Effects of laminate misalignment on macroscopic strength and microscopic damage development of plain-woven laminates

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Abstract
In this study, a multiscale damage analysis of plain-woven laminates with laminate misalignment is conducted based on the homogenization theory for misaligned internal structures developed by the authors. For this, the homogenization theory is reconstructed for plain-woven laminates with arbitrary laminate misalignment in two directions using a novel boundary condition for unit cell analysis. The Hoffman’s failure criterion is then introduced into the theory to determine the failure of fiber bundles and a matrix material, and the damage modes. The damages are expressed as reduction of stiffness according to the damage modes. The present method can efficiently analyze the macroscopic strength and the microscopic damage development of plain-woven laminates with arbitrary laminate misalignment using only one unit cell. This method is then applied to the damage analysis of plain-woven E-glass/vinylester laminates with 13 cases of laminate misalignment including the non-misaligned case under on-axis uniaxial tension. It is shown that the laminate misalignment noticeably affects the macroscopic strength of the laminates, which is attributable to the difference in the microscopic damage development in the laminates depending on the misalignment.

Key words: Plain-woven laminate, Strength, Damage development, Laminate misalignment, Homogenization, Multiscale

1. Introduction

Plain-woven laminates are composite materials in which plain fabrics are stacked and impregnated with polymer materials. Recently, they have been used as primary structural members in aerospace, auto, energy-related industries because of their light weight, high strength and good formability, as a result they encounter severe conditions like high stress. Therefore, the strength analysis of plain-woven laminates is one of the most important issues in the composites field. Such analysis, however, is not straightforward because their highly complex microstructures composed of fiber bundles and polymer materials (Fig. 1) bring about a variety of damages such as fiber breakage, transverse cracking in fiber bundles, and matrix cracking (Bansaku et al., 1999; Zako et al., 2003).

Numerical multiscale analysis based on the homogenization theory (Suquet, 1987) or the finite element method is a very suitable approach to analyze such microscopic damages in plain-woven laminates because it can explicitly consider their microstructures (Takano et al., 1999; Carvelli and Poggi, 2001; Zako et al., 2003; Uetsuji et al., 2004; Fujita and Kurashiki, 2012). In such analysis, we should consider the effects of misalignment between plain fabrics in plain-woven laminates (Fig. 1) (e.g., Mesogitis et al., 2014), which is referred to as the laminate misalignment hereafter, on the strength and damage development of laminates. This is because the laminate misalignment can noticeably affect the strength and damage properties of plain-woven laminates as already shown by Takano et al. (1999) and Uetsuji et al. (2004).

In the above studies, the effects of laminate misalignment on the strength and damage development of plain-woven laminates were investigated by making their unit cells in the both cases without misalignment (Fig. 1(a)) and with misalignment (Fig. 1(b)), and by analyzing the unit cells based on the homogenization theory (Takano et al., 1999) or the finite element method (Uetsuji et al., 2004) under the periodic boundary condition. However, there were some
problems in the studies. First, it was required to define each unit cell for each laminate misalignment, and to generate the geometry and finite element mesh for every unit cell. This could arise not only the complication with preprocessing of analysis, but also the so-called mesh dependence on analysis results. Second, enormous computational costs were needed for the analysis because unit cells containing laminate misalignment basically became much larger than that without laminate misalignment. For example, the case of Fig. 1(b) requires a three times larger unit cell compared with the non-misaligned case shown in Fig. 1(a). Furthermore, the arbitrary misaligned case as illustrated in Fig. 1(c) needs an extremely large unit cell, which is of almost no practical use for numerical computation. For these reasons, the number of the cases of laminate misalignment examined so far is very limited (only 3 or 4 cases). In addition, recently, the requirement for investigating the influence of irregularity such as laminate misalignment on the results of numerical analyses has been increasing in academic and industrial fields (ASME, 2006; NAFEMS, 2007; JSCES, 2014). It is thus worth developing a novel method which can deal with more cases of laminate misalignment in an efficient way.

