Control of the orientational characteristics of disk-like hematite particles by a simple shear flow

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Abstract
We discuss the behavior of oblate spheroidal hematite particles in a simple shear flow. Magneto-rheological properties are strongly dependent on the magnetic field direction. For instance, a rod-like hematite particle suspension exhibits negative viscosity characteristics in a certain direction of an applied magnetic field. From this background, we here consider the case of an external magnetic field applied in the direction of the angular velocity vector of a simple shear flow. If the magnetic field is much more dominant than the thermal motion, the particle can almost freely rotate about the direction of the angular velocity vector. This characteristic rotational motion is suppressed or controlled by the shear flow more significantly with increasing shear rate.

Key words: Magnetic colloidal dispersion, Simple shear flow, Oblate spheroidal particle, Orientational distribution function, Magnetorheology

1. Introduction

Magnetic suspensions that are composed of magnetic particles dispersed in a base liquid exhibit characteristic features in a flow field and an external magnetic field such as aggregation phenomena, characteristic orientational properties, and self-organization structures (Bullough, 1996; Rosensweig, 1985; Satoh, 2008). These characteristics may stimulate studies in a variety of research fields such as fluid engineering, colloid and interface science, and magnetic material engineering. In the field of fluid engineering, the main targets for application may be mechanical dampers and actuators, which make use of the magneto-rheological effect that is dependent on the magnetic field strength, etc. (Bullough, 1996). In the fields of colloid and interface science and magnetic materials, applications are high-density recording materials (Harrell et al., 2005; Verdes et al., 2006), optical units (Iwayama, 2003; Reese, 2000; Mine, 2005; Furumi and Sakka, 2006a; Furumi and Sakka, 2006b), and surface modifying technology (Satoh and Sakuda, 2010). In other fields, there is a magnetically targeted drug delivery in the bioengineering field (Häfeli et al., 1997; Kuznetsov et al., 1999; Weingart et al., 2013), and a recovering technology for specific substances such as hazardous heavy metal molecules (environmental waste and pollutants) or valuable noble metal molecules from water (sea, lake, etc.) in the environmental resources engineering field (Girginova et al., 2010; Lan et al., 2013; Bruce and Sen, 2005; Satoh, 2001). We here focus on phenomena of magnetic particle suspensions from the viewpoint of fluid engineering.

The magneto-rheological effect is strongly dependent on a variety of factors such as particle shape, magnetic particle-particle and particle-field interactions (Satoh, 2001; Satoh, 2003a; Paulus et al., 2001; de Vicente et al., 2009; de Vicente et al., 2010; Piao et al., 2003; Choi et al., 2000; Bell et al., 2007). A suspension composed of hematite particles exhibit characteristic magneto-rheological properties completely different from those of ordinary ferromagnetic particles (Ozaki et al., 1988; Ozaki and Matijević, 1985; Varanda et al., 2001). That is, this suspension exhibits negative magneto-rheological effects in a certain situation of an applied magnetic field, which has already been verified by an experimental study using a cone-plate-type rheometer (Satoh and Ozaki, 2006; Satoh and Sakuda, 2007; Satoh, 2013; Sakuda et al., 2012). From this background, we understand that it is important to conduct a systematic series of studies in order to investigate the behaviors of non-spherical magnetic particles, which may be lead to new
finding of important characteristics of these magnetic suspensions.

As in the previous study (Satoh, 2012; Satoh and Yokoyama, 2014), we consider a suspension composed of disk-like hematite particles in order to investigate the behavior of these particles in a simple shear flow. A uniform external magnetic field is here assumed to be applied in the direction of the angular velocity vector of a simple shear flow. The situations of the different field directions had already been focused on in the previous studies (Satoh, 2012; Satoh and Yokoyama, 2014), so that we here mainly focus on the control of the orientational characteristics by the simple shear flow, which may be a different aspect of a series of studies regarding a disk-like hematite particle suspension. The theory based on the orientational distribution function is basically the same as in the previous studies (Satoh, 2012; Satoh and Yokoyama, 2014), and therefore, in this brief report, we concentrate on the main characteristic results with summarized descriptions of the particle model and the basic equation of the rotational distribution function.

