angle of vehicle body (= $(z_{bf} - z_{br})/l_{wb}$)
θ_{rd}	: pitch angle of the road (= $(z_{rdf} - z_{rdr})/l_{wb}$)

θ_x	: body angle to the road surface at the wheel-base center ($= \theta_b - \theta_{rd}$)
ρ	: air density ($= 1.2 \text{ kg/m}^3$)
τ_{rz}	: time constant of the rear axle at the heave motion with real dimensions ($\tau_{rz} = T_{rz} L/U$)
τ_{wb}	: delay time of rear road input after front road input ($= l_{wb}/U$)

Subscripts in $A, C_A, C_L, c, k, l, m, S, T, \tau$

m	: inertia
c	: damping
k	: spring
f	: front
r	: rear
θ	: pitch motion
z	: heave motion
wb	: wheel base
*	: reversing the sign convention

Subscripts in z, θ

b	: body
rd	: road
x	: difference of body to road
f	: front
r	: rear

2. Formulation of vehicle motions and unsteady aerodynamic forces in road input

2.1 Motion equation in road input considering aerodynamic lifts

To evaluate the influences of unsteady aerodynamic forces caused by pitch and heave motions in road input, the two-degree-of-freedom (2-DOF) models of the pitch and heave motions are defined in Fig. 1, which include the dampers and springs of the front and rear suspensions and the aerodynamic lifts on the front and rear axles. The vertical displacements at the front and rear axles of the vehicle body (z_{bf}, z_{br}), road input (z_{rdf}, z_{rdr}), and suspension (z_{xf}, z_{xr}) are also defined. The suspension forces based on the damper coefficients (c_f, c_r) and spring coefficients (k_f, k_r) act at the front and rear axles (k_f, k_r, c_f , and c_r represent a coefficient for one wheel suspension), respectively. Inertial forces occur with respect to the masses of the vehicle body (m_f, m_r) and vertical accelerations ($\ddot{z}_{bf}, \ddot{z}_{br}$) at the front and rear axles; in addition, the aerodynamic forces (A_f, A_r) act in parallel to the suspension forces. The specifications of the real vehicle used in this study are shown in the Nomenclature.

The motion equations are expressed in Eq. (1) according to the 2-DOF of the pitch moment and vertical force. The suspension displacements (z_{xf}, z_{xr}) are expressed as the difference between vehicle bodies (z_{bf}, z_{br}) and road input displacements (z_{rdf}, z_{rdr}), and converted to the relative pitch angle (θ_x) and heave displacement (z_x) of the body with respect to the road, as shown in Eq. (2). The front road displacement (z_{rdf}) is inputted as the reference variable, and the rear road displacement (z_{rdr}) is inputted with the delay time (τ_{wb}) determined by the speed (U) and wheel base (l_{wb}), as shown in Eq. (3).

$$\begin{bmatrix} m_f \frac{l_{wb}}{2} & -m_r \frac{l_{wb}}{2} \\ m_f & m_r \end{bmatrix} \begin{bmatrix} \ddot{z}_{bf} \\ \ddot{z}_{br} \end{bmatrix} + \begin{bmatrix} 2c_f \frac{l_{wb}}{2} & -2c_r \frac{l_{wb}}{2} \\ 2c_f & 2c_r \end{bmatrix} \begin{bmatrix} \dot{z}_{xf} \\ \dot{z}_{xr} \end{bmatrix} + \begin{bmatrix} 2k_f \frac{l_{wb}}{2} & -2k_r \frac{l_{wb}}{2} \\ 2k_f & 2k_r \end{bmatrix} \begin{bmatrix} z_{xf} \\ z_{xr} \end{bmatrix} = \begin{bmatrix} \frac{l_{wb}}{2} & -\frac{l_{wb}}{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A_f \\ A_r \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} z_{xf} \\ z_{xr} \end{bmatrix} = \begin{bmatrix} z_{bf} \\ z_{br} \end{bmatrix} - \begin{bmatrix} z_{rdf} \\ z_{rdr} \end{bmatrix} = \begin{bmatrix} l_{wb}/2 & 1 \\ -l_{wb}/2 & 1 \end{bmatrix} \begin{bmatrix} \theta_x \\ z_x \end{bmatrix} \quad (2)$$

$$z_{rdr}(t) = z_{rdf}(t - \tau_{wb}) \quad (3)$$

Equations (1)–(3) are converted to Eqs. (4)–(6), respectively, through a Laplace transformation, assuming an initial value of 0.

