Quasi-steady state to non-steady state transition criterion of laminar stagnating lean premixed flame with fuel concentration oscillation

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Abstract
The criterion for transition from a quasi-steady state to a non-steady state of laminar stagnating lean premixed flame in the presence of oscillation in fuel concentration is discussed experimentally. Two types of mixtures with different Lewis numbers are examined: lean methane/air and lean propane/air mixtures. Sinusoidal oscillation in fuel concentration is effected by an oscillator with two cylinder-piston units that supply leaner and richer mixtures alternately. The flame response under fuel concentration oscillation is measured as a function of frequency. The frequency of fuel concentration oscillation varies within 2–20 Hz, and the burner exit velocity varies within 0.6–1.1 m/s. For both mixtures, oscillator characteristics cause the amplitude of oscillation of the flame position to increase with frequency. The increase in amplitude becomes less sharp as frequency increases further. In the present study, two additional Strouhal numbers, one based on heat transfer and another based on mass transfer, are newly introduced in addition to the ordinary Strouhal number which is based on momentum transfer. When any of these Strouhal numbers exceeds unity, the increase in the amplitude starts to fall off. This indicates that the three Strouhal numbers play equally important role on phenomena with momentum, heat and mass transfer, such as combustion.

Key words: Flame dynamics, Fuel concentration oscillation, Laminar premixed flame, Stagnating flame, Strouhal number

1. Introduction
Lean premixed combustion is important in developing combustors with higher efficiency and lower NOx emission. However, lean premixed combustion involves technical issues that have not been solved yet, such as combustion instabilities. Lieuwen and Zinn (1998) suggested that equivalence ratio oscillation plays a key role in driving instability in low-NOx-emission lean premixed combustors. Lieuwen et al. (2001) showed that instabilities of lean premixed combustion result from a feedback process between heat release, acoustic pressure and equivalence ratio oscillations. Cho and Lieuwen (2005) analyzed the flame response to equivalence ratio perturbations and showed that the heat release response is controlled by the superposition of three disturbances: heat of reaction, flame speed and flame area. These disturbances are directly and indirectly generated by equivalence ratio oscillation. Prior research has shown that the equivalence ratio oscillation, that is, the fuel concentration oscillation, plays a key role in the instability of low-emission lean premixed combustion.

The effect on flame motion of fuel concentration oscillation has been numerically investigated. Lauvergne and Egolfopoulos (2000) showed that the flame response is quasi-steady for low-frequency oscillation in mixture composition, whereas the response is substantially attenuated by diffusion at high frequencies. Marzouk et al. (2000) suggested that flame subject to variation in stoichiometric-to-lean equivalence ratio is intensified by a back support effect, which is the result of heat transport from the burnt gas region. Rosdzimin et al. (2012) showed that the motion in flame location create a closed cycle around the location of the flame under the corresponding equivalence ratio in the steady-
state condition, by the back support effect. As for the flammability limit, Sankaran and Im (2002) showed that the flammability limit is expanded to a lower equivalence ratio as the oscillation frequency is increased.

In contrast with numerical analysis, little empirical research has been reported, because it is difficult to make experimental apparatuses that can oscillate the fuel concentration without inducing velocity oscillation. Suenaga et al. (2005, 2010) revealed experimentally that the burning velocity and flame luminosity near the lean flammability limit could become lower under concentration oscillation than those under the steady-state condition.

The present authors, working with others, have investigated the flame motion subject to fuel concentration oscillation with well suppressed velocity oscillation for a methane/air lean mixture, both experimentally (Rosdzimin et al., 2015) and numerically (Miyamae et al., 2014). These studies demonstrated that the flame location moved with concentration oscillation, but the trajectory of the flame location does not follow that under the steady state, mainly due to the back support effect (Tomita et al., 2015). In these papers, the Strouhal number, $St$, was introduced and it was elucidated that the amplitude of the flame position exhibits quasi-steadiness for $St < 1$ and unsteadiness for $St > 1$. Similar role of the Strouhal number has discussed on the flame stretch fluctuation due to flow field oscillation induced by stagnation wall oscillation (Hirasawa et al. (2000)). Hemchandra (2012) and Rosdzimin et al. (2013) discussed the effect of fuel concentration oscillation on the Bunsen-type flame, using the Strouhal number. These studies showed that the Strouhal number is useful for discussing the flame response to fuel concentration oscillation.

In the present study, the flame response is further discussed in the context of fuel concentration oscillation of lean methane/air and propane/air flames. Two types of new Strouhal numbers, one based on heat and another based on mass transfers, are introduced in addition to the well-known Strouhal number which is based on momentum transfer. The criterion for transition from a quasi-steady state to a non-steady state is discussed, using those three Strouhal numbers.