The authors (Matsuda et al., 2007) have performed a multiscale elastic-viscoplastic analysis of plain-woven glass fiber-reinforced plastic (GFRP) laminates using the homogenization theory for nonlinear time-dependent composites (Ohno et al., 2000, 2001). More recently, they have succeeded in developing a homogenization theory for elastic-viscoplastic materials with misaligned internal structures (Matsuda et al., 2011), and have applied the theory to the analysis of elastic-viscoplastic behavior of plain-woven GFRP laminates with laminate misalignment in one direction. The most remarkable merit of the theory is that it enables only one unit cell to deal with arbitrary laminate misalignment as depicted in Fig. 1(c), which avoids the above-mentioned cumbersome procedures to handle the laminate misalignment. Therefore, if the theory is applied to the damage analysis of plain-woven laminates with laminate misalignment, we could efficiently investigate the dependence of laminate misalignment on the damage behavior of plain-woven laminates.

In this study, a multiscale damage analysis of plain-woven laminates with laminate misalignment is conducted based on the homogenization theory for misaligned internal structures developed by the authors (Matsuda et al., 2011). For this, the homogenization theory is reconstructed for plain-woven laminates with arbitrary laminate misalignment in two directions using a novel boundary condition for unit cell analysis. The Hoffman's failure criterion (Hoffman, 1967) is then introduced into the theory to determine the failure of fiber bundles and a matrix material, and the damage modes (Zako et al., 2003; Fujita and Kurashiki, 2012). The damages are expressed as reduction of stiffness according to the damage modes. The present method is applied to the damage analysis of plain-woven E-glass/vinylester laminates with 13 cases of laminate misalignment including the non-misaligned case under on-axis uniaxial tension. Based on the results of analysis, the effects of misalignment on the macroscopic strength and the microscopic damage development of plain-woven GFRP laminates are examined.

2. Homogenization theory for plain-woven laminates with laminate misalignment

2.1 Perturbed velocity field

Let us consider a plain-woven laminate with laminate misalignment, its unit cell $Y$, and the Cartesian coordinates $y_i$ ($i = 1, 2, 3$) as illustrated in Fig. 2, where the amounts of misalignment between adjacent layers in the $y_1$- and $y_2$-directions are set at $a$ and $b$, respectively. Then, the boundary of $Y$, $\Gamma$, is divided into $\Gamma_{\alpha}$ ($\alpha = 1, 2, \ldots, 12$) as depicted in Fig. 2, where the upper and lower boundary facets are divided into 4 parts, respectively, in accordance with the amounts of misalignment. Now, the microscopic velocity field $\dot{u}_i(y, t)$ in $Y$ is expressed as (Suquet, 1987)

2

Fig. 1 Microstructures and unit cells of plain-woven laminates; (a) without misalignment, (b) with misalignment of 1/3 unit cell length, (c) with misalignment of arbitrary (regular) amount.
\[ \dot{u}_i(y, t) = \dot{F}_y(t)y_j + \dot{u}_i^a(y, t), \]

where (') indicates the differentiation with respect to \( t \), \( F_y(t) \) denotes the macroscopic deformation gradient, and \( \dot{u}_i^a \) stands for the perturbed velocity from the macroscopic one, \( \dot{F}_y(t)y_j \). It should be noted that, in the present misaligned case, the microstructure of the laminate possesses the periodicity in the \( \alpha \) - and \( \beta \) -directions as illustrated in Fig. 3, where a cross-section in the inclined direction corresponding to the amount of misalignment \( a \) is depicted.

Thus, \( \dot{u}_i^a \) satisfies the periodicity in these directions, resulting in the periodic distribution of \( \dot{u}_i^a \) on \( \Gamma_1 \) and \( \Gamma_7 \), and \( \Gamma_2 \) and \( \Gamma_8 \), respectively. For the same reason, \( \dot{u}_i^a \) distributes periodically on \( \Gamma_3 \) and \( \Gamma_5 \), and \( \Gamma_4 \) and \( \Gamma_6 \), respectively. In addition, the microstructure possesses the periodicity in the \( y_1 \) - and \( y_2 \)-directions, which allows \( \dot{u}_i^a \) to distribute periodically on \( \Gamma_9 \) and \( \Gamma_{11} \), and \( \Gamma_{10} \) and \( \Gamma_{12} \). Needless to say, the microscopic stress and its rate in \( Y \), \( \sigma_{ij} \) and \( \dot{\sigma}_{ij} \), also satisfy the same periodicity as \( \dot{u}_i^a \).