Fig. 1 Definition of variables at the vehicle motion model of the pitch and heave motions with the vehicle suspension and unsteady aerodynamic forces

$$\begin{bmatrix} m_f s^2 & -m_r s^2 \\ m_f s^2 & m_r s^2 \end{bmatrix} \begin{bmatrix} \hat{z}_{bf}(s) \\ \hat{z}_{br}(s) \end{bmatrix} + \begin{bmatrix} 2c_f s + 2k_f & -2c_r s - 2k_r \\ 2c_f s + 2k_f & 2c_r s + 2k_r \end{bmatrix} \begin{bmatrix} \hat{z}_{xf}(s) \\ \hat{z}_{xr}(s) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{A}_f(s) \\ \hat{A}_r(s) \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \hat{z}_{xf}(s) \\ \hat{z}_{xr}(s) \end{bmatrix} = \begin{bmatrix} \hat{z}_{bf}(s) \\ \hat{z}_{br}(s) \end{bmatrix} - \begin{bmatrix} \hat{z}_{rdf}(s) \\ \hat{z}_{rdr}(s) \end{bmatrix} = \begin{bmatrix} l_{wb}/2 & 1 \\ -l_{wb}/2 & 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} \quad (5)$$

$$\hat{z}_{rdr}(s) = e^{-s\tau_{wb}} \cdot \hat{z}_{rdf}(s) \quad (6)$$

2.2 Formulation of the unsteady aerodynamic forces caused by pitch and heave motions

The coupling analyses of unsteady aerodynamic forces with vehicle motions are enabled by substituting the formulated unsteady aerodynamic forces into the aerodynamic item (A_f, A_r) of Eq. (1). Their aerodynamic formulations have been studied in a previous report (Maeda et al., 2020), and they are defined by summing the aerodynamic forces caused by the pitch motion ($A_{f\theta}, A_{r\theta}$) and heave motion (A_{fz}, A_{rz}) for the front and rear axles, as shown in Fig. 1. The presented pitch and heave motions are equivalent to the relative pitch angle (θ_x) and relative heave displacement (z_x) of vehicle body to the road input, respectively. The flow change around the vehicle is assumed to be affected by these relative motions, because the changes in flow mass or flow velocity are almost decided by the space change between the vehicle body and road surface. Thus, the unsteady aerodynamic forces (A_f, A_r) are expressed in Eq. (7) by the Laplace transformation as the aerodynamic inertia, damping, and spring coefficients occurring in proportion to the acceleration, velocity, and displacement of the vehicle body, respectively. For the rear aerodynamic force due to the heave motion (A_{rz}), the first-order lag (τ_{rz}, T_{rz}) must be considered.

$$\begin{bmatrix} \hat{A}_f(s) \\ \hat{A}_r(s) \end{bmatrix} = \begin{bmatrix} \hat{A}_{f\theta}(s) & \hat{A}_{fz}(s) \\ \hat{A}_{r\theta}(s) & \hat{A}_{rz}(s) \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} = \begin{bmatrix} m_{f\theta} s^2 + c_{f\theta} s + k_{f\theta} & m_{fz} s^2 + c_{fz} s + k_{fz} \\ m_{r\theta} s^2 + c_{r\theta} s + k_{r\theta} & \frac{m_{rz} s^2 + c_{rz} s + k_{rz}}{\tau_{rz} s + 1} \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix}$$

$$= \frac{1}{2} \rho U^2 S \begin{bmatrix} C_{Amf\theta} \left(\frac{L}{U}\right)^2 s^2 + C_{Acf\theta} \frac{L}{U} s + C_{Akf\theta} & \frac{C_{Amfz}}{L} \left(\frac{L}{U}\right)^2 s^2 + \frac{C_{Acfz}}{L} \frac{L}{U} s + \frac{C_{Akfz}}{L} \\ C_{Amr\theta} \left(\frac{L}{U}\right)^2 s^2 + C_{Acr\theta} \frac{L}{U} s + C_{Akr\theta} & \frac{C_{Amrz}}{L} \left(\frac{L}{U}\right)^2 s^2 + \frac{C_{Acrz}}{L} \frac{L}{U} s + \frac{C_{Akrz}}{L} \\ T_{rz} \frac{L}{U} s + 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} \quad (7)$$

