A study of dynamic evaluation of structural buckling

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Abstract
Technology of deployable space structures is necessary for spacecraft to challenge advanced missions. It is important in designing the deployable space structures that they are easily deployable and reliably repeatable. Traditional approach for improving the repeatability was conducted by investigating errors and its effect to the deployment. However, the traditional approach has a problem that results change depending on estimation of the errors. With that background, this study proposes numerical methods to enable selection of robust deployable structures against the errors. The repeatability is decreased due to occurrence of the buckling caused by the errors. Therefore, a structure not occurring the buckling should be selected for designing of a reliably repeatable structure. The buckling is detected by non-positive eigenvalues of a stiffness matrix of the structure in static analysis. However, detection of the buckling in dynamic analysis is difficult because the eigenvalue is also non-positive when the structure has rigid-body motion. This study solved the problem by proposing a method to discriminate the buckling from the rigid-body motion. Furthermore, a method to evaluate instability of the structure quantitatively is desired when only structures occurring the buckling are available for the spacecraft. When the buckling occurs, small disturbance sets off grave displacement. Therefore, this study proposed the method to evaluate the instability quantitatively by calculating disturbance force and buckling displacement as index values of the instability based on the equation of motion. Finally, it was confirmed that the proposed methods are appropriate by the dynamic analyses of truss arch.

Key words: Deployable structures, Repeatability, Buckling, Dynamics, Stiffness matrix, Evaluation method

1. Background
Technology of deployable space structures is necessary for spacecraft to challenge advanced missions. It is important in designing the deployable space structures that they are easily deployable and reliably repeatable. Many studies about the deployment performance are conducted, and structures having superior deployment performance have been proposed (Shimazaki et al., 2013). On the other hand, conventional approach for improving the repeatability was conducted by investigating errors and its effect to the deployment (Saito and Tanaka, 2011). However, the traditional approach has a problem that results change depending on estimation of the errors. With that background, this study proposes numerical methods to enable selection of robust deployable structures against the errors. Using these methods, the error modeling is not necessary, and the selection of the reliably repeatable structures is enabled even if the deployment includes various errors.

The repeatability is decreased due to occurrence of the buckling caused by the errors. Infinitesimal bias of force or displacement set off grave displacement by the buckling. That is to say, structures deploying without the buckling are robust structures against the errors. Therefore, a structure not occurring the buckling should be selected for designing a reliably repeatable structure by detecting the buckling in the simulation of the deployment. In a static analysis, buckling can be detected by appearance of non-positive eigenvalues of a stiffness matrix of the structure (Fujii et al., 2005). On the other hand, a dynamic analysis is required for the deployable space structures because the deployable space structures have large deformation and large momentum in many cases. In the dynamic analysis, there is a problem to
detect the buckling that the non-positive eigenvalues also appear when the structure has rigid-body motion as shown in Fig.1. Using the stiffness matrix of a truss element $K^{ab}$ shown in Eq.(1) as an example, external force $f$ exerted in the translational rigid-body motion is zero as shown by Eq.(2). In this regard, $E$ is Young’s modulus, $A$ is cross-sectional area of the element, $L$ is length of the element before deformation, $l$ is the length of the element after deformation, $\varepsilon$ is the axial strain, $e$ is unit vector from the node 2 to the node 1, $w_1$ and $w_2$ are displacements of the node 1 and the node 2 respectively, and $I_{33}$ is 3 by 3 identity matrix. Thus, the eigenvalue of the stiffness matrix is zero. Calculating the external force exerted in the rotational rigid-body motion, the external force becomes zero or negative value to the direction of displacement according to the axial strain $\varepsilon$, as shown in Eq.(3).