2. Particle model

A disk-like hematite particle (Ozaki et al., 1988) is idealized as an oblate spheroid with a point dipole moment $m$ at the particle center in a direction normal to the particle axis direction $e$. The direction of the magnetic moment is denoted by the unit vector $n = m/|m| = m/m$. The major and minor axis length are denoted by $a$ and $b$, respectively, and the aspect ratio $r_p$ is expressed as $r_p = a/b$. A simple shear flow is described by the rate-of-strain tensor $E$ and the angular velocity vector $\Omega$. The particle will perform the rotational Brownian motion with the translational velocity that is equal to the local velocity of the simple shear flow. As shown in Fig. 1, the orthogonal coordinate system is set in such a way that the origin necessarily coincides with the particle center, and the particle direction $e$ is specified by the zenithal angle $\theta$ and the azimuthal angle $\phi$. Moreover, the magnetic moment direction $n$ is specified by the angle $\psi$, as shown in Fig. 1, which is defined in the coordinate system expressed by the unit vectors $(\delta_\theta, \delta_\phi)$. Employing the symbol $\dot{\gamma}$ for the shear rate, the flow field $U$ of the shear flow is expressed as $U = (\dot{\gamma} y, 0, 0)$ and the angular velocity vector $\Omega$ is described as $\Omega = (0, 0, -\dot{\gamma}/2)$. Since an external magnetic field is assumed to be applied in the $z$-axis direction, the unit vector $h$ denoting the direction of the magnetic field $H (=Hh)$ is expressed as $h = (0, 0, 1)$.

![Fig. 1 Magnetic oblate spheroidal particle in a simple shear flow. A magnetic field is applied in the $z$-direction and a simple shear flow is in the $x$-direction.](image)

3. Basic equation of the orientational distribution function of the magnetic particle

If the motion of the fine particle in a shear flow is dynamically solved with neglecting the rotational Brownian motion, the basic equation will lead to the famous Jeffery equation (Jeffery, 1922; Jeffery, 1923). In the present phenomenon, the rotational Brownian motion is regarded as an important role for determining the orientational behavior of the oblate spheroidal particle in a simple shear flow under a uniform applied magnetic field, and therefore it is quite reasonable to employ the stochastic approach based on the orientational distribution function. This stochastic method, adopted in the present study, is briefly summarized below.

The following three kinds of torques act on the particle in a dilute situation: the torque due to the interaction with
the external magnetic field, the random torque inducing the rotational Brownian motion and the viscous torque due to the ambient flow field. In a colloidal dispersion, the inertial term is usually negligible, so that the equations of the torque balance give rise to the expressions for the time change in the particle direction, \( \mathbf{\dot{e}} \), and also the time change in the magnetic moment direction, \( \mathbf{n} \). The orientational distribution function \( \Psi(t, \theta, \phi, \psi) \) is required to satisfy the following continuum equation:

\[
\frac{\partial \Psi}{\partial t} = -\frac{\partial}{\partial \mathbf{e}} \left( \mathbf{\dot{e}} \Psi \right) - \frac{\partial}{\partial \mathbf{n}} \left( \mathbf{n} \Psi \right) = -\Psi \frac{\partial}{\partial \mathbf{e}} \mathbf{\dot{e}} - \mathbf{\dot{e}} \cdot \frac{\partial \Psi}{\partial \mathbf{e}} - \Psi \frac{\partial}{\partial \mathbf{n}} \mathbf{n} - \mathbf{n} \cdot \frac{\partial \Psi}{\partial \mathbf{n}}
\]

(1)

The ordinary continuum equation does not include the second term on the first right-hand side in Eq. (1), but in the present case this term is necessary for considering the spin rotational Brownian motion about the particle axis. Substitution of \( \mathbf{\dot{e}} \) and \( \mathbf{n} \) into Eq. (1) yields the final basic equation of the orientational distribution function for a time-independent situation as

\[
\Psi \left[ 3 \frac{Y_H}{Y_C} (\mathbf{E}^* : \mathbf{e}) + \frac{\xi}{Pe} \left( \frac{\partial}{\partial \mathbf{e}} (\mathbf{e} \cdot \mathbf{h}) \cdot \mathbf{n} + (\mathbf{e} \cdot \mathbf{h}) \frac{\partial}{\partial \mathbf{e}} \mathbf{n} \right) \right] - \frac{\xi}{Pe} \frac{Y_C}{X_C} \left( \frac{\partial}{\partial \mathbf{n}} |\mathbf{e} \times (\mathbf{n} \times \mathbf{h})| \cdot \mathbf{\delta}_\beta \right)
\]

\[+\left[ -\left( \mathbf{Q}_\perp \times \mathbf{e} \right) - \frac{Y_H}{Y_C} (\mathbf{E}^* : \mathbf{e}) + \frac{\xi}{Pe} (\mathbf{e} \cdot \mathbf{n}) \right] \frac{\partial \Psi}{\partial \mathbf{e}} + \frac{1}{Pe} \left( \frac{\partial}{\partial \mathbf{e}} \mathbf{e} \cdot \frac{\partial}{\partial \mathbf{e}} \mathbf{e} \right) \Psi + \frac{1}{Pe} \frac{Y_C}{X_C} \left( \frac{\partial}{\partial \mathbf{n}} \mathbf{n} \cdot \frac{\partial}{\partial \mathbf{n}} \mathbf{n} \right) \Psi \]

\[= 0
\]