Coefficients C_{Am**} , C_{Ac**} , C_{Ak**} , and T_{rz} in Eq. (7) are non-dimensional owing to the dynamic pressure ($D = 1/2\rho U^2$), model frontal projected area (S), model length (L), and wind velocity ratio to model length (U/L) (the subscripts are defined as follows: f : front, r : rear, θ : pitch motion, z : heave motion). The coefficients expressed as m_{**} , c_{**} , k_{**} , and τ_{rz} are the aerodynamic coefficients of a real vehicle given the vehicle speed, air density, frontal projected area, and vehicle length (where m is the aerodynamic inertia, c is the aerodynamic damping, k is the aerodynamic spring, and τ is the aerodynamic first-order delay).

2.3 Calculation of unsteady aerodynamic coefficients caused by pitch and heave motions on a real vehicle

Because the direct measurement of the unsteady aerodynamic coefficients of a real vehicle is difficult, the real dimension aerodynamic coefficients were converted from the non-dimensional unsteady aerodynamic coefficients obtained from the excitation test of a 1/4 scale model in a small wind tunnel (conducted in our previous study).

A small hatchback of the commercial vehicle was used; it has a rectangular protrusion of 40 mm $\times$ 40 mm at the front end on the roof assuming flow fluctuations. The cases of the vehicle without and with the protrusion are selected for comparison. The vehicle shapes and steady-state aerodynamic coefficients are shown in Fig. 2. The two cases of the real-dimension unsteady aerodynamic coefficients caused by the pitch and heave motions were calculated by substituting the frontal projected area (S), length of the wheelbase (L), and speed (U) of the real vehicle in Eq. (7). The aerodynamic force response functions due to the pitch angle ($A_{f\theta}/\theta$, $A_{r\theta}/\theta$) are shown in Fig. 3 at 60 km/h, and the aerodynamic force response functions due to the heave displacement (A_{fz}/z , A_{rz}/z) are shown in Fig. 4. The concrete coefficient values of their response functions are shown in the figures for cases without and with a protrusion.

The largest difference in the cases without and with a protrusion occurred on the rear aerodynamic force response function due to pitch motion. In the case without a protrusion, the rear aerodynamic force due to pitch motion pushes back the pitch angle because the response phase is approximately 0; this implies that the rear upward force (positive rear force) occurs for the pitch angle in the front (positive pitch displacement). Conversely, in the case with a protrusion, the rear aerodynamic force due to pitch motion promotes the pitch angle because the response phase is almost 180° at a low frequency, implying that the rear downward force (negative rear force) occurs for the pitch angle in the front. In addition, the rear response gain due to pitch motion without a protrusion is larger than the gain with a protrusion. In the next section, we show how the differences in aerodynamic responses affect the vehicle behaviors in actual running conditions.

Body case	C_D	C_{Lf}	C_{Lr}
Without a protrusion	0.304	-0.010	0.069
With a protrusion	0.332	0.012	0.094

C_D : drag coefficient
 C_{Lf}, C_{Lr} : coefficient of the front and rear lifts

(a) Target vehicle and the steady-state aerodynamic coefficients for each case

(b) Model shape without a protrusion

(c) Model shape with a protrusion

Fig. 2 Targeted vehicle, model shapes, and steady-state aerodynamic coefficients in the cases without and with a rectangular protrusion of 40 mm $\times$ 40 mm on the front roof end.

Fig. 3 Response functions of the aerodynamic force by the pitch motion at 60 km/h in the case of the real vehicle without and with a protrusion. (a) front response due to pitch; the difference between the vehicle shapes is almost none. (b) rear response due to pitch; the difference between the vehicle shapes is remarkably large.

Fig. 4 Response functions of aerodynamic forces to the heave motion at 60 km/h in the case of the real vehicle without and with a protrusion. (a) Front response due to heave; no difference is observed. (b) Rear response due to heave; the difference is small.