Thus, the eigenvalue of the stiffness matrix is zero or negative value.

$$K^{ab} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \quad \text{with } k = EA\frac{1}{L}e \otimes e + \varepsilon (I_{33} - e \otimes e)$$ (1)

$$f = K^{ab} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \quad (\therefore w_1 = w_2)$$ (2)

$$f = K^{ab} \begin{bmatrix} v \\ -v \end{bmatrix} = 2 \begin{bmatrix} EA\frac{\varepsilon}{L}v \\ -EA\frac{\varepsilon}{L}v \end{bmatrix} \quad \text{(with } e \cdot v = 0)$$ (3)

Fig.1  Rigid-body motion of a truss element

In view of this, the discrimination between the buckling and the rigid-body motion is necessary for the detection of the buckling in the dynamic deployment analysis. However, the method to solve this problem has not been proposed. In order for us to manage to achieve prediction of the buckling in the dynamic analysis, a method is proposed that the dynamic buckling is given as linear combination of static buckling modes, and the buckling is detected by the translation of coefficients of the linear combination (Nemoto and Kasuya, 1997). In this regard, the deformation from the initial shape must be small. Therefore, this method is unfit for the deployable space structures, of which geometric shape change largely momentarily. With that background, this study proposes a method to detect the buckling in the dynamic analysis by discriminating the buckling from the rigid-body motion. Furthermore, a method to evaluate instability of the structure quantitatively is desired when only structures occurring the buckling are available for the spacecraft. When the buckling occurs, small disturbance sets off grave displacement, but the degree depends on the state of motion. Therefore, we assume that the instability can be measured as the quantity of the disturbance and the quantity of the displacement. And then, this study proposes a method to evaluate quantitatively the instability caused by the buckling by calculating the disturbance force and the buckling displacement as index values of the instability based on the equation of motion. The proposed method enables quantitative evaluation of the repeatability. It is available for the design of the spacecraft.

2. Purpose and procedure

The purpose of this study is proposing numerical methods to enable designing of the reliably repeatable structures by detecting and measuring the buckling.

Firstly, we propose a method to detect the buckling in the dynamic analysis by discriminating the buckling from the rigid-body motion in Section 3. Secondly, we propose a method to evaluate quantitatively the instability caused by the buckling by calculating the disturbance force and the buckling displacement as index values of the instability based on the equation of motion in Section 4. Thirdly, we conduct a dynamic buckling analysis of a truss arch having the rigid-body motion in order to verify the proposed method to detect the buckling. Finally, we calculate the index values acquired by the proposed method for two models of truss arch, and compare the index values in order to verify the proposed method to evaluate the instability quantitatively.

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In this study, the truss element is adopted as a structural element of FEM because the truss element is available for the dynamic analysis of large structures in the aspect of computational cost. Additionally, the Newmark-beta method is adopted as a time integration method because the Newmark-beta method has relatively high numerical stability and is easy to code.

3. Method to detect buckling in a dynamic analysis

In this section, a method to detect the buckling in the dynamic analysis is proposed. The method consists of three procedures as below.

Procedure1: Finding rigid-body modes which the structure in motion has.
Procedure2: Checking whether each mode of the structure is a rigid-body mode or a deformational mode.
Procedure3: Checking the eigenvalue of the deformational mode. If the eigenvalue is non-positive, the mode is determined as the buckling mode.

Details of each procedure are explained below.

Procedure1:

The rigid-body mode cannot be determined by the stiffness matrix \( K_{ab} \) itself. Therefore, we propose the following method. When a structure is moving and deforming, we assume each element of the structure has no strain at the moment. Then, the stiffness matrix, which is given by Eq.(1), is represented as Eq.(4) because \( \varepsilon = 0 \) and \( l = L \).

\[
K_{ab} |_{\varepsilon = 0} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \quad \text{(with \( 1^T K = EA \ell e \otimes e \))}
\]