2. Experimental apparatus and method
2.1 Flow system

Figure 1 shows the flow system of the present study. Methane/air mixture or propane/air mixture is used. The mixture with fuel concentration oscillation is made by mixing a primary flow having constant fuel concentration with a secondary flow having oscillating fuel concentration. In order to effect oscillation of the fuel concentration for the secondary flow, leaner and richer mixtures are supplied alternately to the primary flow by the oscillator described in detail in Sec. 2.3. The mixture is issued from the exit of the burner.

2.2 Burner

The burner is shown in Fig. 2 along with the coordinate system. The primary flow passes through the converging nozzle. The secondary flow with fuel concentration oscillation is supplied to the primary flow through 16 holes after leaner and richer mixtures are supplied alternately at just upstream of the 16 holes, as shown in Fig. 3. Details of the characteristics of the fuel concentration oscillation are given in Sec. 2.3. Two honeycombs installed at the upstream and downstream sides of the converging nozzle along with two screens installed at the burner exit reduce oscillation in the velocity of the mixture at the burner exit.

2.3 Oscillator

The oscillator shown in Fig. 4 theoretically produces sinusoidal oscillation in fuel concentration via two pistons that move in antiphase, which keeps the total flow rate constant as shown by Eq. (1).

$$Q_{\text{total}} = Q_{\text{primary}} + Q_{\text{richer}} + 2\pi f A_{\text{piston}} L_{\text{stroke}} \sin(2\pi ft) + Q_{\text{leaner}} + 2\pi f A_{\text{piston}} L_{\text{stroke}} \sin(2\pi ft - \pi)$$

(1)

Here, $Q_{\text{primary}}$ is the volume flow rate of the primary flow, $f$ is the frequency of the piston motion, $A_{\text{piston}}$ is the cross-sectional area of each piston and $L_{\text{stroke}}$ is the half-stroke length of piston movement. In the actual system, however, a small velocity oscillation is produced because the piston motion is not exactly sinusoidal, due to the crank mechanism. The fuel concentration oscillation in the secondary flow is produced by supplying richer and leaner mixtures alternately. The instantaneous equivalence ratio is calculated as the following.

$$\varphi = \frac{Q_{\text{primary}}}{Q_{\text{total}}} + \frac{Q_{\text{richer,air}} - Q_{\text{leaner,air}}}{Q_{\text{total,air}}} + 2\pi f \left[ \left( A_{\text{piston}} L_{\text{stroke}} \sin(2\pi ft) \right)_{\text{richer}} - \left( A_{\text{piston}} L_{\text{stroke}} \sin(2\pi ft - \pi) \right)_{\text{leaner}} \right] / Q_{\text{total,air}}$$

(2)
Here, $\varphi$ is the equivalence ratio and $\varphi_{\text{primary}}$ is the equivalence ratio of the primary flow. $\varphi_A$ is the initial difference between the equivalence ratios of the secondary flows ($\varphi_A=(\varphi_{\text{richer}}-\varphi_{\text{leaner}})/2$).

It is worth noting that the amplitude of the equivalence ratio increases with frequency, as shown in the third term of Eq. (2), in the present oscillator. The influence of velocity oscillation on flame oscillation is well suppressed compared with that of fuel concentration oscillation by setting two 100-mesh screens at the burner exit, as mentioned in Sec. 2.2.

2.4 Measurement of the flame position

The flame motion was captured by a high-speed video camera. The flame position is defined as the minimum height ($y$ position) of the blue flame zone along the center line, as determined from the video images by using Matlab2015.

2.5 Experimental conditions

In the present study, the oscillator frequency and the equivalence ratio amplitude are varied for methane/air and propane/air mixtures. The experimental conditions of the equivalence ratio of the primary and secondary flows are given in Table 1. In the case without fuel concentration oscillation (“no-oscil” in the table), mixtures with equivalence ratio of 0.70 are issued for both flows. In cases with fuel concentration oscillation (“oscil” in the table), the amplitudes of the fuel concentration can be varied as functions of the initial difference between the equivalence ratio of the leaner and richer mixtures and the frequency $f$, as shown with Eq. (2). The burner exit velocity $v_{\text{exit}}$ is set as shown in Table 2. The oscillator frequency is varied in the range 2–20 Hz.