### 2.2 Homogenization theory

The constitutive equation of the constituents in \( Y \) is expressed in a rate form as
\[ \sigma_y = c_{ijl} \dot{e}_{ijl}, \]  
(2)

where \( c_{ijl} \) represents the elastic stiffness satisfying \( c_{ijkl} = c_{jikl} = c_{klij} = c_{ijlk} \), and \( \dot{e}_{ijl} \) indicates the microscopic strain rate. The equilibrium of \( \sigma_y \) is expressed in a rate form as

\[ \dot{\sigma}_{ij} = 0, \]  
(3)

where \((\quad)_{ij}\) denotes the differentiation with respect to \( y_j \). Let \( v_i(y,t) \) be an arbitrary variation of the perturbed velocity field \( \dot{u}^k \) defined in \( Y \) at \( t \). Then, the integration by parts and the divergence theorem allow Eq. (3) to be transformed to the following weak form (Matsuda et al., 2011):

\[ \int_\Gamma \dot{\sigma}_{ij} v_j d\Gamma - \sum_i \int_{\Gamma_i} \dot{\sigma}_{ij} v_j d\Gamma = 0, \]  
(4)

where \( n_j \) indicates the unit vector outward normal to \( \Gamma \). As already mentioned in Subsection 2.1, \( \dot{u}^k \) \((v_i)\) and \( \sigma_y \) distribute periodically on \( \Gamma_1 \) and \( \Gamma_2 \), \( \Gamma_3 \) and \( \Gamma_4 \), \( \Gamma_5 \) and \( \Gamma_6 \), \( \Gamma_7 \) and \( \Gamma_8 \), respectively, while \( n_j \) takes the opposite direction on these boundaries (Fig. 3). Consequently, the following relations are obtained:

\begin{align*}
&\int_{\Gamma_1} \dot{\sigma}_{ij} v_j d\Gamma + \int_{\Gamma_3} \dot{\sigma}_{ij} v_j d\Gamma = 0, \\
&\int_{\Gamma_2} \dot{\sigma}_{ij} v_j d\Gamma + \int_{\Gamma_4} \dot{\sigma}_{ij} v_j d\Gamma = 0, \\
&\int_{\Gamma_5} \dot{\sigma}_{ij} v_j d\Gamma + \int_{\Gamma_6} \dot{\sigma}_{ij} v_j d\Gamma = 0, \\
&\int_{\Gamma_7} \dot{\sigma}_{ij} v_j d\Gamma + \int_{\Gamma_8} \dot{\sigma}_{ij} v_j d\Gamma = 0.
\end{align*}

(5)

Substitution of Eq. (5) into Eq. (4) makes the boundary integral term become zero, and we obtain

\[ \int_Y \dot{\sigma}_{ij} v_j dY = 0. \]  
(6)

This resulting equation has the same form as that of the original homogenization theory (Suquet, 1987; Ohno et al., 2000, 2001). Therefore, we can derive the following evolution equation of microscopic stress and the relation between macroscopic stress rate \( \dot{\Sigma}_{ij} \) and macroscopic strain rate \( \dot{E}_{ijl} \):

\[ \dot{\sigma}_{ij} = c_{pq} \left( \delta_{pi} \dot{\Sigma}_{qj} + \chi_{pq}^{\mu} \right) \dot{E}_{ijl}, \]  
(7)

\[ \dot{\Sigma}_{ij} = \left( c_{pq} \left( \delta_{pi} \dot{\sigma}_{qj} + \chi_{pq}^{\mu} \right) \right) \dot{E}_{ijl}, \]  
(8)

where \( \delta_{ij} \) indicates the Kronecker's delta, \((\#)\) stands for the volume average in \( Y \), i.e. \((\#) = |Y|^{-1} \int_Y \# dY\) where \( |Y| \) signifies the volume of \( Y \), and \( \chi_{pq}^{\mu} \) is a function to be determined by solving the following boundary value problem for \( Y \):

\[ \int_Y c_{pq} \chi_{pq}^{\mu} v_j dY = -\int_Y c_{ijl} \dot{e}_{ijl} dY. \]  
(9)

Using Eqs. (7) and (8), we can obtain both the macroscopic response of plain-woven laminates and the microscopic stress evolution through the incremental computation (see Ohno et al. (2000, 2001) for more detail on the computational procedure). The microscopic stress is used to determine the failure of fiber bundles and a matrix material, which will be described in the subsequent section.

It is emphasized that the present theory enables one unit cell to deal with arbitrary misalignment shown in Fig. 1(c) by changing the area ratios of \( \Gamma_1 - \Gamma_2 \) and \( \Gamma_5 - \Gamma_6 \).