We define \( \dot{K} \) as a matrix merged \( \dot{K}_{ab} \) over the entire structure. The structure is in a state that all elements are free length and no internal force exists. Therefore, only the constant rigid-body motion can displace the structure without external force. Hence, the rigid-body mode is defined as an eigenvector of \( \dot{K} \) of which eigenvalue \( \lambda \) is zero. The rigid-body mode is defined by the following equation:

\[
\eta_i = \{ w | \lambda w = \dot{K} w = 0 \}
\]

Furthermore, we define rigid-body modal space \( \dot{Q} \) by the following equation:

\[
\dot{Q} = [\dot{q}_1, \ldots, \dot{q}_n]
\]

Procedure2:

We define \( K \) as a matrix merged \( K_{ab} \) over the entire structure. Modes of the structure in motion are the eigenvectors of \( K \). The number of the modes \( n \) is the number of generalized coordinates of the structure. In order to detect the rigid-body mode, we check if each eigenvector of \( K \) is included in the rigid-body modal space \( \dot{Q} \). If an eigenvector \( \eta_i \) is not included in \( \dot{Q} \), the eigenvector can be so determined that it is not a rigid-body mode but a deformational mode. We calculate the orthogonal component \( \eta_i^* \) of each eigenvector \( \eta_i \) to the rigid-body modal space \( \dot{Q} \) by Eq.(7) in order to check if \( \eta_i \) is included in \( \dot{Q} \). Figure2 shows the conceptual diagram of calculation of the orthogonal component.

\[
\eta_i^* = \eta_i - \sum_{k=1}^{n} (\eta_i \cdot \dot{q}_k) \dot{q}_k
\]

Furthermore, we define degree of orthogonality \( \kappa_i \) as below:

\[
\frac{|\eta_i^*|}{|\eta_i|} = \kappa_i
\]

If \( \kappa_i = 0 \), \( \eta_i \) is the rigid-body mode.

Procedure3:

An eigenvector of \( K \) with non-positive eigenvalue is either the rigid-body mode or the buckling mode. Therefore, when the eigenvector is deformational mode, we check its eigenvalue \( \lambda \). If the eigenvalue is less than or equal to zero, the eigenvector is the buckling mode.

Figure3 shows the flowchart of judging a mode. We set \( 10^{-5} \) as a threshold value of \( \kappa_i \) because there are computational errors in a numerical analysis. Each eigenvector is categorized into six types, as shown in Fig.3.
4. Method of quantification of buckling force and displacement

In this section, a method to evaluate instability of the structure quantitatively is proposed. When the buckling occurs, small disturbance sets off grave displacement. However, the degree depends on the state of motion. If the direction of the motion is similar to the direction of the buckling mode, the structure may displace gravely. On the other hand, if the direction of the motion differs from the direction of the buckling mode, the inertial force may hamper the grave displacement by the small disturbance. Therefore, we define the degree of the instability as the quantity of the disturbance and the quantity of the buckling displacement. Assuming that the structure is subjected to external force and the external force causes buckling displacement, we calculate the norm of the disturbance force (DF value) and the norm of the buckling displacement (BD value) as index values of the instability based on the equation of motion in order to evaluate quantitatively the instability caused by the buckling.

The DF value can be obtained as a positive real number. Large DF value indicates that the structure is relatively stable because the buckling displacement hardly occurs without the large disturbance force. In the same way, small DF value indicates that the structure is relatively instable because the buckling displacement easily occurs by the small disturbance force. The BD value can be obtained as a vector consists of positive real numbers. Small component values of BD value indicate that the structure is relatively stable because the buckling displacement by the disturbance is small. In the same way, large component values of BD value indicate that the structure is relatively instable because the buckling displacement by the disturbance is large. The DF value and the BD value are obtained by the following theory.