(2)

in which superscripts \( \perp \) and \( || \) imply quantities (vectors) normal and parallel to the particle axis, and the notation \( \mathbf{\delta}_\beta = \mathbf{e} \times \mathbf{n} \) has been used for simplification. This expression is obtained using the dimensionless quantities that are non-dimensionalized by the shear rate \( \gamma \) and the thermal energy \( kT \), where \( k \) is Boltzmann’s constant and \( T \) is the system temperature: for instance, the angular velocity and the rate-of-strain tensor are non-dimensionalized by the shear rate, and the magnetic particle-field interaction energy is non-dimensionalized by \( kT \). In Eq. (2), the quantities with superscript * are dimensionless, and \( \xi \) and \( Pe \) are the non-dimensional parameters representing the strengths of the magnetic particle-field interaction and the viscous shear force relative to random force (or thermal energy), respectively. \( Pe \) is called the Peclet number. \( X_C^* \), \( Y_C^* \) and \( Y^* \) are resistance functions specifying the resistance coefficients (Satoh, 2003b). The above-mentioned non-dimensional parameters are expressed as

\[
\xi = \mu_0 m H / kT , \quad Pe = \eta_1 \gamma Y_C / kT
\]

(3)

in which \( \mu_0 \) is the permeability of free space and \( \eta_1 \) is the viscosity of base liquid.

Specification of the particle aspect ratio and the non-dimensional parameters basically enables one to solve the basic equation in Eq. (2). As in the previous studies (Satoh, 2013), we employ the iteration method to solve the finite difference expression for the basic equation.

4. Magneto-rheological effect

We consider the viscosity due to the magnetic particle-field interaction for a dilute suspension, where the contribution of magnetic interactions between particles to the viscosity is negligible. The shear stress \( \tau^{(m)} \) due to the torque \( \mathbf{T}^{(m)} \), which arises from the interaction between the magnetic moment and the applied magnetic field, is expressed as

\[
\tau^{(m)} = -\frac{n}{2} \left( \mathbf{e} \cdot \mathbf{T}^{(m)} \right)
\]

(4)

in which \( n \) is the number density of particles, \( \mathbf{e} \) is Eddington’s epsilon and \( \langle \mathbf{e} \cdot \mathbf{T}^{(m)} \rangle \) is the average using the solution of the orientational distribution function.

The present theoretical analysis of the shear stress in Eq. (4) gives rise to such an important result regarding the viscosity that, in the case of an external magnetic field applied in the direction of the angular velocity vector of the simple shear flow, the viscosity due to the magnetic particle-field interaction gives rise to zero. This is quite in contrast to the results for the previous cases of the different directions of an external magnetic field, where large viscosity significantly arises in strong magnetic field situations (Satoh, 2012; Satoh and Yokoyama, 2014).
5. Results and discussion
5.1 Dependence of the orientational distribution function on the magnetic field strength

Since the orientational distribution function $\Psi$ is a function of the particle direction ($\theta, \phi$) and the magnetic moment direction $\psi$, it is quite difficult to visualize results of the orientational distribution in a three-dimensional way. Hence, we show here the maximum value of $\Psi(\theta, \phi, \psi)$ for the change in $\psi$ for a given set of ($\theta, \phi$). Unless specifically notified, the following results were obtained for the particle aspect ratio $r_p=5$.

Figure 2 shows results of the orientational distribution function for a weak shear flow $Pe=1$: Figs. 2(a) and 2(b) are for the magnetic field strength $\xi=5$ and 10, respectively. In the case of a weak magnetic field $\xi=1$, not shown as a figure, the orientational distribution function does not exhibit a specific favored direction, but a uniform orientational characteristic. As the magnetic field strength is increased from $\xi=1$ to 5, shown in Fig. 2(a), a linear peak appears along the $\phi$-direction at $\theta=90^\circ$, and this characteristic becomes much clearer in the case of the further strong magnetic field $\xi=10$, shown in Fig. 2(b). It is seen that a slightly high peak is observed to appear at $\phi \approx 135^\circ$, which corresponds to the tendency that the oblate plane weakly inclines along the direction $\phi \approx 45^\circ$ ($=135^\circ-90^\circ$). This linear peak-type distribution along the $\phi$-direction implies that the particle almost freely rotate about the $z$-axis with the magnetic moment maintaining inclining in the magnetic filed direction in the situation of a weak shear flow. This orientational characteristic is in significantly contrast to that for the previous other field directions where the distribution shows only a single peak-type distribution with a high peak (Satoh, 2012; Satoh and Yokoyama, 2014).