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Figure 1. Flow system.  
Figure 2. Burner.  
Figure 3. Supply unit of secondary flow.  
Figure 4. Cross section of the oscillator.
3. Results and discussion

3.1 Flame motion with fuel concentration oscillation

Direct images of the flame at different phases with fuel concentration oscillation are shown in Fig. 5. In the figures, \( T \) is the time of one period and \( t/T \) is the time normalized by the period. The yellow line indicates the mean flame position. Figure 5 (a) shows the flame at the mean flame position. Figure 5 (b) shows the flame nearest the stagnation plate. These show that the flame moves, keeping the flame flat. Figure 6 shows the variations in flame position for methane/air premixed flame in one period with \( \phi_A = 0.00 \) and 0.30; \( \phi_m = 0.70 \); \( v_{exit} = 0.6 \text{ m/s} \); and \( f = 5 \text{ Hz} \). Here, the amplitude of the flame position in the oscillating case (\( \phi_A = 0.30 \)) shows clear sinusoidal motion. In contrast, without concentration oscillation (\( \phi_A = 0.00 \)) the variation is negligibly small, which indicates that the effect of velocity oscillation is well suppressed compared with concentration oscillation.

<table>
<thead>
<tr>
<th></th>
<th>Primary flow</th>
<th>Secondary flow</th>
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<tbody>
<tr>
<td></td>
<td>( \phi_{primary} )</td>
<td>( \phi_{riche} )</td>
</tr>
<tr>
<td>no-oscil</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>oscil (( \phi_A = 0.30 ))</td>
<td>0.70</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 1: Equivalence ratios of primary and secondary flows.

Table 2: Velocity of each fuel at burner exit.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Methane</th>
<th>Propane</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{exit} ) [m/s]</td>
<td>0.60–0.80</td>
<td>0.90–1.10</td>
</tr>
</tbody>
</table>

Figure 5. Flame shape and position variation (methane): \( v_{exit} = 0.6 \text{ m/s}, f = 5 \text{ Hz}, \phi_A = 0.30 \), flame rate of 125 fps, and shutter speed of 1/125 s.

Figure 6. Flame position variation, one period (methane): \( v_{exit} = 0.6 \text{ m/s}, f = 5 \text{ Hz} \).
3.2 Introduction of three Strouhal numbers

The overall momentum conservation equation is

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \mathbf{v} \cdot (\nu \nabla \mathbf{v}) \quad (3)$$

Here, $\mathbf{v}$ is a velocity vector, $p$ is pressure, and $\nu$ is kinematic viscosity. Equation (3) is made dimensionless by letting $t^* = 2\pi f_0 t$, $\mathbf{v}^* = \mathbf{v} L$, $p^* = \nu U/\nu_0$, and $\nu^* = \nu_0^2/2\pi f_0^2$ to obtain Equation (4).

$$\frac{\partial \mathbf{v}^*}{\partial t^*} + \frac{1}{2\pi f_0 L} (\mathbf{v}^* \cdot \nabla)\mathbf{v}^* = -\frac{1}{4\pi f_0^3 \nu_0^2} \nabla p^* + \frac{1}{2\pi f_0^3 \nu_0^2 U} \nu^* \cdot (\nu^* \nabla v^*) \quad (4)$$

Here, the Strouhal number is defined as

$$St = \frac{2\pi f_0 L}{U} \quad (5)$$

The Strouhal number is usually regarded as the ratio of the non-steady term (the first term of the left-hand side of Eq. (4)) and the convective term (the second term of the left-hand side of Eq. (4)) and taken to indicate the effect of non-steady motion compared with convective motion. The Strouhal number also appears in the viscous term (the second term of the right-hand side of Eq. (4)).

Similar to Eq. (4) non-dimensional conservation equations of chemical species and energy are obtained as below:

$$\frac{\partial Y_i}{\partial t^*} + \frac{1}{2\pi f_0 L} (\mathbf{v}^* \cdot \nabla) Y_i = -\frac{1}{2\pi f_0 \nu_0 U} \nabla \cdot (\nu^* Y_i) + \frac{\omega_{k,0}}{2\pi f_0 \rho i \rho i^*} \quad (6)$$

$$\frac{\partial T^*}{\partial t^*} + \frac{1}{2\pi f_0 L} (\mathbf{v}^* \cdot \nabla) T^* = \frac{1}{2\pi f_0 \nu_0 U} \nabla \cdot (\nu^* T^*) + \frac{1}{2\pi f_0 \rho^* c_p} \quad (7)$$

From Eqs. (5), (6), and (7), three Strouhal numbers are defined:

$$St_v = St = \frac{2\pi f_0 d}{U} \quad (8)$$

$$St_D = St \cdot Sc = \frac{2\pi f_0 d}{U} \frac{\nu}{D} \quad (9)$$

$$St_a = St \cdot Pr = \frac{2\pi f_0 d \nu}{U} \frac{\nu}{a} \quad (10)$$

Here, $St_v$ indicates the well-known Strouhal number for momentum transport. $St_D$ indicates the Strouhal number for mass transport, which is defined as the product of the Strouhal number for momentum transfer and the Schmidt number, as seen in Eq. (9). $St_a$ indicates the Strouhal number for heat transport, which is defined as the product of the Strouhal number for momentum transport and the Prandtl number, as seen in Eq. (10).