When a structure is moving and buckling, the infinitesimal displacement \( \Delta \mathbf{x} \), as shown in Fig.4, is represented as

\[
\Delta \mathbf{x} = \alpha \mathbf{e}^* + p \mathbf{q}_{\text{c}} \tag{9}
\]

where \( \mathbf{e}^* \) is a unit vector of the buckling mode, \( \alpha \) is magnitude of displacement in the direction of the buckling mode, \( \mathbf{q}_{\text{c}} \) is a unit vector of each rigid-body mode, \( p \) is magnitude of displacement in the direction of each rigid-body mode.

\[
\mathbf{x} = \mathbf{x}_0 + \frac{1}{2} (\mathbf{x}_0 + \dot{\mathbf{x}}_0) M - \frac{\gamma}{2} (\ddot{\mathbf{x}}_0 - \dot{\mathbf{x}}_0) M^2 \tag{10}
\]

\[
\dot{\mathbf{x}} = \dot{\mathbf{x}}_0 + (1 - \gamma) \dot{\mathbf{x}}_0 + \gamma \ddot{\mathbf{x}}_0 M \tag{11}
\]

where subscript \( 0 \) means the quantity before displacement, and superscript \( \cdot \) means the second derivative with respect to time. \( M \) is the infinitesimal time. \( \beta \) and \( \gamma \) are the constant parameters of the Newmark-beta method. Furthermore, \( \mathbf{x} \) is represented as
\[ x = x_0 + \Delta x \]  

In the same way, the internal force \( F \) and the external force \( F_e \) after the displacement are represented as below:

\[ F = F_0 + \alpha \Delta F \]  
\[ F_e = F_{e0} + \alpha \Delta F \]

where symbol \( \alpha \) means the increment of each quantity. Using Eq.(11) and Eq.(12), Eq.(10) can be deformed as below:

\[ \bar{x} = \frac{1}{\beta \Delta t} \Delta x - \frac{1}{\beta \Delta t} \bar{x}_0 - \frac{1}{2 \beta} \left\{ \frac{1}{\beta} \right\} \bar{x}_0 \]

The equation of motion is written as

\[ M \ddot{x} + F = F_{e0} \]  

Substituting Eq.(15) into Eq.(16), the following relation is obtained:

\[ \frac{1}{\beta \Delta t} M \Delta x - \frac{1}{\beta \Delta t} M \bar{x}_0 - \frac{1}{2 \beta} M \bar{x}_0 = \Delta F_{e0} \]  

Moreover, the following formulas hold:

\[ M \ddot{x} + K \bar{x} = F_{e0} \]  

Substituting Eq.(18) and Eq.(19) into Eq.(17), the following equation is obtained:

\[ \frac{1}{\beta \Delta t} M \Delta x + K \Delta \bar{x} = \Delta F_{e0} \]  

Considering that \( \Delta t \) is enough small, we introduce the following approximate equation:

\[ \Delta F_{e0} = p q + \Delta F_{e0} \]

where

\[ d + p q = \Delta F_{e0} \]  

The increment vector of the external force \( \Delta F_{e0} \) is the disturbance force to set off the displacement \( \Delta x \). Thus, we determine the DF value as the norm of \( \Delta F_{e0} \). \( \Delta F_{e0} \) is a function of \( p, \alpha \) and \( \Delta t \). Therefore, we calculate the value of \( p, \alpha \) and \( \Delta t \) minimizing \( \| F_{e0} \| \) because the minimum \( \| F_{e0} \| \) is the norm of the smallest disturbance force to set off the displacement \( \Delta x \). Using Eq.(22), the square of \( \| F_{e0} \| \) is written as below:

\[ \| F_{e0} \|^2 = (p q + d) \cdot (p q + d) \]

\[ = pq \cdot q + pq \cdot d + pq \cdot d + d \cdot d \]

The value of \( p \) minimizing \( \| F_{e0} \| \) is obtained by the following equation:

\[ \frac{d}{dp} \| F_{e0} \|^2 = 2pq \cdot q + 2q \cdot d = 0 \]

\[ \therefore p = -[q^T q]^{-1} q^T d \]