3. Damage development analysis

3.1 Failure criterion

In this study, the Hoffman's failure criterion (Hoffman, 1967) is introduced into the above-mentioned homogenization theory to determine the failure of fiber bundles and a matrix material (Takano et al., 1999; Zako et al., 2003; Fujita and Kurashiki, 2012). This criterion is a generalized Tsai-Hill criterion, and can consider the anisotropic damage behavior. In the present study, fiber bundles in plain-woven laminates are regarded as unidirectional fiber-reinforced composites, and are modeled as transversely isotropic elastic materials having strongly anisotropic damage properties. Hence, the Hoffman's criterion is suitable for the failure determination of fiber bundles. Also, this criterion is applied to the failure determination of a matrix material which is isotropic in elastic properties but anisotropic in strength.

The Hoffman's criterion is described using the current stress state as follows:

\[ F = C_1 (\sigma_{11} - \sigma_{22})^2 + C_2 (\sigma_{22} - \sigma_{33})^2 + C_3 (\sigma_{33} - \sigma_{11})^2 + C_4 \sigma_{12} + C_5 \sigma_{23} + C_6 \sigma_{31}^2 + C_7 \sigma_{21}^2 + C_8 \sigma_{13}^2, \]  
(10)

where the lower indices \( L, T \) and \( Z \) respectively denote the local coordinates for the fiber direction, the transverse
direction, and the perpendicular direction to the L- and T-directions, and $\sigma_{\text{T}}$ stands for the stress in the local coordinates. Moreover, $C_i$ - $C_9$ in Eq. (10) indicate the material parameters defined as

$$
C_1 = \frac{1}{2} \left( \frac{1}{F_{11}^L} + \frac{1}{F_{11}^T} - \frac{1}{F_{12}^L} - \frac{1}{F_{12}^T} \right), \quad C_2 = \frac{1}{2} \left( \frac{1}{F_{11}^L} + \frac{1}{F_{22}^T} - \frac{1}{F_{12}^L} - \frac{1}{F_{12}^T} \right), \quad C_3 = \frac{1}{2} \left( \frac{1}{F_{11}^L} + \frac{1}{F_{22}^L} - \frac{1}{F_{22}^T} - \frac{1}{F_{12}^T} \right),
$$

(11)

$$
C_4 = \frac{1}{F_{11}^L}, \quad C_5 = \frac{1}{F_{12}^L}, \quad C_6 = \frac{1}{F_{12}^T}, \quad C_7 = \frac{1}{F_{11}^T}, \quad C_8 = \frac{1}{F_{12}^T}, \quad C_9 = \frac{1}{F_{11}^T},
$$

where $F_{11}^L$, $F_{11}^T$ and $F_{12}^L$ denote the tensile, compressive, and shear strength in the local coordinates, respectively. At each time step in the incremental homogenization analysis described in Section 2, $F$ at every reference point, practically at every finite element in a unit cell, is calculated based on Eq. (10), and the element in which $F$ reaches 1 is considered to be damaged.

3.2 Failure expression

Fiber bundles are considered to have 6 kinds of damage modes as listed in Table 1, which are classified into 4 types of damages (Fig. 4) (Zako et al., 2003; Fujita and Kurashiki, 2012). Thus, if $F$ reaches 1 at a finite element in a fiber bundle, we have to determine which damage mode has occurred at the element. For this, we calculate the stress to strength ratios listed in Table 1 at the element. Then, the damage mode corresponding to the highest ratio is determined to occur, and the corresponding elastic moduli as shown at the lowest row in Table 1 are reduced to 1/100 (Zako et al., 2003, Fujita and Kurashiki, 2012), where $E_\text{LT}$ and $G_\text{LT}$ respectively indicate the Young's modulus and the shear stiffness of fiber bundles in the local coordinates. Finally, we recalculate the elastic stiffness $c_{ijkl}$. It is noted that each finite element in fiber bundles is allowed to experience multiple damage modes through the analysis. For a matrix, on the other hand, its Young's modulus is reduced to 1/100 regardless of damage modes, and $c_{ijkl}$ is recalculated. For more detail on the computational procedure, see Zako et al. (2003) and Fujita and Kurashiki (2012).

4. Analysis

Using the present method, a multiscale analysis of the damage behavior of plain-woven GFRP laminates with laminate misalignment under on-axis uniaxial tension was performed.