3. Coupling analysis of the unsteady aerodynamic forces with the vehicle motions in the road input

3.1 Derivation of the motion response functions in the road input

By using the motion equations in Eqs. (4)–(6) and the unsteady aerodynamic response function in Eq. (7), the response functions, including unsteady aerodynamic effects, of vehicle motion in the road input are formulated as Eq. (8).

$$\begin{aligned}
& \begin{bmatrix} m_f s^2 & -m_r s^2 \\ m_f s^2 & m_r s^2 \end{bmatrix} \begin{bmatrix} l_{wb}/2 & 1 \\ -l_{wb}/2 & 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} + \begin{bmatrix} m_f s^2 & -m_r s^2 \\ m_f s^2 & m_r s^2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-s\tau_{wb}} \end{bmatrix} \hat{z}_{rdf}(s) \\
& = - \begin{bmatrix} 2c_f s + 2k_f & -(2c_r s + 2k_r) \\ 2c_f s + 2k_f & 2c_r s + 2k_r \end{bmatrix} \begin{bmatrix} l_{wb}/2 & 1 \\ -l_{wb}/2 & 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} \\
& + \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} m_{f\theta} s^2 + c_{f\theta} s + k_{f\theta} & m_{fz} s^2 + c_{fz} s + k_{fz} \\ m_{r\theta} s^2 + c_{r\theta} s + k_{r\theta} & \frac{m_{rz} s^2 + c_{rz} s + k_{rz}}{\tau_{rz} s + 1} \end{bmatrix} \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix}
\end{aligned} \tag{8}$$

Because Eq. (8) is only expressed according to the variables of the relative pitch angle (θ_x), relative heave displacement (z_x), and road input displacement (z_{rdf}), the equation can be rewritten as Eq. (9).

$$\begin{aligned}
\hat{M}(s) \begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} & = - \begin{bmatrix} m_f s^2 & -m_r s^2 \\ m_f s^2 & m_r s^2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-s\tau_{wb}} \end{bmatrix} \hat{z}_{rdf}(s) \\
\hat{M}(s) & := \begin{bmatrix} \hat{M}_{11}(s) & \hat{M}_{12}(s) \\ \hat{M}_{21}(s) & \hat{M}_{22}(s) \end{bmatrix} \\
& = \begin{bmatrix} m_f s^2 + 2c_f s + 2k_f & -m_r s^2 - 2c_r s - 2k_r \\ m_f s^2 + 2c_f s + 2k_f & m_r s^2 + 2c_r s + 2k_r \end{bmatrix} \begin{bmatrix} l_{wb}/2 & 1 \\ -l_{wb}/2 & 1 \end{bmatrix} \\
& - \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} m_{f\theta} s^2 + c_{f\theta} s + k_{f\theta} & m_{fz} s^2 + c_{fz} s + k_{fz} \\ m_{r\theta} s^2 + c_{r\theta} s + k_{r\theta} & \frac{m_{rz} s^2 + c_{rz} s + k_{rz}}{\tau_{rz} s + 1} \end{bmatrix}
\end{aligned} \tag{9}$$

Accordingly, the response functions of the relative pitch angle (θ_x) and relative heave displacement (z_x) in the road input displacement (z_{rdf}) are given by Eq. (10).

$$\begin{bmatrix} \hat{\theta}_x(s) \\ \hat{z}_x(s) \end{bmatrix} = -\hat{M}(s)^{-1} \begin{bmatrix} m_f s^2 & -m_r s^2 \\ m_f s^2 & m_r s^2 \end{bmatrix} \begin{bmatrix} 1 \\ e^{-s\tau_{wb}} \end{bmatrix} \hat{z}_{rdf}(s) \tag{10}$$

3.2 Influence analyses of the unsteady aerodynamic forces using the motion response functions

Because all the coefficients are numerical constants in Eq. (10), the response functions are provided concretely by substituting the values of suspension spring, suspension damping, vehicle length, vehicle mass, running speed, and unsteady aerodynamic coefficient. The response functions in the road input are set to 60 km/h for the three cases, i.e., without a protrusion, with a protrusion, and without considering the aerodynamic effect. The response functions of the pitch and heave motions in road input are shown in Fig. 5, and the response functions of the vertical displacement at the front and rear axles are shown in Fig. 6. These response functions do not have unstable poles in both cases.