Setting \( Q \) as a matrix arranging \( q \), in a row sideways, and setting \( P \) as a vector arranging \( p \), in a row vertically, Eq.(25) is written as below:

\[ p = -[Q^T Q]^{-1} Q^T d \]

On the other hand, Eq.(22) can be represented as below using \( Q \) and \( p \):

\[ \alpha a^* - b^* - c^* + Q p = \Delta F_{e0} \]

Therefore, Eq.(27) is rewritten as below:

\[ \Delta F_{e0} = (I - Q Q^T)(\alpha a^* - b^* - c^*) \]

\[ = (I - Q Q^T)(\alpha a^* - b^* - c^*) \]

Next, we calculate the value of \( \alpha \) minimizing \( \| F_{e0} \| \). Let us introduce parameters \( a, b \) and \( c \) defined as Eq.(29).
\begin{align}
(I - \tilde{Q}^{T} \tilde{Q}^{-1}) \tilde{Q}^{-1} \alpha' &= a,
(I - \tilde{Q}^{T} \tilde{Q}^{-1}) \beta' = b,
(I - \tilde{Q}^{T} \tilde{Q}^{-1}) \gamma' &= c
\end{align}

Using Eq.(29), $\|F_{\alpha}\|$ is represented as the following equation:

\begin{align}
\|F_{\alpha}\| = \sqrt{\alpha^2 |\alpha|^2 + |\beta|^2 + |\gamma|^2 - 2\alpha(a \cdot b + a \cdot c) + 2b \cdot c}
\end{align}

\begin{align}
= \sqrt{\alpha^2 \left( \frac{a \cdot b + a \cdot c}{|\alpha|^2} \right)^2 + |\beta|^2 + |\gamma|^2 - 2b \cdot c - \frac{(a \cdot b + a \cdot c)^2}{|\alpha|^2}}
\end{align}

Thus, the value of $\alpha$ minimizing $\|F_{\alpha}\|$ is obtained as below:

\begin{align}
\alpha = \frac{a \cdot b + a \cdot c}{|\alpha|^2}
\end{align}

Substituting Eq.(31) into Eq.(30), the following equation holds:

\begin{align}
\|F_{\alpha}\| = \sqrt{\frac{1}{\beta |\alpha|^2} |\alpha|^2 + \frac{2}{\beta |\alpha|^2} b \cdot c - \frac{(1/\beta |\alpha|^2) a \cdot b + a \cdot c}{|\alpha|^2}}
\end{align}

Then, we introduce parameters $a_1$, $a_2$, $n$ and $b_2$ defined as Eq.(33) in order to separate $\Delta t$.

\begin{align}
a_1 = (I - \tilde{Q}^{T} \tilde{Q}^{-1}) \tilde{Q}^{-1} \alpha' = \alpha n, \quad b_2 = (I - \tilde{Q}^{T} \tilde{Q}^{-1}) \tilde{Q}^{-1} \gamma' = n
\end{align}

Inserting Eq.(33) into Eq.(32), $\|F_{\alpha}\|$ is represented as the following equation:

\begin{align}
\|F_{\alpha}\| = \sqrt{\left( \frac{1}{\beta |\alpha|^2} |\alpha|^2 + \frac{2}{\beta |\alpha|^2} b \cdot c - \frac{(1/\beta |\alpha|^2) a \cdot b + a \cdot c}{|\alpha|^2} \right)^2}
\end{align}

Furthermore, we introduce a parameter $\theta$ as the angle between $n$ and $b_2$. In addition, we define parameters $T$, $A$ and $B$ as Eq.(35) in order to calculate the value of $\Delta t$ minimizing $\|F_{\alpha}\|$.

\begin{align}
T = \frac{1}{\beta |\alpha|^2}, \quad A = \frac{b \cdot c - (n \cdot b_2)(n \cdot c)}{|\alpha|^2 \sin^2 \theta}, \quad B = \frac{|\alpha|^2 - (n \cdot c)^2}{|\alpha|^2 \sin^2 \theta}
\end{align}