![Fig. 2 Dependence of the orientational distribution function on the magnetic field strength for the Peclet number $Pe=1$: (a) $\xi=5$ and (b) $\xi=10$. For the case of a weak shear flow, the orientational distribution exhibits a linear peak-type characteristics along the $\phi$-direction.](image)

![Fig. 3 Dependence of the orientational distribution function on the Peclet number (shear rate) for the magnetic field strength $\xi=10$: (a) $Pe=5$ and (b) $Pe=10$. The shear flow functions as a mechanics for changing the regime of the distribution from a linear peak to a single peak distribution.](image)
5.2 Dependence of the orientational distribution function on the shear rate

We now discuss the influence of the shear rate (Peclet number) on the orientational distribution function. Figure 3 shows results of the orientational distribution function for a strong magnetic field \( \xi = 10 \): Figs. 3(a) and 3(b) are for \( Pe=5 \) and 10, respectively. As the shear rate (Peclet number) is increased from the situation in Fig. 2(b), the distribution changes from a linear peak into one peak distribution. The peak becomes higher and the peak point approaches \( \phi = 90^\circ \), as shown in Fig. 3(a) or 3(b), although the peak arises at \( \phi = 114^\circ \) for the case of \( Pe=10 \). The peak at \( \phi = 90^\circ \) implies that the oblate plane inclines along the shear flow direction (x-direction) in a strong shear flow situation such as \( Pe=10 \). In contrast to the previous weak shear flow case, it is noted that the particle cannot freely rotate due to the influence of the strong shear flow. Hence, this clearly implies that the rotational motion of the disk-like particles about the magnetic moment direction in a strong magnetic field can be controlled by an increase in the shear rate of the flow field.

5.3 Characteristics of the orientational distribution function in each angle direction

In order to consider the orientational characteristics in more detail, we here consider the profile of the orientational distribution \( \Psi(\theta, \phi, \psi) \) by addressing the change in \( \Psi(\theta, \phi, \psi) \) along each direction through the peak point with the other two variables being fixed. Figure 4 shows results of these profiles for a strong magnetic field \( \xi = 10 \): Figs. 4(a) and 4(b) are for \( Pe=1 \) and 10, respectively. It is noted that a peak of the \( \psi \)-changing profile at \( \psi = 180^\circ \) implies that the magnetic moment inclines in the magnetic field direction (z-direction).

In the case of a weak shear flow, shown in Fig. 4(a), the Brownian motion is much more dominant than the shear flow. Since the magnetic field is more dominant, the curves regarding the change in \( \theta \) and \( \psi \) show a high peak within a narrow range of each angle. This implies that the magnetic moment is much more strongly restricted to the field direction and also the oblate plane significantly inclines along the magnetic field direction. In contrast, the \( \phi \)-changing curve maintains the sinusoidal characteristic with shallow depth. This implies that the particle almost freely rotate about the magnetic moment direction in a weak shear flow situation. We next consider the results for a strong shear flow \( Pe=10 \), shown in Fig. 4(b). It is seen that the \( \phi \)-changing curve is dramatically changed from a sinusoidal curve in Fig. 4(a) into a curve with deep valley similar to that for the \( \theta \)-changing and \( \psi \)-changing curves. This characteristic of the \( \phi \)-changing curve implies that the oblate plane tends to incline along the direction of \( \phi = 24^\circ \) (114°–90°) with maintaining the tendency of the alignment of the magnetic moment to the field direction. Moreover, it is seen that the shear flow enhances the tendency of the directional characteristics of both the particle and the magnetic moment. That is, as the Peclet number is increased, the particle direction remains in the xy-plane, and cannot freely rotate about the magnetic field direction with the magnetic moment aligning in the magnetic field direction.

![Figure 4: Dependence of the orientational distribution function on the angle \( \theta, \phi, \psi \) for \( \xi = 10 \); (a) \( Pe=1 \) and (b) \( Pe=10 \). As the Peclet number is increased, the particle direction cannot freely rotate about the magnetic field direction with the magnetic moment aligning in the magnetic field direction.](image-url)
6. Conclusion

We address a dilute suspension composed of oblate spheroidal hematite particles in a simple shear flow in order to investigate the orientational characteristics and magneto-rheological properties by means of an analytical approach based on the orientational distribution function. In the present study, an external magnetic field is assumed to be applied in the direction of the angular velocity vector of a simple shear flow. Characteristic features, which are significantly different from those for the previous cases of the different magnetic field directions, are summarized as follows. If the magnetic field is much more dominant than the thermal motion, the particle can almost freely rotate about the direction of the angular velocity vector with maintaining the magnetic moment inclining in the magnetic field direction. This orientational characteristic is in significantly contrast to that for the previous other field directions where the distribution shows only a single peak-type distribution with a high peak. The above-mentioned characteristic rotational motion is suppressed by the shear flow more significantly with increasing shear rate. Hence, this clearly implies that the rotational motion of the disk-like particles about the magnetic moment direction can be controlled by an increase in the shear rate of the flow field.

References