As shown in Fig. 5, almost no difference exists between the case with a protrusion and the case in which the aerodynamic effect was not considered; however, differences in the response gain and phase are observed between the cases without and with a protrusion. For the pitch motion response (θ_x/z_{rdf}), the gain in the case without a protrusion is slightly smaller than that with a protrusion at less than 1.6 Hz, and is larger at over 1.6 Hz in the opposite direction. For the heave response (z_x/z_{rdf}), the gain in the case without a protrusion is larger than that with a protrusion at over 1.2 Hz. As shown in Fig. 6, the differences occur almost entirely with the response of the rear vertical motion, as the frequency and magnitude of the rear gain peak without a protrusion are higher and larger as the rear phase advances.

Thus, the coupling analyses of unsteady aerodynamic forces with vehicle motions in road input are necessary, because the unsteady aerodynamic influences are sufficiently large and change owing to the road input frequencies.

Fig. 5 Transfer functions of the vehicle motion response (θ_x, z_x) in the road input considering the unsteady aerodynamic forces at 60 km/h in the case of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

Fig. 6 Transfer functions of the vehicle motion response (z_{xf}, z_{xr}) in the road input considering the unsteady aerodynamic forces at 60 km/h in the case of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

3.3 Analyses of the influence of unsteady aerodynamic forces by using timeline simulations

The influences of the unsteady aerodynamic forces on the vehicle motions in the road inputs were analyzed by executing timeline simulations using the response functions in Eq. (10). First, the timeline responses in the step road input, assuming that the vehicle is riding up on a 20-mm step at 60 km/h, was simulated. Three cases, i.e., without a protrusion, with a protrusion, and without considering aerodynamics, were compared (Fig. 7) with respect to the

timeline of road inputs (z_{rdf}, z_{rdr}), aerodynamic forces (A_f, A_r), suspension forces (S_f, S_r), and relative displacements ($\theta_x, z_x, z_{xf}, z_{xr}$). The displacements of road step were inputted in the front and rear axles with a delay of the wheelbase, resulting in the occurrence of aerodynamic and suspension forces. Consequently, the vehicle motions occurred and converged while fluctuating.

The difference is almost not observed between the cases with a protrusion and without considering aerodynamics (which are almost piled up in the figure). Between the cases without and with a protrusion, a difference is observed in the rear aerodynamic force (A_r). The amplitude of relative pitch displacement (θ_x) is larger, and its phase advances in the case without a protrusion. The relative heave displacement (z_x) also has a similar tendency.

These differences in pitch and heave motions occur almost with respect to the rear vertical motion (z_{xr}), similar to that in the previous analyses of response functions. In addition, because the change in rear suspension force occurs in the case without a protrusion, the unsteady aerodynamic and suspension forces are confirmed to be the results for the coupled effects.

Next, to confirm the change in the aerodynamic influence according to the road input frequency, the simulated responses in the sine input road at constant frequencies of 1.4 and 1.8 Hz are shown in Figs. 8 and 9, for the three aforementioned cases (Fig. 7). With an input frequency of 1.4 Hz, few differences were observed between the cases with a protrusion and without considering aerodynamics. The rear vertical displacement response (z_{xr}) for the case without a protrusion is slightly fast and the amplitude is larger. Therefore, the pitch angle response (θ_x) is fast but the amplitude is small. By contrast, with an input frequency of 1.8 Hz, the rear vertical displacement response (z_{xr}) of the case without a protrusion is fast and the amplitude is larger; therefore, the pitch angle response (θ_x) is fast and thus the amplitude becomes larger. This phenomenon where the pitch angle response direction changes with the road input frequency is attributed to the amplitude of rear vertical motion being larger than that of the front.

As mentioned above, the unsteady aerodynamic forces caused by vehicle motions affect the suspension forces, and the coupled effects occur by the interplay of unsteady aerodynamic forces and suspension responses. In addition, by understanding the phenomenon that the pitch angle response changes with the road input frequencies, we confirm that the coupling analyses of unsteady aerodynamic forces with suspension characteristics and road input are necessary.