Substituting Eq.(35) into Eq.(34), $\|F_{\alpha}\|$ is rewritten as below:

\begin{align}
\|F_{\alpha}\| = \frac{|\alpha|^2 \sin \theta \sqrt{(T + A)^2 + B - A^2}}
\end{align}

Hence, $\|F_{\alpha}\|$ is minimized when $T = -A$, that is $\Delta t$ is given as Eq.(37).

\begin{align}
\Delta t = \frac{|\alpha|^2 \sin \theta \sqrt{(T + A)^2 + B - A^2}}
\end{align}

When Eq.(37) holds, $\|F_{\alpha}\|$ is obtained as below:

\begin{align}
\|F_{\alpha}\| = \left[ \frac{|\alpha|^2 - (n \cdot c)^2}{|\alpha|^2 \sin \theta} \right]^{1/2}
\end{align}

In this regard, $\Delta t$ is scrutinized in the end. We set $\Delta t_{\text{max}}$ as a maximum value of $\Delta t$ in order to confine $\Delta t$ to a pragmatic value as the disturbance. And then, we scrutinize $\Delta t$ so that $\Delta t$ holds $0 < \Delta t < \Delta t_{\text{max}}$. Moreover, $\Delta t$ is so scrutinized that $\|F_{\alpha}\|$, calculated by Eq.(36), is real number. Finally, Eq.(36) is calculated using the scrutinized $\Delta t$, and the DF value is defined as the obtained $\|F_{\alpha}\|$. We define BD value as a vector consists of norm of buckling displacement of each node. The BD value is represented as Eq.(39).

\begin{align}
(BD \text{ value}) = \alpha \begin{bmatrix} e_{\text{node1}}^* & e_{\text{node2}}^* & \cdots & e_{\text{nodeN}}^* \end{bmatrix}^T
\end{align}

where $\alpha$ is obtained by Eq.(31), and $N$ means the number of the nodes of the structure. For example, the BD value of a structure consisting of three nodes is defined as Eq.(41) when $e^*$ of the structure is given as Eq.(40).

\begin{align}
e^* = \begin{bmatrix} x_{\text{node1}}^* & y_{\text{node1}}^* & z_{\text{node1}}^* & x_{\text{node2}}^* & y_{\text{node2}}^* & z_{\text{node2}}^* & x_{\text{node3}}^* & y_{\text{node3}}^* & z_{\text{node3}}^* \end{bmatrix}^T
\end{align}

\begin{align}
(BD \text{ value}) = \alpha \sqrt{\sum_{i=1}^{N} x_{\text{nodei}}^2 + y_{\text{nodei}}^2 + z_{\text{nodei}}^2}
\end{align}

5. Verification of the method of discrimination between buckling and rigid-body motion

We confirmed if the buckling mode was detected using the method proposed in Section3 by a dynamic buckling analysis of a truss arch. The truss arch was loaded as shown in Fig.5 and Table1. The first buckling was detected at Step1926. Table2 shows the result of the eigenvalue, the eigenvector, the degree of orthogonality and the type.
degree of orthogonality and the type are obtained by the method proposed in Section 3. The eigenvectors of mode No.1 in both time steps show rotational rigid-body modes about the y axis, and their degree of orthogonality are 0. It means that the proposed method can judge the rigid-body mode correctly. In the same way, the eigenvectors of mode No.8 and No.9 in both time steps show deformational modes, and their degree of orthogonality are 1. It means that the proposed method can judge the deformational mode correctly. Moreover, the types of mode No.1 in both time steps are 11 (compressive rotational rigid-body mode) because their degree of orthogonality are 0 and its eigenvalues are negative. On the other hand, the type of mode No.8 changes from 23 (Step1925) to 13 (Step1926). The type of mode No.8 in Step1926 is 13 because its degree of orthogonality is 1 and its eigenvalue is negative. It means the buckling mode can be detected by the proposed method. In regard to the modes No.2-7, the modes are caused by the constraint. They can be discriminated mechanically because their eigenvalues are just 1. For the reasons mentioned above, we confirmed that the buckling mode was detected in a dynamic analysis using the method introduced in Section 3.