Table 3 indicates the non-dimensional magnitudes of molecular transport for methane/air mixture and propane/air mixture. Values of the Prandtl number and Schmidt number are nearly the same and less than unity for methane/air mixture. Since $Le = Sc/Pr$, this implies that $Le$ is almost unity. The value of the Prandtl number is less than unity, while that of Schmidt number is larger than unity for propane/air mixture, which results in $Le > 1$.

<table>
<thead>
<tr>
<th></th>
<th>Methane/air mixture</th>
<th>Propane/air mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Pr$</td>
<td>0.71</td>
<td>0.73</td>
</tr>
<tr>
<td>$Sc$</td>
<td>0.73</td>
<td>1.33</td>
</tr>
<tr>
<td>$Le$</td>
<td>1.02</td>
<td>1.82</td>
</tr>
</tbody>
</table>
As discussed in previous papers, in the case of a property constant single component flow, transitions from the quasi-steady state to the non-steady state takes place when the Strouhal number of momentum transfer is unity. In the present study, heat and mass transfer play an important role alongside momentum transfer in the property variable multi-component fluid flow. Thus, we examine three Strouhal numbers here. Equations (9) and (10) and the values of the Prandtl and Schmidt numbers in Table 3 indicate that, for the methane/air mixture, the frequencies at $St_D = 1$ and $St_\alpha = 1$ are nearly the same because the Lewis number is almost unity and they are larger than that the frequency at $St_\nu = 1$. For the propane/air mixture, the frequency at $St_\alpha = 1$ is larger than that at $St_\nu = 1$, while that at $St_D = 1$ is smaller than that at $St_\nu = 1$.

3.3 Flame response

Figure 7 shows the variation in the amplitude of the flame position amplitude for different frequencies of fuel concentration oscillation for methane/air and propane/air flames. In all cases, the amplitude of the flame position increases with the frequency of fuel concentration oscillation. This is due to the effect of the oscillator characteristics, as mentioned in Sec. 2.3. The variation in the amplitude of the flame position starts to fall off as frequency increases further. This is due to the effect of non-steady phenomena as discussed in previous papers (Tomita et al., 2015). In the present study, the frequencies at which the three Strouhal numbers are unity are also shown. In the case of methane/air mixture, $St_D$ and $St_\alpha$ are larger than $St_\nu$ because $Sc$ and $Pr$ are larger than unity. It is also interesting to note that $St_D$ and $St_\alpha$

![Figure 7. Amplitude of flame position against frequency of fuel concentration oscillation.](image-url)
are nearly the same since $Le$ is almost unity. In contrast, $St_D$ for propane/air mixture is smaller than $St_\nu$, while $St_\alpha$ is larger than $St_\nu$. Figure 7(a) shows that the transition from a quasi-steady state to a non-steady state takes place when $St_\nu = 1$. Figure 7(b) shows that the transition takes place when $St_D = 1$. These indicates that for both mixtures, the transition takes place when one of the Strouhal numbers reaches unity.

So far, the unsteadiness of single-component fluid flow has been discussed in terms of the Strouhal number for momentum transport, $St_\nu$. In property variable multi-component fluid flows, such as combustion, heat and mass transfers play important roles, as does momentum transfer. In steady-state phenomena, the importance of heat and mass transports has been widely discussed as a Lewis number effect (Law, 2006). The present study has shown that heat and mass transports play important roles for non-steady state phenomena as well as for the steady-state phenomena of property variable multi-component fluid flow, and that the three Strouhal numbers defined here are useful in discussing non-steady phenomena of multi-component fluid flows.

4. Concluding remarks

The flame response with fuel concentration oscillation was discussed experimentally. The velocity oscillation was well suppressed compared with fuel concentration oscillation. The flame position response with fuel concentration oscillation for methane/air and propane/air mixtures has been measured. Strouhal numbers for heat transfer, $St_\alpha$, and that of mass transfer, $St_D$, were introduced as well as Strouhal number for momentum transfer, $St_\nu$. The Strouhal number of heat transfer, $St_\alpha$, is defined as the product of the Strouhal number of momentum transfer and the Prandtl number; that the Strouhal number of mass transfer, $St_D$, is as the product of the Strouhal number of momentum transfer and the Schmidt number.

The flame response to oscillation in fuel concentration changes from quasi-steady to non-steady state when one of the three Strouhal numbers reaches unity. These results indicate that heat and mass transport play important roles for non-steady state phenomena in property variable multi-component fluid flow and that the three Strouhal numbers defined in the present study are useful in discussing non-steady phenomena of property variable multi-component fluid flow.

References


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