4.1 Unit cell and laminate misalignment

A unit cell $Y$ for the laminates was defined as shown in Fig. 5 by referring Matsuda et al. (2011), and was divided into eight-node isoparametric elements (8192 elements, 9603 nodes). It has 4 fiber bundles, two of which are the warps and the others are the wefts. For $Y$, the volume fraction of fiber bundles in the laminates and that of fibers in the fiber bundles were respectively set to be 44% and 57% (Matsuda et al., 2011; Uetsuji et al., 2004).

In the present analysis, the following expression for laminate misalignment was considered (Fig. 6):

$$
[a, b] = [nl/8, nl/8] \quad (n = 0, 1, 2, ..., 7),
$$

(12)

where $l$ indicates the side length of $Y$. The above expression offers 64 cases of laminate misalignment including the non-misaligned case, but some cases are geometrically equivalent one another (e.g., $[0, 0]=[l/2, l/2]$, $[l/8, 0]=[3l/8, l/2]=[5l/8, l/2]=[7l/8, 0]$, $[l/8, l/2]=[l/2, 7l/8]=[3l/8, 3l/8]=[5l/8, 5l/8]=[7l/8]$, and so on). Consequently, 13 cases of laminate misalignment including $[0, 0]$, which are indicated by the red points in Fig. 6, are independent and sufficient to be examined. Thus, the 13 cases of misalignment were considered in this study.

<table>
<thead>
<tr>
<th>Damage modes</th>
<th>L</th>
<th>T</th>
<th>LT</th>
<th>Z</th>
<th>ZL</th>
<th>TZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress to strength ratios</td>
<td>$\sigma_{\text{T}}^2/F_{11}^L$</td>
<td>$\sigma_{\text{T}}^2/F_{11}^T$</td>
<td>$\sigma_{\text{T}}^2/F_{12}^L$</td>
<td>$\sigma_{\text{T}}^2/F_{12}^T$</td>
<td>$\sigma_{\text{T}}^2/F_{11}^T$</td>
<td>$\sigma_{\text{T}}^2/F_{12}^T$</td>
</tr>
<tr>
<td>Stiffness to be reduced</td>
<td>$E_\text{L}$</td>
<td>$E_\text{T}$, $G_{1\text{T}}$</td>
<td>$E_\text{T}$, $G_{1\text{T}}$</td>
<td>$E_\text{Z}$, $G_{\text{ZL}}$</td>
<td>$E_\text{Z}$, $G_{\text{ZL}}$</td>
<td>$E_\text{Z}$, $E_\text{Z}$, $G_{\text{ZL}}$, $G_{\text{ZL}}$</td>
</tr>
</tbody>
</table>

Fig. 4 4 types of damages in fiber bundles: (a) mode L, (b) mode T and LT, (c) mode Z and ZL, (d) mode TZ.
Fiber bundles (Warps)  
Fiber bundles (Wefts)

Fig. 5 Unit cell $Y$ of plain-woven GFRP laminates and finite element mesh; (a) full view, (b) fiber bundles (wefts and warps) in $Y$.

4.2 Material properties and loading condition

The fiber bundles were regarded as E-glass/vinylester unidirectional composites and as transversely isotropic elastic materials. On the other hand, the matrix was vinylester, which was regarded as an isotropic elastic material. The elastic constants and strength of the fiber bundles and matrix were set as listed in Table 2 by referring Uetsuji et al. (2004). The plain-woven laminates were subjected to macroscopically uniaxial tension in the $y_i$-direction (weft direction) at a constant strain rate of $10^{-5}$ s$^{-1}$ at room temperature until final fracture.

4.3 Results of analysis: macroscopic stress-strain relations

Figure 7 shows the macroscopic stress-strain relations in the loading direction (weft direction) for the plain woven GFRP laminates with the 13 cases of laminate misalignment. From the figure, two peaks and subsequent sudden drops in stress are observed in all the stress-strain relations. The first one at the area (A) is attributable to the transverse cracks in the warps (fiber bundles perpendicular to the loading direction), while the second one at the area (B) corresponds to the final fracture of the laminates due to the failure of the wefts (fiber bundles in the loading direction). Such mechanisms will be mentioned from the microscopic perspective in the next subsection. At the first peak, the maximum stress, 147 MPa at $[0, 0]$, is about 12% higher than the minimum one, 132 MPa at $[0, 0]$. At the second peak, the maximum stress, 163 MPa at $[0, 0]$, is about 14% higher than the minimum one, 142 MPa at $[0, 0]$. These results reveal that the macroscopic strength of the plain-woven GFRP laminates noticeably depends on the laminate misalignment.