Table 1 Description of the truss arch model 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>time step size</td>
<td>(dt)</td>
<td>(5\times10^{-5})</td>
<td>[s]</td>
</tr>
<tr>
<td>stiffness of an element</td>
<td>(E_A)</td>
<td>(2.8\times10^{7})</td>
<td>[N]</td>
</tr>
<tr>
<td>density of an element</td>
<td>(\rho)</td>
<td>2.7</td>
<td>[g/cm³]</td>
</tr>
<tr>
<td>increment of external force</td>
<td>(\Delta f)</td>
<td>(2.5\times10^{-4})</td>
<td>[-]</td>
</tr>
<tr>
<td>condition of constraint</td>
<td>(x,y,z) of node①</td>
<td></td>
<td></td>
</tr>
<tr>
<td>condition of loading</td>
<td>Increasing (f) step-by-step by (\Delta f). node③ is subjected to (-E_Af) in the direction of (x) axis.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Result of the eigenvalue, the eigenvector, the degree of orthogonality and the type. The orthogonality and the type are obtained by the method proposed in Section 3. Comparing the types and the eigenvectors, we can confirm that the types are categorized correctly, and that the buckling mode is detected correctly by the proposed method.

<table>
<thead>
<tr>
<th>time step</th>
<th>mode No.</th>
<th>Type</th>
<th>eigenvalue (\lambda)</th>
<th>eigenvector node①</th>
<th>eigenvector node②</th>
<th>eigenvector node③</th>
<th>degree of orthogonality (\kappa)</th>
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</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>(x) (y) (z)</td>
<td>(x) (y) (z)</td>
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<table>
<thead>
<tr>
<th>time step</th>
<th>mode No.</th>
<th>Type</th>
<th>eigenvalue (\lambda)</th>
<th>eigenvector node①</th>
<th>eigenvector node②</th>
<th>eigenvector node③</th>
<th>degree of orthogonality (\kappa)</th>
</tr>
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6. Verification of the method of quantification of buckling force and displacement

We confirmed if the index values proposed in Section 4 are appropriate by comparison of dynamic buckling analyses of truss arches. One model of the truss arches is the model 1 shown in Fig.5 and Table1, and another model is the model 2 shown in Fig.6 and Table3. The model 2 is given the velocity in the direction of y axis at Step1926, that is the first time step of occurring of the buckling. The two models are the same shape and in the different motion. The results are shown in Fig.7, Fig.8 and Fig.9. Figure7 shows that eigenvalues of two models are almost same. It indicates the two models are the same shape. All DF values of the model 1 are zero, but DF values of the model 2 are positive values, as shown in Fig.8. Furthermore, BD values of the model 1 are larger than BD values of the model 2 generally, as shown in Fig.9. It indicates that the model 2 needs larger external force to set off the buckling displacement, and that the buckling displacement of the model 2 is smaller. That is to say, the model of which direction of motion is differs from the direction of buckling is resistant to the grave displacement caused by the buckling. For the reasons mentioned above, we confirmed that DF value and BD value are appropriate as the index values of instability.
7. Conclusions

This study proposed the following two numerical methods in order to design a reliably repeatable space structure by observing the buckling:

1. The method to detect the buckling in a dynamic analysis by discriminating the buckling from the rigid-body motion.
2. The method to evaluate quantitatively the instability caused by the buckling by calculating the disturbance force and the buckling displacement based on the equation of motion.

It must be noted that we have already confirmed the availability of the proposed methods for the analyses of presumed actual space structures, like spinning membrane deployment and so on, in the same way as the truss arches stated above.

References