Fig. 7 Simulation results of the vehicle motions with the unsteady aerodynamic forces on a 20 mm step response at 60 km/h in the cases of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

Fig. 8 Simulation results of the vehicle motions with the unsteady aerodynamic forces on a 10 mm sine road response of 1.4 Hz at 60 km/h in the cases of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

Fig. 9 Simulation results of the vehicle motions with the unsteady aerodynamic forces on a 10 mm sine road response of 1.8 Hz at 60 km/h in the cases of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

4. Verification of the unsteady aerodynamic effects using actual running tests

4.1 Measuring the vehicle behavior affected by unsteady aerodynamic forces in actual running tests

For the verification of the coupling analysis results of unsteady aerodynamic forces with vehicle motions, the vehicle response behaviors in the road input are measured in the actual running tests. Their tests are conducted in the proving ground with road irregularities in the cases without and with a rectangular protrusion of $40 \text{ mm} \times 40 \text{ mm}$ on the front end of the roof. The relative vertical displacements to the road surface were measured by laser displacement meters equipped at the front and rear end of the vehicle body, as shown in Fig. 10(a). The values were measured at a sampling rate of 100 Hz with the data logger system, and they were converted to the relative vertical displacements at the front and rear axles (z_{xf}, z_{xr}), according to which the relative pitch angle (θ_x) and relative heave displacement (z_x) were calculated. The profile of the road with irregularities was measured beforehand, as shown in Fig. 10(c). When the vehicle runs at 60 km/h, the road input spectrum becomes almost flat between 0.6 and 2.0 Hz, as shown in Fig. 10 (d).

At the same time, in the measurement, the subjective evaluations of the vehicle behavior were performed by the drivers, and their evaluation results are shown in Fig. 10(b). In the case without a protrusion, better body response ability to the road input, solid steering, and straight-line stability were obtained. By contrast, in the case with a protrusion, better rugged vibrations, i.e., the softness of road shock, was observed, along with better trembling vibrations and seesaw shaking. The vehicle behaviors were affected by the aerodynamic effects caused by the vehicle shape, and the drivers felt these differences in the vehicle behaviors.

Fig. 10 Tested vehicle and measurement sensors, subjective evaluation results, configuration, and spectrum of the road irregularity in the proving ground at the actual running test.

4.2 Comparison of the vehicle behaviors between the measurements in the actual running test and the analysis simulation

The measured timeline data and amplitude spectrums in the actual running tests for the relative pitch angle (θ_x) and relative heave displacement (z_x), beside the road input (z_{rdf}) of the cases without and with a protrusion are shown in Fig. 11. The relative timeline position of the road inputs is decided by the advance time of the first positive peak in road input from the first positive peak in heave displacement as per the simulation results, because they are not simultaneous

Fig. 11 Measured vehicle motions in the timeline and spectrums at the real vehicle running test in the proving ground at 60 km/h over three measurement trials in each case of a real vehicle without and with a protrusion.

Fig. 12 Simulated vehicle motions in the timeline and spectrums by the simulation considering the unsteady aerodynamic forces at 60 km/h in the case of a real vehicle without a protrusion, with a protrusion, and no aerodynamic effect.

measurements. The figure shows measurement results obtained over three trials in each case without and with a protrusion, and the timeline data of the case with a protrusion are displayed with 0.1 s delay for easy comparison. These results appear almost piled up in the timeline, and the gains around the spectrum peak also appear to be piled up. Therefore, these results are thought to be measured with reproducibility. The comparison of the two cases showed that the relative heave displacement (z_x) and relative pitch angle (θ_x) without a protrusion are larger.

The analysis simulations corresponding to these measurement results are performed, and the simulation results of the three cases, i.e., without a protrusion, with a protrusion, and no consideration for aerodynamics are shown in Fig. 12. The timeline data of the case with a protrusion is displayed with 0.1 s delay, and the data of the case with no consideration for aerodynamics is displayed with 0.1 s faster. Although the simulated responses in timeline do not necessarily agree with the measured results, the tendency of the difference in measured results in actual running tests is represented, where the relative heave displacement (z_x) and relative pitch angle (θ_x) of the case without a protrusion are larger than those in the case with a protrusion. Meanwhile, for the gain of spectrums, the agreement of the tendency between the measured results and simulated results is also represented. When the high-frequency input until 2 Hz in this testing road, the relative heave displacement (z_x) and pitch angle (θ_x) of the case without a protrusion become larger as the same to the previous analyses by response functions. Therefore, the validity of the analyses in this study is confirmed by the measured results in actual running tests.