4.4 Results of analysis: microscopic damage development

Figures 8(a) and (b) respectively show the microscopic damages in the unit cells for $[0, 0]$ and $[0, 0]$ at the first drop of the macroscopic stress, which provided the maximum and minimum peak stresses at the area (A) in Fig. 7. In the figures, the damaged elements are colored in accordance with their damage modes. We can see almost no difference between the full views of the unit cells shown in the left figures of Figs. 8(a) and (b), because damages have not occurred significantly in the matrix yet. However, the fiber bundles in the unit cells shown in the right figures of Figs.

Table 2 Material constants of fiber bundles and vinylester (Uetsuji et al., 2004).

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Young's module [GPa]</th>
<th>Shear stiffness [GPa]</th>
<th>Poisson's ratio</th>
<th>Tensile strength [MPa]</th>
<th>Compressive strength [MPa]</th>
<th>Shear strength [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber bundles (E-glass/vinylester)</td>
<td>$E_t$ 48.20</td>
<td>$G_{tx}$ 4.872</td>
<td>$v_{yx}$ 0.254</td>
<td>$F_{t}^{u}$ 2024.0</td>
<td>$F_{t}^{l}$ 2982.0</td>
<td>$F_{t}^{s}$ 121.3</td>
</tr>
<tr>
<td></td>
<td>$E_r$ 12.22</td>
<td>$G_{rx}$ 4.775</td>
<td>$v_{rx}$ 0.067</td>
<td>$F_{r}^{u}$ 242.3</td>
<td>$F_{r}^{l}$ 242.3</td>
<td>$F_{r}^{s}$ 121.3</td>
</tr>
<tr>
<td></td>
<td>$E_z$ 12.22</td>
<td>$G_{xz}$ 4.775</td>
<td>$v_{zx}$ 0.233</td>
<td>$F_{z}^{u}$ 242.3</td>
<td>$F_{z}^{l}$ 121.3</td>
<td>$F_{z}^{s}$ 121.3</td>
</tr>
<tr>
<td>Vinylester</td>
<td>$E$ 3.334</td>
<td>$G$ 1.282</td>
<td>$v$ 0.300</td>
<td>$F_{v}^{u}$ 88.26</td>
<td>$F_{v}^{l}$ 117.7</td>
<td>$F_{v}^{s}$ 88.26</td>
</tr>
</tbody>
</table>
8(a) and (b) clearly show the difference between the two cases. Quite a lot of damages in the mode T (transverse cracks) through the warps are observed in the both cases, which brought about the first stress drop in Fig. 7, but the distributions of the damages are considerably different from each other. When $[0, l/2]$, the damages distribute in the middle and at the edge of the warps, whereas with $[l/8, 0]$, all the damages are located at the edge of the warps.

Figures 9(a) and (b) respectively depict the microscopic damages for $[l/4, l/4]$ and $[0, 0]$ at the second stress drop (final fracture), which provided the maximum and minimum peak stresses at the area (B) in Fig. 7. In this case, the difference between the two cases becomes clearer. The second drop, i.e. the final fracture, takes place due to the mode L damages in the wefts. This mechanism is common in the both cases. As seen from Figs. 9(a) and (b), however, the damage process leading to the fracture is markedly different from each other. When $[l/4, l/4]$, the distribution of damages and their modes are quite complex, and the damages in the mode L distribute between the warps as indicated by the circles in the right figure of Fig. 9(a). By contrast, when $[0, 0]$, the mode L damages occur at the relatively narrow areas near the warps as indicated by the circles in the right figure of Fig. 9(b).

Such difference in the damage development depending on the laminate misalignment resulted in the difference in the macroscopic behavior and strength of the plain-woven GFRP laminates as already shown in Fig. 7, although we cannot show the microscopic damage distributions in all the cases examined due to space limitation.
5. Conclusions

In this study, a multiscale damage analysis of plain-woven GFRP laminates with laminate misalignment was performed using the homogenization theory for misaligned internal structures in conjunction with the Hoffman's failure criterion. The present method was able to efficiently analyze the damage development in plain-woven laminates with arbitrary laminate misalignment using only one unit cell. In the analysis, 13 cases of laminate misalignment including no misalignment were considered. It was shown that the laminate misalignment noticeably affected the macroscopic strength of the laminates; the maximum strength was about 14% higher than the minimum strength. This was attributable to the difference in microscopic damage development in the laminates depending on the laminate misalignment.

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