In addition, when the subjective evaluation results by the drivers were compared to the vehicle motions measured or simulated, a better body response ability in the case without a protrusion is equivalent to larger relative heave and pitch motions, and a softness of road shock is equivalent to smaller relative heave and pitch motions. Although the vehicle stability cannot be judged only by the difference in these motions, the aerodynamic influence on the vehicle motion due to the vehicle shapes that drivers feel can be represented quantitatively for a part.

5. Influence rates of the unsteady aerodynamic forces for the vehicle specifications

In the coupled equation of the unsteady aerodynamic forces and vehicle motions formulated in this study, the influence rates of the unsteady aerodynamic forces on the vehicle specifications can be evaluated, by comparing each unsteady aerodynamic coefficient (aerodynamic inertia, damping, and spring) to the equivalent vehicle specifications (vehicle mass, suspension damping, and spring). The aerodynamic coefficients of the rear due to pitch motion ($m_{r\theta}$, $c_{r\theta}$, $k_{r\theta}$) are replaced by the modified coefficients ($m_{r\theta}^* = -m_{r\theta}$, $c_{r\theta}^* = -c_{r\theta}$, $k_{r\theta}^* = -k_{r\theta}$) to obtain the defined sign convention for the directions according to the vehicle specification factors. (reversing the axis definition, where positive aerodynamic rear lift results from the positive pitch motion before here.) The response functions of the pitch and heave motions in road input are expressed in Eq. (10), where $\hat{M}(s)^{-1}$ is converted to Eq. (11) by arranging each term in the matrix.

$$\begin{aligned} \hat{M}(s)^{-1} &:= \frac{1}{\hat{M}_{11}(s) \cdot \hat{M}_{22}(s) - \hat{M}_{12}(s) \cdot \hat{M}_{21}(s)} \cdot \begin{bmatrix} \hat{M}_{22}(s) & -\hat{M}_{12}(s) \\ -\hat{M}_{21}(s) & \hat{M}_{11}(s) \end{bmatrix} \\ &= \frac{1}{\hat{M}_{11}(s) \cdot \hat{M}_{22}(s) - \hat{M}_{12}(s) \cdot \hat{M}_{21}(s)} \cdot \\ &\quad \left(\begin{bmatrix} (m_f + m_r)s^2 + 2(c_f + c_r)s + 2(k_f + k_r) & -(m_f - m_r)s^2 - 2(c_f - c_r)s - 2(k_f - k_r) \\ -\frac{l_{wb}}{2}(m_f - m_r)s^2 - l_{wb}(c_f - c_r)s - l_{wb}(k_f - k_r) & \frac{l_{wb}}{2}(m_f + m_r)s^2 + l_{wb}(c_f + c_r)s + l_{wb}(k_f + k_r) \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} m_{fz}s^2 + \left(c_{fz} + \frac{m_{rz}}{\tau_{rz}}\right)s + \left(k_{fz} + \frac{c_{rz}}{\tau_{rz}} - \frac{m_{rz}}{\tau_{rz}^2}\right) + E_{rz}(s) & -m_{fz}s^2 - \left(c_{fz} - \frac{m_{rz}}{\tau_{rz}}\right)s - \left(k_{fz} - \frac{c_{rz}}{\tau_{rz}} + \frac{m_{rz}}{\tau_{rz}^2}\right) + E_{rz}(s) \\ -(m_{f\theta} - m_{r\theta}^*)s^2 - (c_{f\theta} - c_{r\theta}^*)s - (k_{f\theta} - k_{r\theta}^*) & (m_{f\theta} + m_{r\theta}^*)s^2 + (c_{f\theta} + c_{r\theta}^*)s + (k_{f\theta} + k_{r\theta}^*) \end{bmatrix} \right) \\ E_{rz}(s) &= \frac{m_{rz}/\tau_{rz}^2 - (c_{rz}/\tau_{rz}) + k_{rz}}{\tau_{rz}s + 1} \\ m_{r\theta}^* &= -m_{r\theta}, \quad c_{r\theta}^* = -c_{r\theta}, \quad k_{r\theta}^* = -k_{r\theta} \end{aligned} \quad (11)$$

Fig. 13 Rates of the unsteady aerodynamic coefficients to the equivalent vehicle specification factors (from vehicle mass, damping, and spring of suspension) at 60 km/h in the case of a real vehicle without and with a protrusion.

Each term of the inverse matrix $\hat{M}(s)^{-1}$ is divided into the vehicle specification factors in the first term and the aerodynamic coefficients in the second term, which are arranged in terms of the acceleration, velocity, and displacement for each degree of Laplace variable (s). Thereafter, the factors and coefficients are contrasted with each other. The first-order lag in the aerodynamic coefficients is reduced to common denominators, and the remaining term of $E_{rz}(s)$ is excluded from the comparison because the force that occurs by its remaining term is small.

The rates of unsteady aerodynamic coefficients to the factors with vehicle specifications are calculated at 60 km/h in Fig. 13, by comparing each term between the vehicle specifications and the aerodynamic coefficients in Eq. (11). The contrasted vehicle specification factors and aerodynamic coefficients are shown as denominators and numerators, respectively, for each comparison in the quadrature axis label of Fig. 13. These comparisons are made on the inertia, damping and spring, with those for the front and rear aerodynamic effects due to heave and pitch motion separately.

As shown in Fig. 13(a), in the case of heave motion, the aerodynamic effects due to the damping terms have a larger influence to the vehicle specification factors than others do. The front aerodynamic damping coefficient due to heave motion (c_{fz}) suppresses the damping force, while the rear aerodynamic inertia (m_{rz}) and time constant (τ_{rz}) promote the damping force. However, there are few differences between the cases with and without a protrusion.

As shown in Fig. 13(b), in the case of pitch motion, the aerodynamic effects due to the rear inertia term significantly influence the vehicle specification factors. In particular, the rear aerodynamic inertia due to pitch motion ($m_{r\theta}^*$) in the case without a protrusion reduces its effective rear mass and is 39% of the rear mass inertia ($l_{wb}/2 \cdot m_r$) of the vehicle. Thus, this effect almost produces the differences in vehicle behavior between the cases without and with a protrusion, although its impact depends on the road input frequency.

Therefore, by comparing the unsteady aerodynamic coefficients with the equivalent vehicle specification factors, we can clarify which unsteady aerodynamic coefficients have effective influences.

6. Conclusions

To clarify the unsteady aerodynamic influence on the vehicle behaviors in road input, coupling analysis was conducted by modeling the response functions and the timeline simulations. The vehicle shapes attaching without and with a protrusion were compared for a detail difference of body shape. Moreover, measuring vehicle behaviors through actual running tests were done for the verification of these analyses. The obtained results are as follows:

1. According to the derived response functions, pitch and heave motion in road input are affected significantly by the change of unsteady aerodynamic forces, which are caused by vehicle motion in a detail difference of body shape. In addition, their affected motions are changed by the difference of frequency in road input. By attaching a protrusion on the roof, the pitch motion increases in low-frequency road input (<1.6 Hz) and decreases in high-frequency (>1.6 Hz). Meanwhile, the heave motion decreases over 1.2 Hz.
2. According to the timeline simulation results, the coupled motion occurs with respect to the unsteady aerodynamic forces and vehicle suspension responses. This was confirmed by the generation of the suspension forces because of the difference between the unsteady aerodynamic forces. Additionally, the change of pitch motion with respect to the difference in frequency in the road input is attributed to the change in the amplitude of suspension displacements between the front and rear axles.
3. By measuring the vehicle behavior in actual running tests at high frequencies road, the decreased pitch and heave motion in the case of a protrusion attached to the vehicle were verified with a good agreement. Moreover, their motions were consistent with what the drivers felt in the actual running test.
4. The comparison of the unsteady aerodynamic coefficients with the equivalent vehicle specification factors of the inertia, damping, and spring showed that the rear aerodynamic inertia caused by the pitch motion contributes to 39% of the rear vehicle mass inertia effect in pitch motion. This aerodynamic effect has the impact equivalent to changing suspension specifications in vehicle design.

The findings of this study show the significance of considering the unsteady aerodynamic force caused by vehicle motion for improving the vehicle stability with respect to the vehicle body shape. Moreover, the coupling analyses with unsteady aerodynamic forces, suspension responses, and road input are also of great importance.

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